Directed Search

Lecture 2: Matching Patterns and Inequality

Lectures at Osaka University (2012)

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Main sources for this lecture:

- Shi, S., 2001, "Frictional Assignment I: Efficiency," JET 98, 232-260.
- Shi, S., 2005, "Frictional Assignment, Part II: Infinite Horizon and Inequality," RED 8, 106-137.
- Shi, S., 2002, "A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Biased Technology," RES 69, 467-491.

1. Motivation and Issues

- many markets have heterogeneity on both sides:
  - labor market:
    workers differ in skills, firms in capital and size
  - loan market:
    borrowers differ in project quality, lenders in funds
  - marriage market:
    men and women differ in income, beauty, etc.

- positive assortative matching (PAM):
  individuals are matched according to their ranking:
  - workers with higher skills match with better firms;
  - projects with higher quality match with better loans;
  - rich people marry rich people;
    handsome men marry beautiful women, etc.

- two questions about the matching pattern:
  - positive: is PAM an equilibrium?
  - normative: is PAM socially efficient?

- answer by Gary Becker (73, JPE) and Tinbergen (51):
  - PAM is an equilibrium and it is socially efficient when markets are frictionless
  - necessary and sufficient condition for this result: joint surplus of a match is complementary (supermodular) in the two sides' attributes

- think again:
  - most matching markets are frictional
  - not all observed matching patterns are PAM

Main questions: **when there are search frictions,**

- does the efficient allocation have PAM?
- how to decentralize the efficient allocation?
- how does matching affect inequality?

With undirected search, Shimer and Smith (00) find that complementarity is not enough for PAM to arise in eqm

- but their equilibrium is inefficient, generically; is this inefficiency responsible for non-PAM?
- still need to answer other questions above

Directed search:

- makes sense with homogeneous individuals
- makes more sense with heterogeneity:
  observable heterogeneity helps directing search
  - job ads typically specify worker qualifications;
    workers can observe firms' attributes
  - differentiated loan terms target different borrowers
  - people may date selectively

Roadmap:

- analyze a market with matching between

workers who differ in skill levels	+	machines that differ in qualities

- eqm and efficient allocation with no friction
- with search friction and directed search, characterize:
  efficient allocation
  decentralization, inequality
- extend to infinite horizon; dynamics
- calibrate to examine effects of skill-biased technology

2. Frictionless Economy and Assignment

One-period environment

- risk-neutral workers: exogenous supply;
  observable skill $s \in S \subset \mathbb{R}_+$: number $= n(s)$;
- machine quality $k \in \mathcal{K} \subset \mathbb{R}_+$: costs $C(k)$;
  endogenous supply determined by free entry
- one worker operates one machine;
- output of the pair (k, s): $F(k, s)$

Assumptions on F:

- complementarity (supermodularity): $F_{ks} > 0$

- both inputs are necessary: $F(0, s) = F(k, 0) = 0$

- every skill is employable with some machine quality: $F(k, s_L) - C(k) > 0$ for some $k \in \mathcal{K}$

- regularity condition: F concave; $C_{kk} > 0$, $(F_k C_{kk} - C_k F_{kk})F > (F_k - C_k)F_k^2$

Frictionless assignment

- no frictions: all pairs are matched instantaneously
- efficient assignment ϕ^p: $S \rightarrow \mathcal{K}$

$$\max_k \left[F(k,s) - C(k)\right], \quad \text{for each } s \in S,$$

$$\text{i.e.,} \quad F_k(\phi^p(s), s) = C_k(\phi^p(s))$$

- $\phi^p(s)$ exists and is unique for each s
- PAM:

$$\phi^{p\prime}(s) = \frac{F_{ks}}{C_{kk} - F_{kk}} > 0 \quad \text{iff } F_{ks} > 0$$

Decentralization:

- wage function:

$$W(k,s) = \begin{cases} F(k,s) - C(k), & \text{if } F(k,s) - C(k) \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

- a firm solves: $\max_{k \in \mathcal{K}} W(k,s) \Longrightarrow$ solution $k = \phi^p(s)$
- equilibrium wage: $w^p(s) = F(\phi^p(s), s) - C(\phi^p(s))$

- assignment pattern has NO first-order effect on wage:

$$w^{p\prime}(s) = \underbrace{F_s(\phi^p(s), s)}_{\text{direct effect}} + \underbrace{[F_k(\phi^p(s), s) - C_k(s)]\phi^{p\prime}(s)}_{\text{a better machine (but } = 0)}$$

3. Efficient Assignment with Frictions

Frictional economy

unit	qualities in subset $\phi(s)$		skill s workers
(k_1, s)	k_1 $\# : M(k_1, s)$	$\longrightarrow$	$\# : M(k_1, s)B(k_1, s)$ B : workers/machines
$\vdots$	$\vdots$		$\vdots$
(k_j, s)	k_j $\# : M(k_j, s)$	$\longrightarrow$	$\# : M(k_j, s)B(k_j, s)$

Matching probability in a unit (k, s):

$$\text{for a machine: } 1 - e^{-B(k,s)}; \quad \text{for a worker: } \frac{1-e^{-B(k,s)}}{B(k,s)}$$

Efficient allocation:

The planner chooses

- $\phi^o(s) \subseteq \mathcal{K}$: machine qualities assigned to $s \in S$
- $M^o(k,s)$: # of machines created for the unit (k,s)
- $B^o(k,s)$: worker/machine ratio in the unit (k,s)

$$\max_{(\phi,M,B)} \sum_{s\in S} \sum_{k\in\phi(s)} M(k,s) \underbrace{\left[\left(1-e^{-B(k,s)}\right) F(k,s) - C(k)\right]}_{\text{expected surplus of a match } (k,s)}$$

$$\text{s.t.} \qquad \underbrace{\sum_{k\in\phi(s)} M(k,s)B(k,s)}_{\#\text{ of skill } s \text{ workers assigned to } k} \qquad = n(s)$$

Component problem of the efficient allocation:

For each $s \in S$, the efficient allocation $(\phi^o(s), B^o(k,s))$ solves:

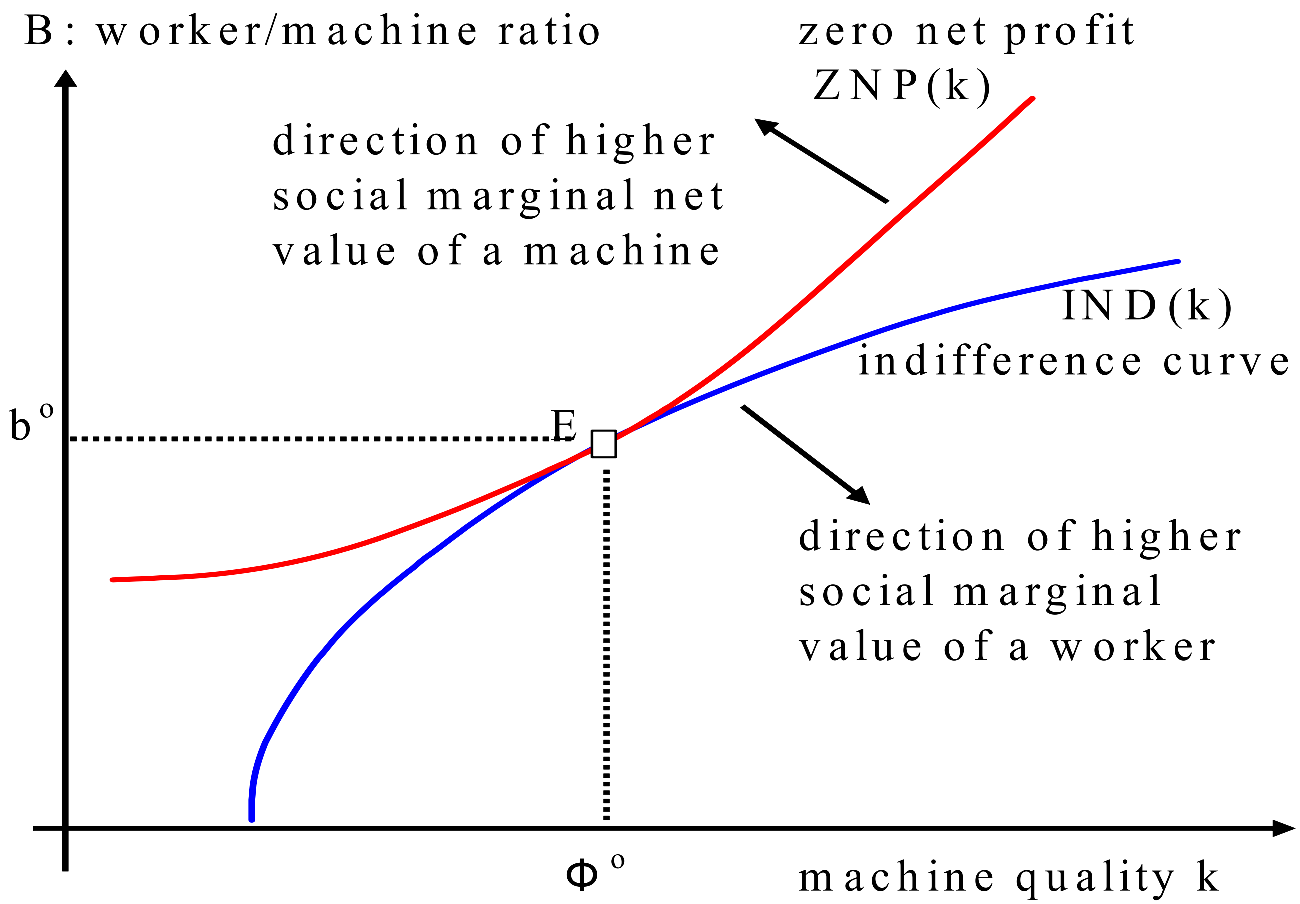
$$(P^o) \qquad \max_{(k,B)} \; e^{-B(k,s)} F(k,s) \quad \boxed{\text{social value of a worker } s}$$

$$\text{s.t.} \quad \underbrace{\left[1 - (1 + B(k,s)) e^{-B(k,s)}\right] F(k,s)}_{\text{social value of a machine in unit } (k,s)} \; = \; C(k)$$

- FOC of $M^o(k,s)$ leads to the constraint in (P^o)
- FOC of B^o coincides with that of (P^o)
- if $k_1 \in \phi^o(s)$ does not solve (P^o), welfare can be increased

Why can the planner's problem be decomposed so?

- The planner chooses machines for each s separately; there is no direct interaction between different s
- For each s, the planner should
  - maximize the worker's social marginal value, which is the objective function in (P^o)
  - create as many machines in each unit (k, s) as to equate: social marginal value of a machine = the cost; (this is the constraint in (P^o))



Efficient allocation

Efficient allocation: solution

- Assignment is distinct: $\phi^o(s_1) \cap \phi^o(s_2) = \varnothing$ if $s_1 \neq s_2$
  - suppose s_1 and s_2 are both assigned to k, with $s_2 > s_1$. Let $b_i = B(k, s_i)$ and $F_i = F(k, s_i)$. Then,

$$\underbrace{e^{-b_1} F_1 = e^{-b_2} F_2}_{\text{social value of } s_1 \text{ and } s_2} \qquad \Longrightarrow b_2 > b_1$$

  - contradiction: net value of using skill s_2 is higher:

$$\left[1 - (1 + b_2)e^{-b_2}\right] F_2 - C(k) > \left[1 - (1 + b_1)e^{-b_1}\right] F_1 - C(k)$$

- assignment is one-to-one: $\phi^o(s)$ is unique for each s if

$$(F_k C_{kk} - C_k F_{kk})F > (F_k - C_k)F_k^2$$

Efficient allocation: solution (continued)

- efficient choice of k for s (where $b^o(s) = B^o(\phi^o(s), s)$):

$$\underbrace{\left[1 - e^{-b^o(s)}\right] F_k(\phi^o(s), s)}_{\text{expected marginal product of } k} \quad = \quad \underbrace{C_k(\phi^o(s))}_{\text{marginal cost}}$$

recall: frictionless assignment

$$F_k(\phi^p(s), s) = C_k(\phi^p(s)) \Longrightarrow \phi^o(s) < \phi^p(s)$$

- efficient choice of b for s:

$$\underbrace{\left[1 - (1 + b^o(s))e^{-b^o(s)}\right] F(\phi^o(s), s)}_{\text{social value of a machine } \phi^o(s)} = C(\phi^o(s))$$

Efficient allocation: solution (continued)

Write these conditions more explicitly:

$$b^o(s) = -\ln\left[1 - \frac{C_k(\phi^o(s))}{F_k(\phi^o(s), s)}\right]$$

$$\ln\left[1 - \frac{C_k(\phi^o(s))}{F_k(\phi^o(s), s)}\right] = \frac{\frac{C(\phi^o(s))}{F(\phi^o(s),s)} F_k(\phi^o(s), s) - C_k(\phi^o(s))}{F_k(\phi^o(s), s) - C_k(\phi^o(s))}$$

Efficient allocation: properties

- efficient assignment is PAM iff

$$F_{ks} > \frac{CF_s F_k^2 (F_k - C_k)}{FC_k (FC_k - CF_k)} \equiv A_1$$

why does PAM fail when $F_{ks} < A_1$?

– take the highest skill, $\bar{s}$. Tension between:
(a) matching $\bar{s}$ with high k so as to increase output
(b) utilizing $\bar{s}$ with high probability

– if k and s are only slightly complementary, (b) $\succ$ (a)

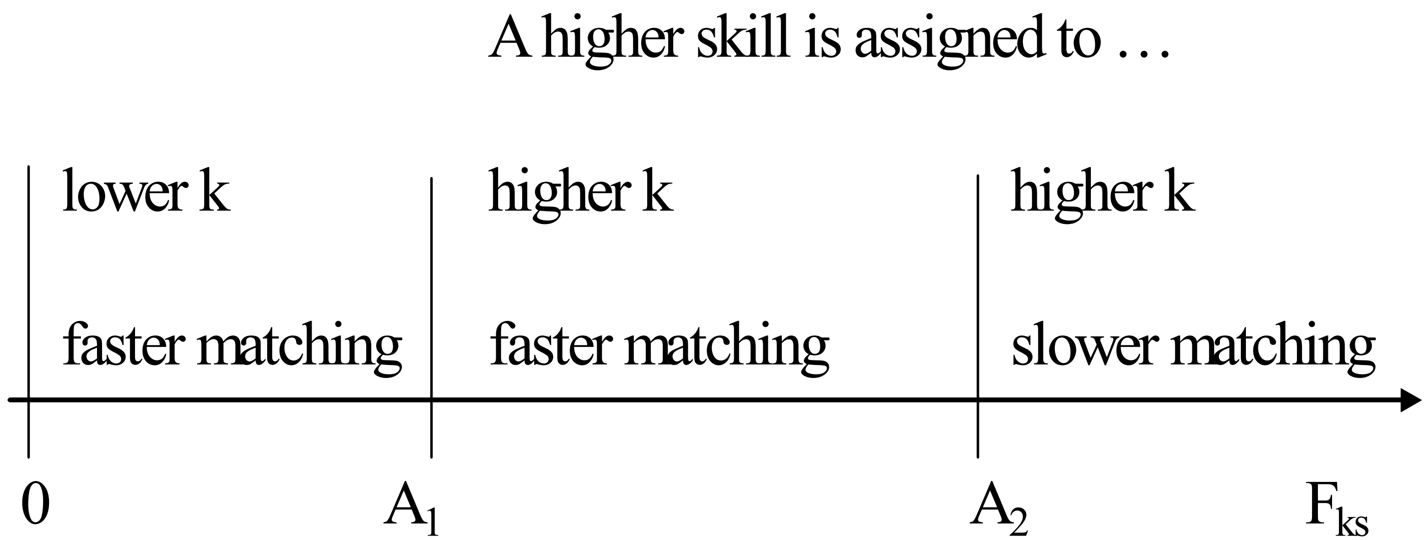
– in this case, it is efficient to create many low k machines to match with $\bar{s}$ to utilize $\bar{s}$ more

- A higher skill has a higher matching rate ($b_s < 0$) iff

$$F_{ks} < \frac{CF_sF_k(F_kC_{kk} - C_kF_{kk})}{F_kC_k(FC_k - CF_k)} \equiv A_2$$

why $b_s > 0$ when $F_{ks} > A_2$?

  - when $F_{ks} > A_2$, complementarity (a) $\succ$ utilization (b)
  - efficient to create high k to match with high s
  - but high k machines are expensive, and so
    - few high k machines are made
    - matching rate for high s is low



Efficient allocation

4. Market Assignment with Frictions

Sequence of actions with directed search:

- perceive a market tightness $B(k, s)$ for each (k, s)
- taking $B(k, s)$ as given, a firm chooses $\phi(s)$ and wage $W(k, s)$
- simultaneously announce the skill to hire and wages
- workers apply after observing all firms' choices
- if a firm gets the skill, chooses one randomly and produces; otherwise remains unmatched.

Wage $W(k,s)$

- Consider a firm D's deviation to $W^d(k,s)$
  - workers' response: application probability $p^d(k,s)$
  - $W^d(k,s)$ solves:

$$\max \left[1-(1-p^d(k,s))^{n(s)}\right]\left[F(k,s)-W^d(k,s)\right]$$

$$\text{s.t. } \underbrace{\frac{1-(1-p^d(k,s))^{n(s)}}{np^d(k,s)}}_{\text{worker's matching prob.}} W^d(k,s) = \underbrace{EW(s)}_{\text{market wage}}$$

- in equilibrium: $p^d(k,s) = p(k,s)$
  - FOC and constraint imply:

$$p(k,s) = \frac{1}{M(k,s)} = \frac{B(k,s)}{n(s)}$$

$$W(k,s) = \underbrace{\frac{B(k,s)}{e^{B(k,s)}-1}}_{\text{worker's share decreasing in } B(k,s)} \times F(k,s)$$

  - expected wage:

$$EW(k,s) = e^{-B(k,s)} F(k,s)$$

Market assignment

A firm chooses the machine quality $\phi(s)$ to target s:

$$\max_{\phi(s)} \; EW(k,s) = e^{-B(k,s)} F(k,s)$$

$$\text{s.t.} \quad \begin{cases} EP(k,s) = C(k), & \text{if } C(k) \leq F(k,s) \\ B(k,s) = \infty, & \text{otherwise,} \end{cases}$$

where expected value of k is:

$$EP(k,s) = \left[1 - (1 + B(k,s))e^{-B(k,s)}\right] F(k,s)$$

Market assignment: properties

- efficiency: market assignment coincides with (ϕ^o, b^o)
- why efficiency?

$$EW(s) = \text{social marginal value of worker } s$$

$$EP(k,s) = \text{social marginal value of machine } k \text{ in unit } (k,s)$$

- more general elements for efficiency:
  - decision rights are allocated correctly
  - competition through directed search
  - commitment to the skill and wage $W(k,s)$

Properties of wages

- actual wage for skill s:

$$w(s) = W(\phi(s), s) = \frac{B(\phi(s), s)}{e^{B(\phi(s),s)} - 1} F(\phi(s), s)$$

  - $w(s)$ is not necessarily increasing:
    higher s can be compensated with higher matching prob
  - machine assignment has first-order effect on wage:
    * PAM $\Rightarrow w'(s) > 0$:
      PAM can increase wage inequality
    * $w_s < F_s$ if and only if $b_s < 0$.

- expected wage $Ew(s) = e^{-B(\phi(s),s)} F(\phi(s), s)$:

$$Ew'(s) = \quad e^{-B(\phi(s),s)} \times$$

$$\Big\{ \underbrace{F_s(\phi(s), s)}_{\substack{\text{direct} \\ \text{effect}}} \underbrace{-B_s(\phi(s), s) F}_{\substack{\text{effect in} \\ \text{mat. prob}}} + \underbrace{\phi'(s) \left[F_k(\phi(s), s) - B_k(\phi(s), s) F \right]}_{\substack{\text{effect through} \\ \text{assigned machine}}} \Big\}$$

  – higher skill has higher expected wage ($Ew'(s) > 0$): efficient allocation has to compensate higher skill with either higher k or higher matching rate, or both

5. Infinite Horizon: Efficient Assignment

Motivation:

- robustness of non-PAM:
  - with one period, utilization concern may dominate PAM
  - with infinite horizon, temporary match failure is not costly; can efficient assignment still be non-PAM?
- intertemporal tradeoff:
  - current match destroys opportunity value of future match
  - is the efficient assignment dynamically stable?
- how does skill-biased technological progress (SBTP) affect assignment pattern, skill premium, wage inequality?

Modifications of the environment

- infinite horizon; discount factor: $\beta \in (0,1)$
- machine breaks down with prob ρ in each period
- exogenous separation (including ρ) is $\sigma(s)$: $\sigma'(s) \leq 0$
- unemployed workers in period t: $u_t(s)$;
  only unemployed workers can be assigned to matching
- $C(k)$: cost of a machine per period

Frictionless assignment ϕ^p still solves: $F_k(k_t, s) = C_k(k_t)$

- intertemporal tradeoff is not important for ϕ^p:
  - any desirable match can be formed instantaneously
  - current match does not destroy opp. value

Efficient allocation: formulation

$$\max \sum_{t=0}^{\infty} \beta^t \sum_{s \in S} \sum_{k_t \in \phi_t(s)} M_t(k_t, s) \left[\begin{array}{c} \left(1 - e^{-B_t(k_t,s)}\right) PV(k_t, s) \\ -C(k_t) \end{array} \right]$$

$$\text{present value: } PV(k_t, s) = F(k_t, s) + \frac{F(k_t, s) - C(k_t)}{1 - \beta[1 - \sigma(s)]}$$

subject to the following constraints for each s:

$$\sum_{k_t \in \phi_t(s)} M_t(k_t, s) B_t(k_t, s) \leq u_t(s)$$

$$u_{t+1}(s) = [u_t(s) - \Sigma_t] + \sigma(s) [n(s) - u_t(s) + \Sigma_t]$$

$$\text{new matches} \quad \Sigma_t = \sum\nolimits_{k_t \in \phi_t(s)} M_t(k_t, s) \left[1 - e^{-B_t(k_t,s)}\right]$$

Efficient allocation: recursive formulation

- planner can solve the problem for each s separately
- for each unit (k_t, s), total expected social surplus is:

$$EV(k_t, s) \equiv M_t(k_t, s) \left\{ \left[1 - e^{-B_t(k_t, s)} \right] PV(k_t, s) - C(k_t) \right\}$$

- $L(u(s))$: total social value of unemployed, skill s workers
  The recursive problem is:

$$(P') \quad L(u(s)) = \max_{(\phi, M, B)} \left[\sum_{k \in \phi(s)} EV(k, s) + \beta \; L(u_{+1}(s)) \right]$$

$$\text{s.t.} \quad \text{two constraints in the original problem.}$$

Efficient allocation: decomposition

- only link between current and future assignment for s is the marginal future value of unemployed s:

$$\lambda(s) \equiv \beta[1-\sigma(s)]L'(u_{+1}(s))$$

- $\lambda(s)$ is the opportunity cost of matching today; gain from a match today: $PV(k,s) - \lambda(s)$
- given $\lambda(s)$, the efficient allocation solves:

$$(P'') \qquad \max_{(k,B)} e^{-B(k,s)} \left[PV(k,s) - \lambda(s) \right]$$

$$\text{s.t.} \quad 1 - [1 + B(k,s)]\, e^{-B(k,s)} = \frac{C(k)}{PV(k,s) - \lambda(s)}$$

Efficient allocation: decomposition (continued)

- (P'') is the same as the one-period problem, with $[PV(k,s) - \lambda(s)]$ replacing $F(k,s)$
- thus, $\phi^o(s)$ and $b^o(s) = B^o(\phi^o(s), s)$ satisfy:

$$1 - e^{-b^o(s)} = \frac{[1 - a(s)]C_k(\phi^o(s))}{F_k(\phi^o(s), s) - a(s)C_k(\phi^o(s))}$$

$$1 - [1 + b^o(s)]\, e^{-b^o(s)} = \frac{C(\phi^o(s))}{PV(\phi^o(s), s) - \lambda(s)}$$

$$\text{where} \quad a(s) = \beta[1 - \sigma(s)]$$

- write the solution for $\phi^o(s)$ as $\phi(\lambda, s)$

Efficient allocation: intertemporal link (through λ)

Recall: λ is the opportunity value of future match.

higher λ reduces net gain from current match, and hence

- increases ϕ: current match must have a higher quality to justify the destruction of opp value of future match
- increases b: higher quality machines are worth creating only if they are matched more quickly

Efficient allocation: dynamics

- future social value λ satisfies the envelope condition:

$$\lambda_{-1} = \Psi(\lambda) \equiv a \times \left\{ \lambda + \underbrace{e^{-b(\phi(\lambda))} \left[PV(\phi(\lambda)) - \lambda \right]}_{\text{expected social gain}} \right\}$$

$$a = \beta(1-\sigma)$$

- unemployment rate $ru(s) \equiv \frac{u(s)}{n(s)}$ satisfies:

$$ru_{+1} = \sigma + (1-\sigma) \left[1 - \frac{1 - e^{-b(\phi(\lambda))}}{b(\phi(\lambda))} \right] ru$$

- initial condition: $ru_0(s) = \frac{u_0(s)}{n_0(s)}$ is given

Efficient allocation: dynamics (continued)

- $\exists$ a unique, saddle-path stable steady state
- along the saddle path,
  (λ_{-1}, b, ϕ) jumps to steady state immediately;
  ru approaches the steady state monotonically

- every machine in every period before its breakdown

is used in either production or matching

Efficient allocation: properties

- ϕ is PAM iff

$$F_{ks} > \frac{CF_s(F_k - C_k)\,(F_k - aC_k)\,[F_k - a(2-a)C_k]}{(1-a)^2\,(F - aC)\,C_k(FC_k - CF_k)}$$

so, sufficient complementarity is needed

- higher skill has a higher matching rate ($b'(s) < 0$) iff

$$F_{ks} < \frac{CF_s(F_kC_{kk} - C_kF_{kk})}{C_k(FC_k - CF_k)}$$

- $\exists$ an interval of F_{ks} in which a higher skill has both a higher machine assignment and higher matching rate

Efficient allocation: decentralization

extend directed search from one period to infinite horizon; see Shi (05, RED)

- firm posts the entire path of wages for the match
- commitment is still key to decentralization
- assignment has first-order effect on wages

6. Numerical Exercises

Functional forms and parameter values

$$C(k) = C_0 k^{\gamma} + C_1$$

$$F(k, s(i)) = F_0 k^{\alpha} s(i)^{1-\alpha}$$

Classification of workers:

$i = 1$: less than 4 years of high school;

$i = 2$: high school but no college education;

$i = 3$: some college but no degree;

$i = 4$: bachelor or higher degree.

Calibration:

- length of a period = one quarter $\Longrightarrow \beta = 1.04^{-1/4}$
- normalize: $F_0 = 1, \quad \phi(2) = 100, \quad \Sigma_s n(s) = 1$
- skill distribution in the labor force $\Longrightarrow n(i)$ for each i
- unemployment rate $\Longrightarrow ru(i)$ for each i
- other targets:
  - unemployment duration of group 2 workers = one quarter
  - relative wage rate of group i to group 2 workers, $RW(i)$
  - overall wage/output ratio = 0.64
  - minimize deviation of the capital/output ratio from 3.32.

Identified parameter values

$s(1)$	$s(2)$	$s(3)$	$s(4)$	β	F_0
30.2111	39.6965	47.7135	75.3342	0.9902	1
$\sigma(1)$	$\sigma(2)$	$\sigma(3)$	$\sigma(4)$	C_0	α
0.0676	0.0355	0.0272	0.0153	0.01287	0.1946
$n(1)$	$n(2)$	$n(3)$	$n(4)$	C_1	γ
0.1091	0.3275	0.2796	0.2839	12.7144	1.3564

Features of the baseline economy

	group i				std. dev. in
	1	2	3	4	log values
$\phi(i)$	82.14	100.00	113.79	156.61	
$w(i)$	18.28	27.65	35.38	61.42	
$\mu(i)$	0.592	0.631	0.653	0.692	
$\theta(i)$	0.530	0.582	0.608	0.657	
$RS(i)$	0.76	1	1.20	1.90	0.305
$RW(i)$	0.66	1	1.28	2.22	0.392
$RV(i)$	0.64	1	1.29	2.25	0.402

$\mu(i) = \frac{1-e^{-b(i)}}{b(i)}$, $\theta(i) = \frac{b(i)}{e^{b(i)}-1}$, $RY(i) = \frac{Y(i)}{Y(2)}$,
$V(i) = ru(i)V_u(i) + [1 - ru(i)] V_e(i)$.

Skill-biased technological progress

$$C(k) = \begin{cases} C_0 k^\gamma + C_1, & \text{if } \phi_0(i) < \underline{k}(\kappa) \\ C_{0d} k^\gamma + C_1,\ C_{0d} < C_0 & \text{if } \phi_0(i) \geq \underline{k}(\kappa). \end{cases}$$

new marginal cost parameter:

$$C_{0d}[\phi_0(4)]^\gamma + C_1 = 0.8\left(C_0[\phi_0(4)]^\gamma + C_1\right)$$

threshold to utilize new tech:

$$\underline{k}(\kappa) = \phi_0(1)\,|1-\kappa| + \phi_0(4)\,|\kappa|$$

$\kappa = 0,\ 0.2,\ 0.4,\ 0.6$: degree of skill bias

threshold skill $i_0 = ceil\left[\phi_0^{-1}(\underline{k}(\kappa))\right]$

Responses to a skill-biased progress

	group i			
	1	2	3	4
$\Delta\phi(i,0)$ (%)	56.70	56.87	56.92	56.99
$\Delta w(i,0)$ (%)	15.50	13.40	12.49	11.09
$\Delta\mu(i,0)$ (%)	2.32	1.68	1.40	0.92
$\Delta\theta(i,0)$ (%)	3.38	2.33	1.89	1.19
$\Delta V(i,0)$ (%)	15.79	13.50	12.55	11.12

$$\Delta y(i,\kappa) \equiv \left(\frac{y(i,\kappa)}{y(i,base)} - 1\right) \times 100$$

For $y = \phi, w, \mu, \theta, V$, the change $\Delta y(i,\kappa)$ is 0 if $i \leq \kappa/0.2$ and is equal to $\Delta y(i,0)$ otherwise.

Responses to a skill-biased progress

	$RW(i,\kappa)$ $(= \frac{w(i,\kappa)}{w(2,\kappa)})$					
	$i = 1$	2	3	4	DW	DV
$\kappa = 0$	0.67	1	1.27	2.18	0.380	0.390
0.2	0.58	1	1.27	2.18	0.408	0.419
0.4	0.66	1	1.44	2.47	0.435	0.445
0.6	0.66	1	1.28	2.47	0.434	0.444
base	0.66	1	1.28	2.22	0.392	0.402

Effects of skill-biased technological progress:

- for workers who can use the new technology, machine quality assignments, wages, matching rates, surplus shares and welfare all go up
- for worker who cannot use the new technology, these variables do not change
- among the skills that can use the new technology, lower-skill workers benefit more from the progress
  - expected net profit with low-quality machines is smaller and, hence, more sensitive to cost reduction
- inequality does always increase with degree of skill bias