1. Consider a constant returns to scale production function $Y = F(K, L)$.

   (a) What is the definition of the constant returns to scale?

   (b) Show that a firm cannot earn economic profits.

   (c) In reality, we observe profits in a market. How can we reconcile the theory with this evidence?

2. Show whether or not the following production function are constant return to scale in K and L?

   (a)
   $$Y = K^{\alpha} L^{\beta}, 0 < \alpha + \beta < 1$$

   (b)
   $$Y = [F(K, L)]^{\alpha}, \ 0 < \alpha < 1,$$
   where $F(K, L)$ is constant return to scale in K and L.

   (c)
   $$Y = [(1 - \theta) K^{\rho} + \theta L^{\rho}]^{\frac{1}{\rho}}, \ 0 < \theta < 1$$

   (d)
   $$Y = F(K, L) + G(K, L)$$
   where $F(K, L)$ and $G(K, L)$ are constant return to scale in K and L.

   (e)
   $$Y = [F(K, L)]^{\alpha} L^{(1-\alpha)}, \ 0 < \alpha < 1$$
   where $F(K, L)$ is constant return to scale in K and L.

3. Answer the following questions.

   (a) When a firm maximizes its profits by choosing the amount of labor, the marginal product of labor is equal to the real wage rate. Explain its economic (intuitive) reason behind this mathematical result.

   (b) When a labor market is competitive, the demand for labor is equal to the supply of labor in a market equilibrium. Explain a rationale for this definition of the equilibrium.

   (c) When a financial market is competitive (and any risk can be diversified), then the returns on investment are expected to be the same across investment opportunities in a market equilibrium. Explain a rationale for the definition of the equilibrium.

   (d) In our lecture note, the price of goods produced is assumed to be 1. Explain a rationale for this assumption.

4. An economy described by the neoclassical growth model has the following production function:

$$Y = K^{\alpha} (TL)^{1-\alpha}, \ 0 < \alpha < 1$$

where Y is GDP, K is capital stock, T is productivity and L is the number of labor.

(a) Assume that a firm maximizes its profit given wage, w and the rental cost of capital, r. Show that $\alpha = \frac{rK}{Y}, 1 - \alpha = \frac{wL}{Y}$