

Directed Search

Lecture 4: Business Cycles

Lectures at Osaka University (2012)

© **Shouyong Shi**
University of Toronto

Main sources for this lecture:

- Menzio, G. and S. Shi, 2011, “Efficient Search on the Job and the Business Cycle,” JPE 119, 468-510.
- Menzio, G. and S. Shi, 2010a, “Block Recursive Equilibria for Stochastic Models of Search on the Job,” JET 145, 1453-1494.
- Menzio, G. and S. Shi, 2010b, “Directed Search on the Job, Heterogeneity, and Aggregate Fluctuations,” AER: P&P 100, 327-332.

1. Motivation

Facts about the US labor market:

- large monthly flows
 - from unemployment to employment: UE rate = 42%;
 - from employment to unemployment: EU rate = 2.6%;
 - from one employer to the other: EE rate = 2.9%
- these flows vary with the business cycle
- these flows are volatile relative to labor productivity
- the stocks (of unemployment and vacancies) are volatile

Table 1. US data (CPS), 1951:I - 2006:II

	u	v	h^{ue}	h^{eu}	h^{ee}	π
monthly average	0.056	63.9	0.452	0.026	0.029	
relative std	9.56	10.9	5.96	5.48	5.98	1
quarterly acr	0.872	0.909	0.822	0.698	0.597	0.760
cross correlation						
u	1	-0.902	-0.916	0.778	-0.634	-0.283
v		1	0.902	-0.778	0.607	0.423
π			0.299	-0.528	0.208	1

Question: How much do labor productivity shocks explain these?

To address this question, we need:

- aggregate shocks to labor productivity
- **on-the-job search** (OJS) to explain EE flow:
 - most search models rule out OJS, but in data:

$$\frac{\text{EE flow}}{\text{UE flow}} = \frac{2.9}{45} \times \frac{1-u}{u} \approx 1$$

- on-the-job search (OJS) to explain volatility:
 - without OJS, a search model implies:
 - weak incentive for job creation
 - low volatility in unemployment (Shimer 05)

To address this question, we need (continued):

- **match heterogeneity** to explain the EU flow:
 - job separation is exogenous in most models,
but it is counter-cyclical and volatile in the data
- match heterogeneity to rationalize EE flow:
 - EE flow is inefficient if matches are homogeneous

What if we add match heterogeneity but not OJS?

- economic booms are times to search
- if workers can search only when unemployed, then unemployment goes up in booms (counterfactual!)

Theoretical challenge:

Analyze business cycles with OJS and match heterogeneity.

- standard models (DMP, Shimer 05, etc.) have ignored OJS:
 - OJS endogenously generates wage distribution
 - with undirected search, distribution affects decisions
 - dynamic, two-way interaction between distribution and decisions is intractable
- directed search models offer hope: eqm is block recursive
 - decisions are independent of the distribution

Roadmap:

- extend a directed search model to incorporate:
OJS, match heterogeneity, aggregate shocks
- characterize efficient allocation and equilibrium:
 - block recursive eqm (BRE) exists and is unique
 - BRE is socially efficient
 - all equilibria are BRE
- calibrate the model to quantitatively answer:
how much do labor productivity shocks explain the
observed cyclical features of the US labor market?

2. The Model

Workers and jobs

- workers (risk neutral):
 - unemployed worker can search with prob $\lambda_u (= 1)$
 - employed worker can search with prob λ_e
- worker flows:
 - **quits** for other jobs (due to OJS)
 - **endogenous destruction**
 - exogenous destruction: δ

- productivity of a job: $y + z$
 - aggregate productivity: $y \in Y \quad \sim \phi(\hat{y}|y)$
 - match specific productivity: $z \in Z \quad \sim f(z)$
 permanent in a match; iid across matches

- signal on match-specific productivity:

$$s = z \text{ with prob } \alpha; \quad s \sim f \text{ with prob } 1 - \alpha$$

- job is pure **experience good**: $\alpha = 0$
- job is pure inspection good: $\alpha = 1$

Directed search:

- workers can directly go to specific submarkets

x : index of submarket (to be specified)

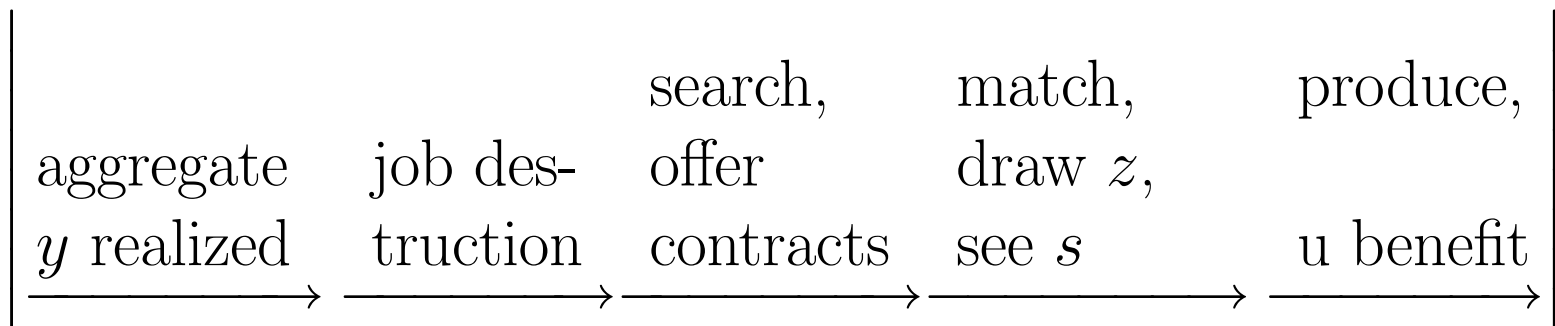
$\theta(x)$: vacancy/applicant (tightness)

- frictions summarized by matching prob:

worker: $p(\theta)$;

firm: $q(\theta) = p(\theta) / \theta$;

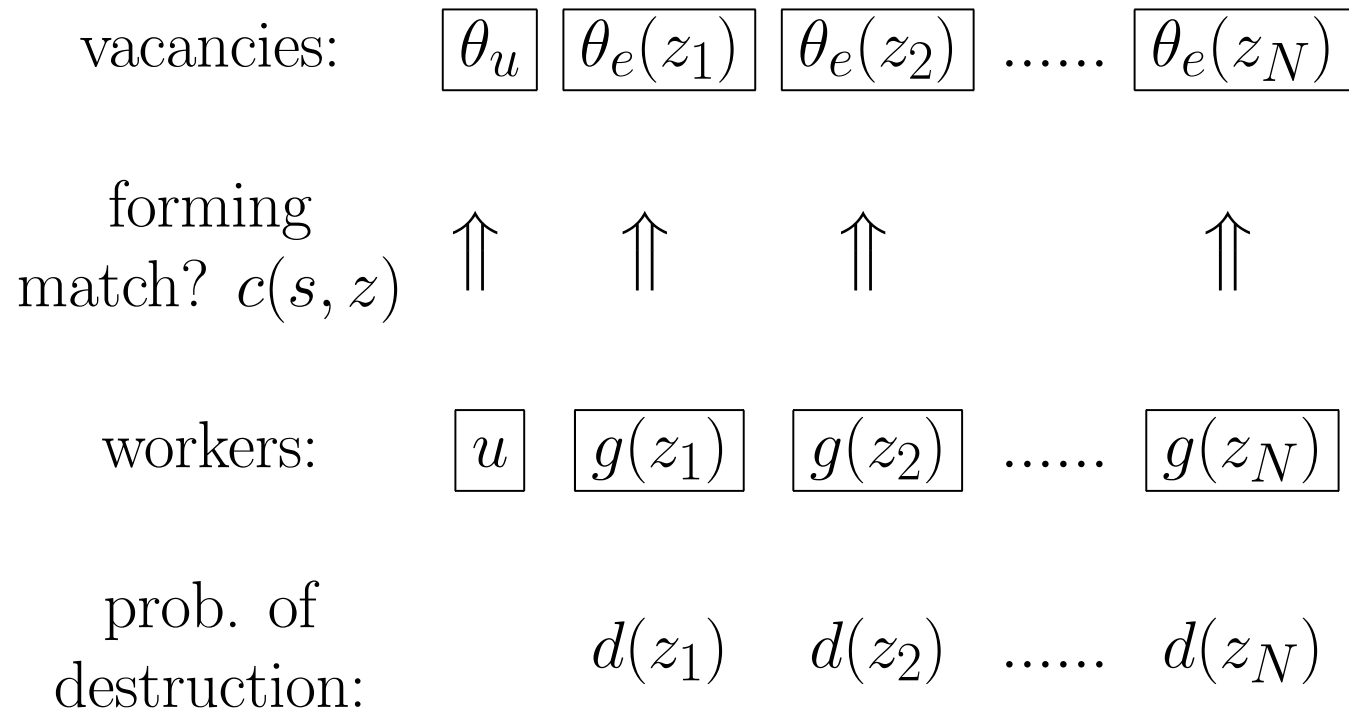
$0 < p'(\theta) < p(\theta) / \theta$ (trade-off between x and θ)



Timing of events in a period

Planner's Problem:

- directly targeting workers at each z with tightness $\theta(z)$:



Planner's chooses:

- vacancy creation: tightness
for unemployed: θ_u ,
for employed at z : $\theta_e(z)$
- match formation probability:
for unemployed with match signal s : $c_u(s)$
for employed at z with signal s : $c_e(s, z)$
- job destruction prob $d \in [\delta, 1]$:
 $d = \delta$: Nature destroys a match

State of economy: $\psi \equiv (y, u, g) \in \Psi$

- y : aggregate labor productivity
- distribution of workers (large dimension):
 - u : measure of unemployed workers
 - $g(z)$: measure of workers employed at z .

- social planner's value function:

$$W(\psi) = \max \left[F(d, \theta_u, \theta_e, c_u, c_e | \psi) + \beta \mathbb{E} W(\hat{\psi}) \right]$$

- net output in a period: $F(d, \theta_u, \theta_e, c_u, c_e | \psi) =$

$$\hat{u} b + \sum_z (y + z) \hat{g}(z) \quad (\text{home and market output})$$

$$-k \left\{ \lambda_u u \theta_u + \sum_z [1 - d(z)] g(z) \lambda_e \theta_e(z) \right\} \quad (\text{vacancy cost})$$

- constraints on $\hat{u}$ and $\hat{g}$ (see next 2 pages)

- measure of unemployed workers next period:

$$\hat{u} = \underbrace{u [1 - \lambda_u p(\theta_u) m_u]}_{\text{hiring from } u} + \underbrace{\sum_z d(z) g(z)}_{\text{job destruction}}$$

prob that a meeting with u turns into a match:

$$m_u = \sum_s \underbrace{c_u(s)}_{\text{creation prob}} f(s)$$

- measure of workers employed at z :

$$\begin{aligned}\hat{g}(z) = & \text{inflow from } u \text{ into } z \\ & + \text{workers employed at } z \text{ who stay put} \\ & + \sum_{z'} g(z') \times \text{prob}(\text{each worker at } z' \text{ moves to } z)\end{aligned}$$

prob(each worker at z' moves to z):

$$\left[1 - d(z')\right] \underbrace{\lambda_e p(\theta_e(z'))}_{\text{search}} \underbrace{\left[\alpha c_e(z, z') + (1 - \alpha)m_e(z')\right] f(z)}_{\text{prob of forming match at } z}$$

$$m_e(z') = \sum_s c_e(s, z') f(s)$$

Potential difficulty: dynamics of distribution $g(z)$

- OJS $\implies$ distribution of matches
- distribution can affect choices/allocation and market tightness
- choices/allocation + distribution $\implies$ new distribution

Theorem 1:

- social value function $W(\psi)$ is unique
- social value function is linear in (u, g) :

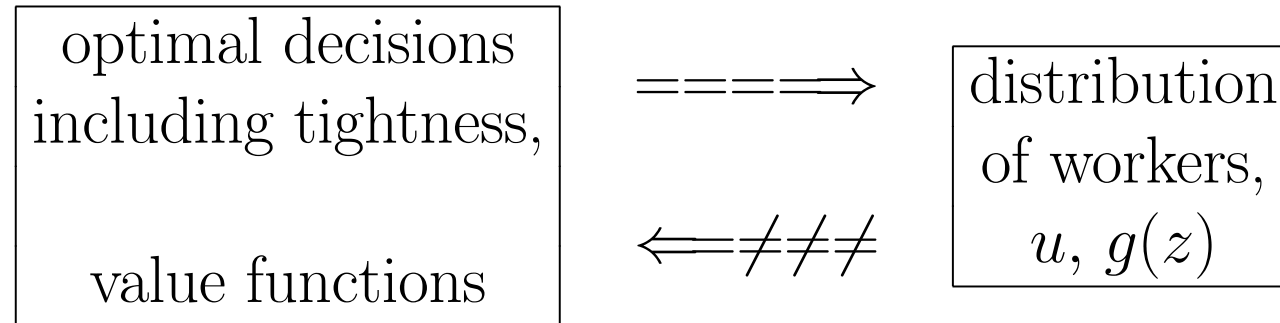
$$W(\psi) = \underbrace{W_u(y)}_{\text{value of unemployed}} \times u + \sum_z \underbrace{W_e(z, y)}_{\text{employed at } z} \times g(z)$$

- **block recursivity:**
 - $W_u(y)$ and $W_e(z, y)$ are independent of (u, g)
 - efficient choices are all independent of (u, g)

$$\begin{aligned}
& W_u(y) \\
= & \max_{(\theta_u, c_u)} \left\{ -k\lambda_u\theta_u + (1 - \lambda_u p(\theta_u)m_u) [b + \beta \mathbb{E}W_u(\hat{y})] \right. \\
& \left. + \lambda_u p(\theta_u) \mathbb{E}_{z'} \left\{ \frac{[\alpha c_u(z') + (1 - \alpha)m_u]}{\times [y + z' + \beta \mathbb{E}W_e(z', \hat{y})]} \right\} \right\}
\end{aligned}$$

$$\begin{aligned}
& W_e(z, y) \\
= & \max_{(d, \theta_e, c_e)} \left\{ d [b + \beta \mathbb{E}W_u(\hat{y})] - (1 - d) k\lambda_e\theta_e \right. \\
& + (1 - d) (1 - \lambda_e p(\theta_e)m_e) [y + z + \beta \mathbb{E}W_e(z, \hat{y})] \\
& \left. + (1 - d) \lambda_e p(\theta_e) \mathbb{E}_{z'} \left\{ \frac{[\alpha c_e(z') + (1 - \alpha)m_e]}{\times [y + z' + \beta \mathbb{E}W_e(z', \hat{y})]} \right\} \right\}
\end{aligned}$$

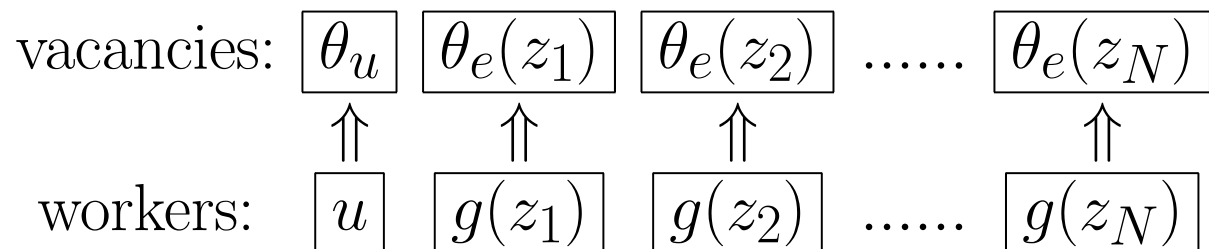
Block Recursivity (Shi 09):



- We can solve the left block first, and then the right block
- eliminate complexity generated by dynamics of distribution

Why does directed search produce block recursivity?

- able to target specific group of workers:



- free-entry generates the right tightness for each submarket:

$$k = p'(\theta_e(z)) \underbrace{\{\text{value of future job relative to current job}\}}_{\text{INDEPENDENT of distribution of workers}}$$

- no need to consider how other types of workers are distributed

Block recursivity FAILS when search is undirected:

$$k = p'(\theta_e(z)) \text{ [acceptance prob]} \text{ [value added by match]}$$

Because applicant is random draw from distribution,

- acceptance prob depends on distribution;
- value added by a match depends on distribution

Block recursivity does NOT rely on:

- risk neutrality of workers
- completeness of contracts

Examples:

- Risk averse workers and wage-tenure contracts (Shi 09)
- Dynamic contracts or fixed wage contracts (Menzio and Shi 10b)

Properties of efficient allocation:

- efficient choices are unique
- match formation – cutoff rule
 - form match for unemployed iff $s \geq r_u^*(y)$
 - form match for employed iff $s \geq r_e^*(z, y)$
 - cutoff $r_e^*(z, y)$ is increasing in z

Efficient choices of forming matches:

- form match for unemployed worker iff

$$b + \beta \mathbb{E} W_u(\hat{y}) \quad (\text{value when unemployed})$$

$$\leq \alpha[y + s + \beta \mathbb{E} W_e(s, \hat{y})] \quad (\text{employed with correct signal})$$

$$+ (1 - \alpha) \mathbb{E}_{z'}[y + z' + \beta \mathbb{E} W_e(z', \hat{y})] \quad (\text{with random signal})$$

- cutoff rule: form match iff signal $s \geq r_u^*(y)$

- form match for employed worker at z iff

$$y + z + \beta \mathbb{E} W_e(z, \hat{y}) \quad (\text{value of staying at } z)$$

$$\leq \alpha[y + s + \beta \mathbb{E} W_e(s, \hat{y})] \quad (\text{employed at } s)$$

$$+(1 - \alpha)\mathbb{E}_{z'}[y + z' + \beta \mathbb{E} W_e(z', \hat{y})] \quad (\text{at random } z')$$

- cutoff rule: form match iff $s \geq r_e^*(z, y)$

- $r_e^*(z, y)$ is increasing in z

Properties of efficient allocation (continued):

- vacancy creation:
 - tightness is $\theta_u^*(y)$ for unemployed
 - tightness is $\theta_e^*(z, y)$ for employed at z
 - tightness $\theta_e^*(z, y)$ is decreasing in z :
(more jobs are created for lower z)

Efficient choices of job creation:

- tightness of market for unemployed: $\theta_u^*(y) \geq 0$ and

$$k \geq p'(\theta_u^*(y)) \sum_{s \geq r_u^*(y)} \left\{ \begin{array}{l} \text{expected social surplus of} \\ \text{employment relative to } u \end{array} \right\} f(s)$$

- expected social surplus of employment:

$$\alpha [y + s - b + \beta \mathbb{E} [W_e(s, \hat{y}) - W_u(\hat{y})]]$$

$$+(1 - \alpha) \mathbb{E}_{z'} \{ y + z' - b + \beta \mathbb{E} [W_e(z', \hat{y}) - W_u(\hat{y})] \}$$

- tightness of market for employed at z : $\theta_e^*(z, y) \geq 0$ and

$$k \geq p'(\theta_e^*(z, y)) \sum_{s \geq r_e^*(z, y)} \left[\alpha [\text{surplus of emp at } s \text{ relative to } z] \right. \\ \left. + (1 - \alpha) \mathbb{E}_{z'} [\text{emp at } z' \text{ relative to } z] \right] f(s)$$

- expected surplus of employment at s relative to z

$$\alpha [s - z + \beta \mathbb{E} (W_e(s, \hat{y}) - W_e(z, \hat{y}))] \\ + (1 - \alpha) \mathbb{E}_{z'} \{ z' - z + \beta \mathbb{E} (W_e(z', \hat{y}) - W_e(z, \hat{y})) \}$$

Properties of efficient allocation (continued):

- job destruction: $d^*(z, y) = 1$ whenever
value of unemployed

> joint value of keeping the match

otherwise, $d^*(z, y) = \delta$ (exogenous separation)
- cutoff rule: $d^*(z, y) = 1$ iff $z < r_d^*(y)$

3. Decentralization:

Markets:

- continuum of submarkets indexed by (x, r) :
 - x : lifetime utility of offer to a worker
 - r : match is formed iff signal $s \geq r$
- tightness $\theta(x, r, \psi)$: determined by free entry of v
- tradeoff between offer and matching prob $p(\theta)$

Directed search

- surplus from search by a worker in match V :

$$D(x, r, V, \psi) = \underbrace{p(\theta(x, r, \psi))}_{\text{meeting prob}}, \underbrace{m(r)}_{\text{match}}, \underbrace{(x - V)}_{\text{gain}}$$

- prob of forming a match: $m(r) = \sum_{s \geq r} f(s)$
- workers are endogenously separated according to V :
higher $V \implies$ less concerned about p than the gain $(x - V)$
 $\implies$ searching for higher x .

Employment contracts:

- specify job separation: $d \in [\delta, 1]$
and submarket for OJS: (x, r)
- contingent on history (z, ψ^t) , $\psi^t = (\psi_1, \dots, \psi_t)$
- implicit assumption: contracts are bilaterally efficient
(V in search problem is match's joint value)

Justifications for the assumption on contracts:

- example: wage-tenure contracts
- benchmark: uniqueness and efficiency of eqm
- not necessary for block recursivity

Value functions:

- unemployed worker's value:

$$V_u(\psi) = b + \beta \mathbb{E} \max[V_u(\hat{\psi}) + \lambda_u D(x, r, V_u(\hat{\psi}), \hat{\psi})]$$

- joint value of a match at z :

$$\begin{aligned} V_e(z, \psi) = & y + z + \beta \mathbb{E} \max_{(d, x, r)} \{ d V_u(\hat{\psi}) \\ & + (1 - d)[V_e(z, \hat{\psi}) + \lambda_e D(x, r, V_e(z, \hat{\psi}), \hat{\psi})] \} \end{aligned}$$

Market tightness: $\theta(x, r, \psi)$

- determined by free entry of vacancies: for all (x, r) ,

$\theta(x, r, \psi) \geq 0$ and

$$k \geq \underbrace{q(\theta(x, r, \psi))}_{\text{filling prob}} \sum_{s \geq r} \left[\underbrace{\left(\frac{\alpha V_e(s, \psi) + (1 - \alpha) \mathbb{E}_z V_e(z, \psi)}{(1 - \alpha) \mathbb{E}_z V_e(z, \psi)} \right)}_{\text{expected joint value of a match}} - x \right] f(s)$$

Theorem:

- All equilibria are block recursive:
i.e., the following elements are independent of (u, g) :
 - optimal choices of (d, x, r) ;
 - value functions: $V_u(y)$ and $V_e(z, y)$
 - market tightness function: $\theta(x, r, y)$
- There exists a unique block recursive equilibrium (BRE)
- The BRE is socially efficient

Recap: Why is equilibrium Block Recursive?

- Directed search
 $\implies$ endogenous separation of workers into submarkets
- Free entry of firms into each submarket
 $\implies$ each submarket's tightness independent of other submarkets

Block recursivity does NOT rely on:

- risk neutrality of workers (see Shi 09)
- efficient contracts (see Menzio and Shi 10b)

4. Data and Calibration

Data:

- HP-filtered CPS, 1951:I - 2006:II

1992 index of average labor productivity = 100

- 1987 CPS Tenure Supplement;
- Conference Board Help-Wanted Index:
2000 vacancy index = 100.

Functional forms:

- matching function: $p(\theta) = \min\{1, \theta^\gamma\}$, $\gamma \in (0, 1)$;
- match specific productivity, $f(z) \sim Weibull(\mu_z, \alpha_z, \sigma_z)$:
 $\mu_z = 0$; α_z : shape parameter; σ_z : scale parameter;
- aggregate productivity $y \in Y$: 3-state Markov
mean: $\mu_y = 1$, std: σ_y , autocorrelation: ρ_y .

Table 2. Identification of parameters
(experience-goods: $\alpha = 0$)

	description	target	data	value
β	discount	real int. rate		0.996
λ_u	off the job search prob	normalize		1
μ_y	mean of average prod π	normalize		1
σ_y	std. of agg. shock	std. of agg. productivity	CPS	0.0152
ρ_y	persistence of π	persis of prod	CPS	0.76

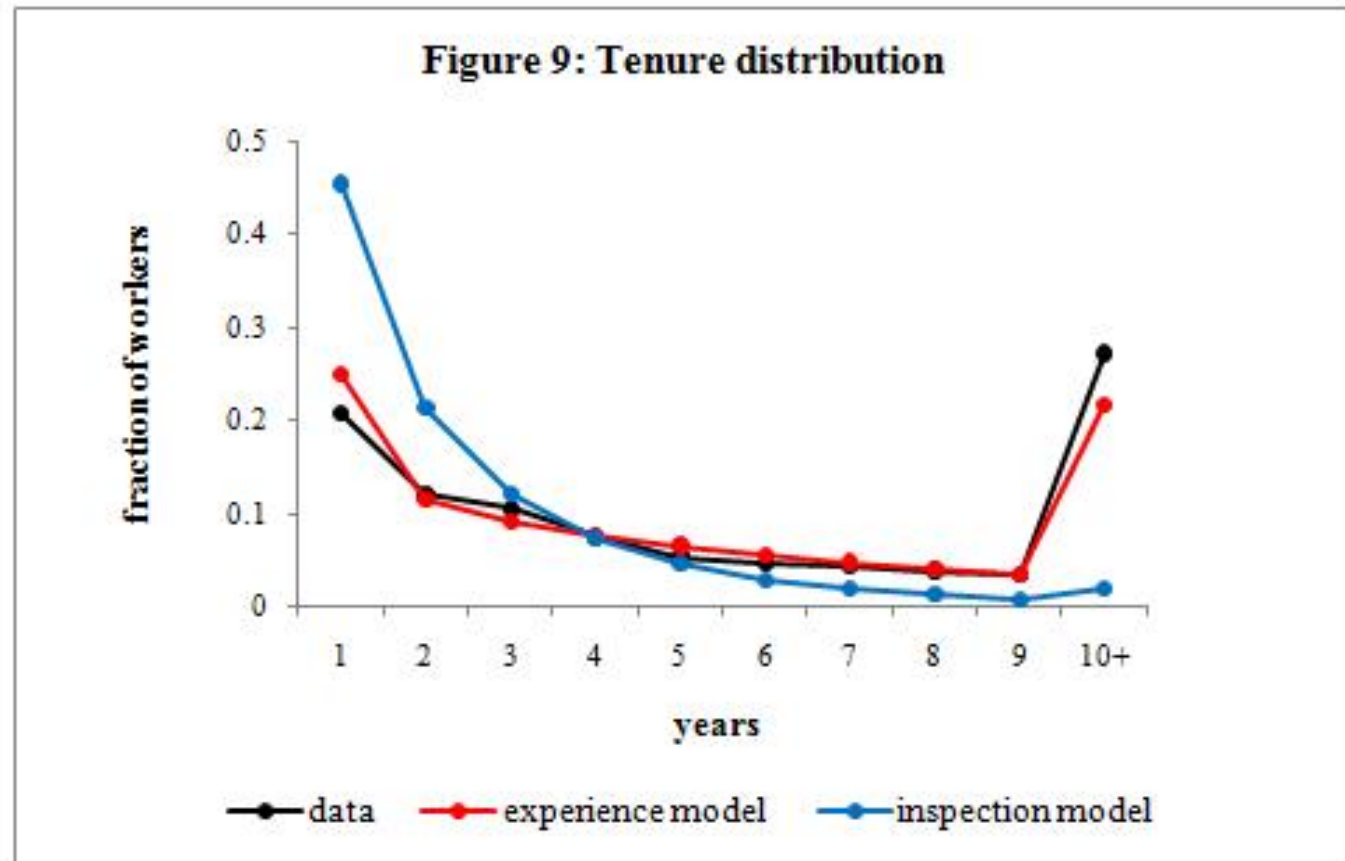
Table 2. Identification of parameters (continued)
(experience-goods: $\alpha = 0$)

	description	target	data	value
k	vacancy cost	UE rate =0.45	CPS	1.550
b	unem benefit	EU rate =0.026	CPS	0.907
λ_e	OJS prob	EE rate =0.029	CPS	0.735
γ	parameter in matching	elasticity of h^{ue} to v/u (=0.27)	CPS	0.600

Transition rates are monthly

Table 2. Identification of parameters (continued)
(experience-goods: $\alpha = 0$)

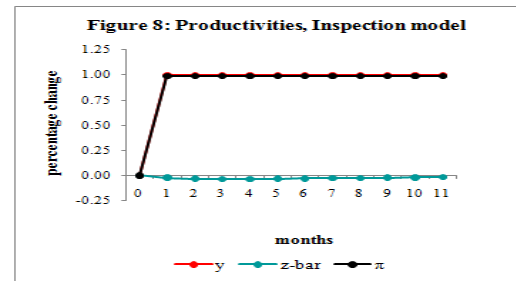
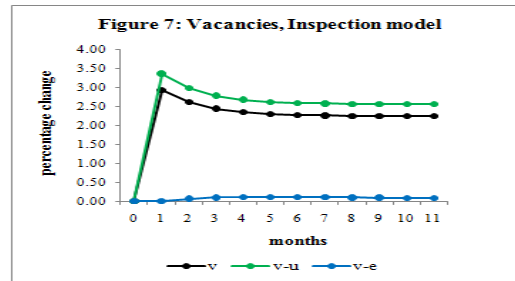
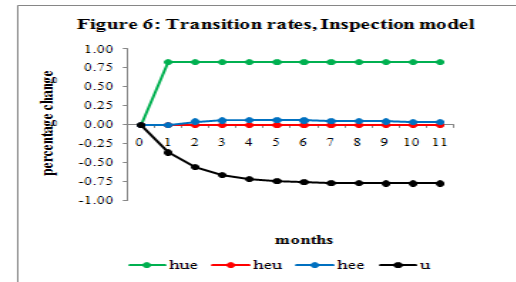
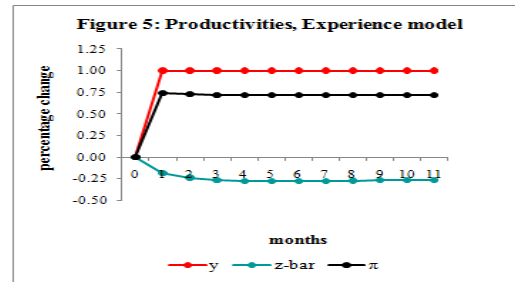
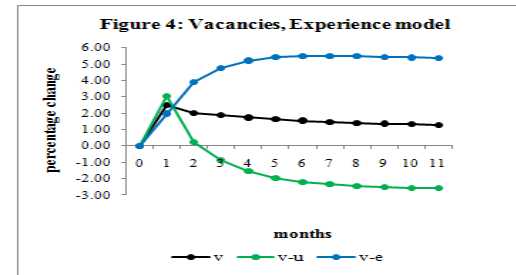
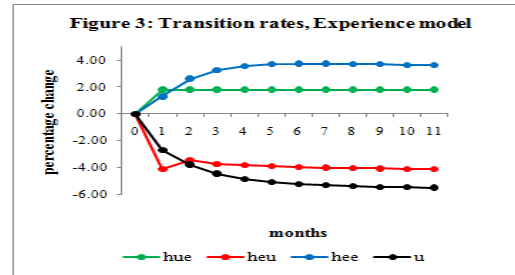
	description	target	data	value
σ_z	scale of specific prod.	ratio of home to market prod.	Hall & M. (= 0.71)	0.952
α_z	shape of specific prod.	tenure distribution	Diebold et al. 97	4.000
δ	exogenous destr. rate	same as above		0.012



5. Model's Predictions (experience goods)

$(\alpha = 0)$

	u	v	h^{ue}	h^{eu}	h^{ee}	π
relative std	7.88 (9.56)	2.54 (10.9)	2.51 (5.96)	6.23 (5.48)	5.59 (5.98)	1
quarterly acr	0.850	0.637	0.799	0.772	0.823	0.762
cross correlation						
u	1	-0.807 (-0.902)	-0.976 (-0.916)	0.972 (0.778)	-0.979 (-0.634)	-0.977
v		1	0.897 (0.902)	-0.898 (-0.778)	0.858 (0.607)	0.894
π			0.999	-0.979	0.983	1



Canonical model: no OJS ($\lambda_e = 0$); no heterogeneity ($\sigma_z = 0$)						
	u	v	h^{ue}	h^{eu}	h^{ee}	π
relative std	0.820 (9.56)	2.690 (10.9)	0.910 (5.96)	0 (5.48)	— (5.98)	1
quarterly acr	0.815	0.677	0.994	1	—	0.745
cross correlation						
u	1	-0.932 (-0.902)	-0.936 (-0.916)	0 (0.778)	— (-0.634)	-0.972
v		1	0.994 (0.902)	0 (-0.778)	— (0.607)	0.990
π			0.999	0	—	1

What if OJS is prohibited?

- y shocks have no effect on vacancies for employed
- incentive to create vacancies weak
 $\implies EU$ rate not volatile
- Beveridge curve may or may not be negatively sloped
(when there is endogenous separation)

What if OJS is prohibited? (continued)

- under-estimate γ in matching function $[p(\theta) = \theta^\gamma]$
 $\implies UE$ rate less responsive to θ (hence less volatile)

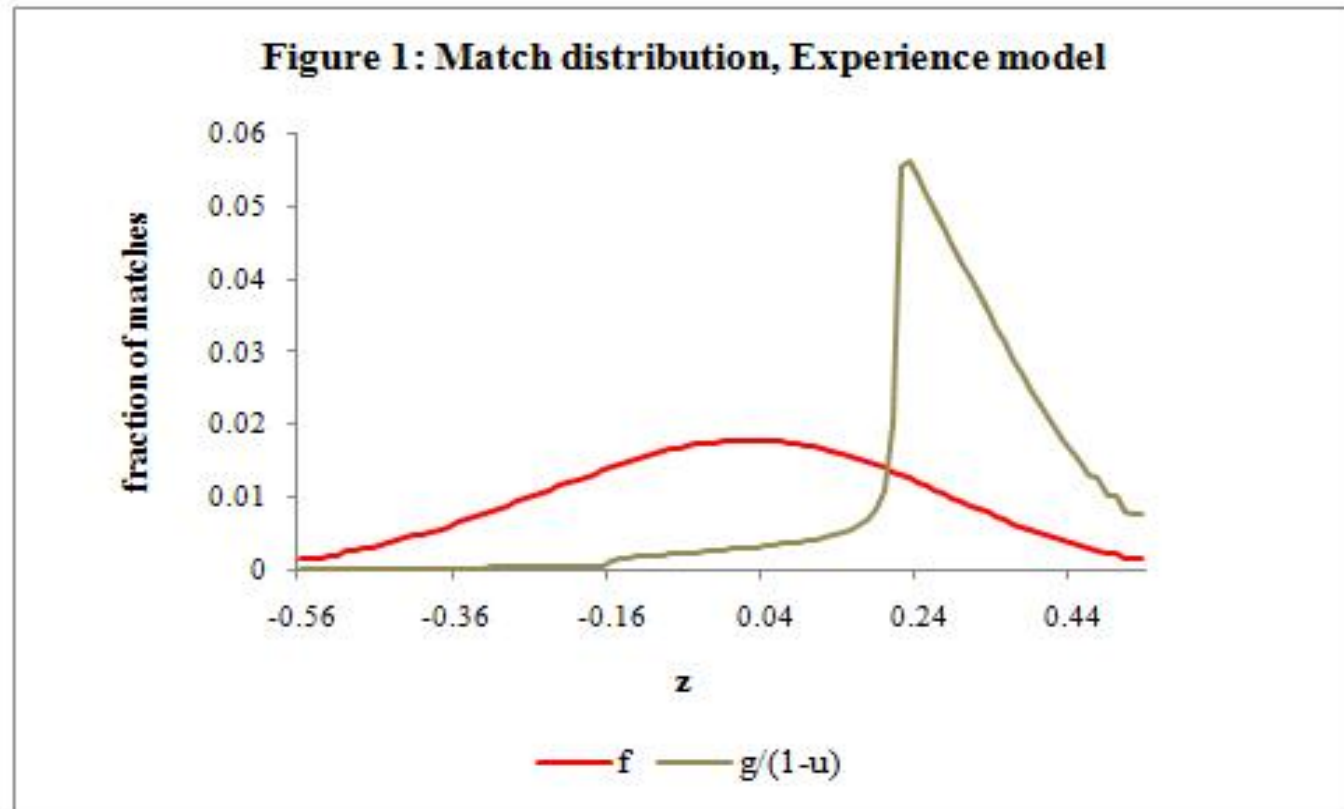
$$\gamma = \frac{\Delta \log h^{ue}}{\Delta \log \theta_u} = \frac{\Delta \log h^{ue}}{\Delta \log(v/u)} \times \frac{\Delta \log(v/u)}{\Delta \log \theta_u} \quad | \quad std(h^{ue})$$

this model	0.27	$\times$	2.22		2.51
no OJS	0.27	$\times$	1		0.91

What if matches is homogeneous?

- EE flows not part of efficient allocation
- EU rate not countercyclical or volatile
 - missing mechanism:
 $y \uparrow \implies$ critical level of s for job destruction falls
- underestimate volatility in y shocks
(match heterogeneity $\implies$ selection in match formation
 $\implies$ dispersion in observed $p <$ dispersion in y)

Return to “matching capital”



Cleansing effect of recessions:

- on job creation:
 - negative y -shocks raise cutoff levels r_u and r_e above which matches are formed
- on job destruction:
 - negative y -shocks raise cutoff level r_d below which matches are destroyed

How important is cleansing effect?

Consider inspection-good version ($\alpha = 1$)

- cleansing effect on job destruction not operational
reason: creation cutoffs $r_u, r_e >$ destruction cutoff r_d
- recalibrate model to the same targets
- model fails to
 - fit calibration target on tenure distribution
 - generate large volatility in labor market

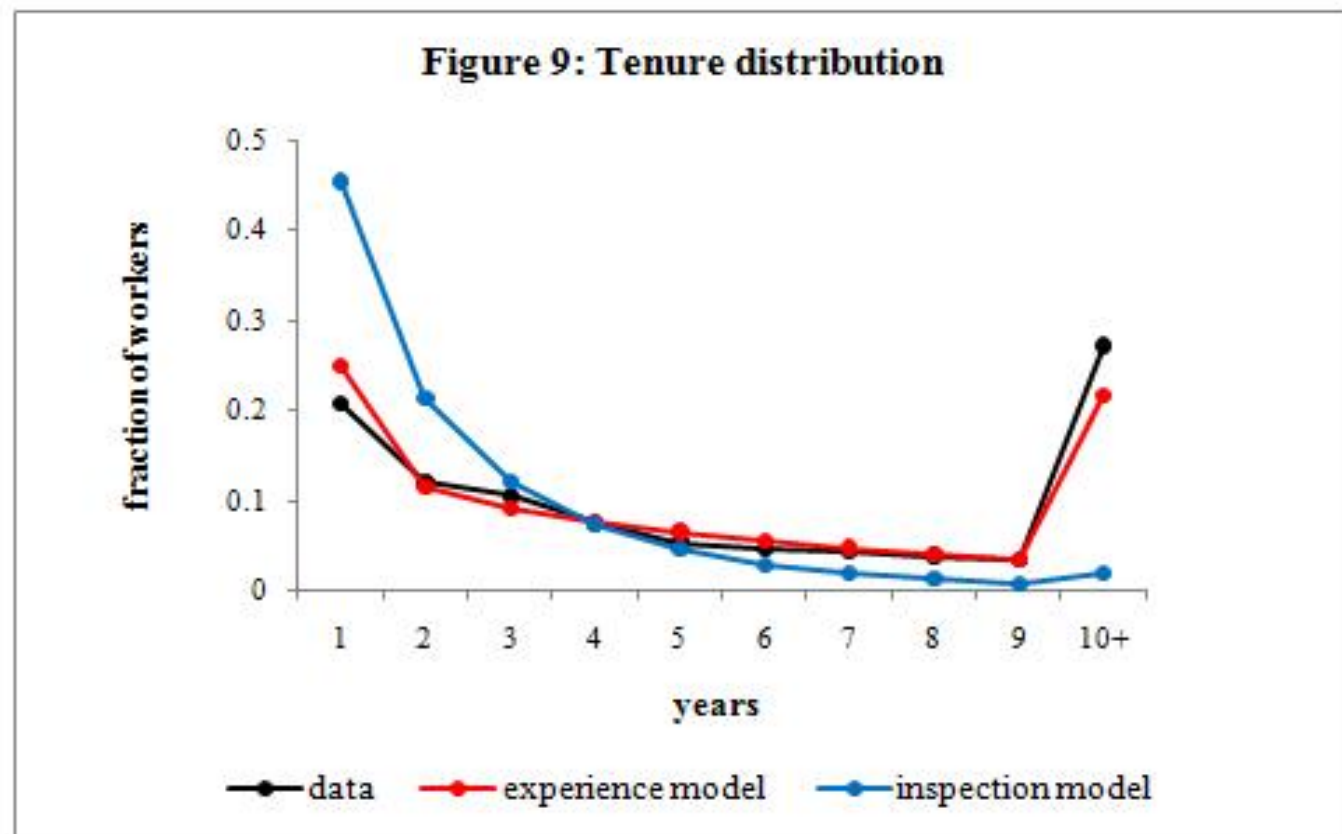
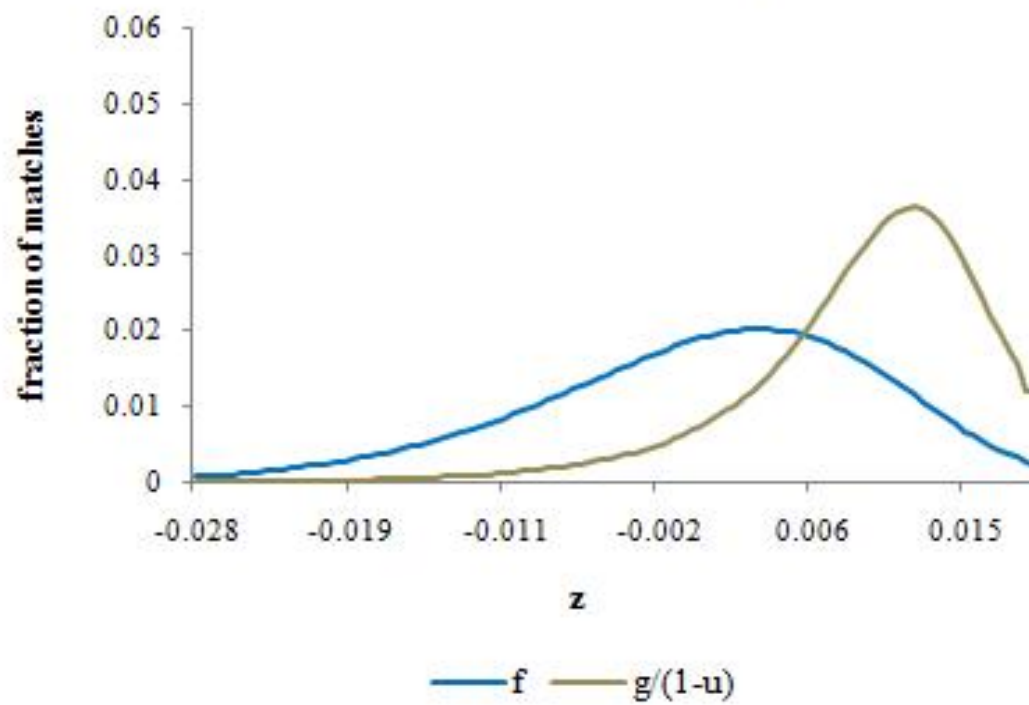


Figure 2: Match distribution, Inspection model



Model's predictions: inspection goods ($\alpha = 1$)

	u	v	h^{ue}	h^{eu}	h^{ee}	π
relative std	0.75 (9.56)	2.40 (10.9)	0.84 (5.96)	0 (5.48)	0.06 (5.98)	1
quarterly acr	0.829	0.686	0.747	1	0.743	0.750
u	1	-0.935 (-0.902)	-0.971 (-0.916)	0 (0.778)	-0.817 (-0.634)	-0.977
v		1	0.992 (0.902)	0 (-0.778)	0.824 (0.607)	0.992
π			0.999	0	0.833	1

6. Conclusion

- Tractable framework for studying business cycles with OJS and match heterogeneity
- BRE exists, is unique and socially efficient
- OJS + match heterogeneity account for
 - 80% of volatility in unemployment
 - strong Beveridge relationship
 - cyclical features of UE, EU and EE flows
 - experience-goods and tenure distribution