Macroeconomics: Modern Macroeconomics 1

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OSIPP

Introduction

- **Purpose:** The Course is designed to help you understand the basic concepts and framework of modern macroeconomics. The theories are supplemented by relevant empirical evidence.
- **Office hour:** Room 602, 9:50-10:20, 12:15-12:45 on Monday. Appointment is required for other time.
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Introduction

- **Grading Policy:** 42% on assignments and 58% on a final exam.

   1. I will give you 7 assignments. Students must hand them in at the following lecture. If students turn an assignment in by the due date, I will give them 6 points. If students turn an assignment in late, I will give them 3 points. If students submit all assignments, you will receive 42 points. Students must write their answers with a pen. I don't allow the typed answers for this assignment.

   2. The full score of final exam is 58 points. I guarantee that 30 points out of 58 points will come from the assignment. If you hand in all assignments and you perfectly answer the questions appeared in assignments, you can certainly receive B.

Introduction

- **Remarks:**
  1. I assume that students have already taken Microeconomics 1.
  2. This course is comparable to the junior or senior undergraduate course in the economics department.
  3. I will mainly teach this course in Japanese. However, I will not prevent students from asking questions in English. I can discuss your questions and comments in Japanese or English at my office hour.

Introduction

- **Course Outline**
  1. The Data of Macroeconomics (2 lectures):
  2. The Framework of Macroeconomics (1 lecture):
  3. Economic Growth and Nation's income (4 lectures):
  4. Stabilization Policy (6 lectures):
  5. Lucas's Critique and Micro Foundation (1 lecture): Consumption.
  6. Final Exam (1 lecture).

What is Macroeconomics?

- Macroeconomics is a study to explain the behavior of aggregate data such as GDP per capita, inflation rate, and unemployment rate.

    1. Observing Statistics, macroeconomists examine the health of our economy and make policy suggestions.
    2. For this purpose, we must infer the structure of the economy that brings the observable data.
    3. Macroeconomics is the current consensus on the inferences about the economic structure.

The Data of Macroeconomics

- Three main statistics
  1. Gross Domestic Product...the measure of richness.
  2. Consumer Price Index...the measure of cost of living
  3. Unemployment Rate..the measure of joblessness

Gross Domestic Product

- Definition: Gross Domestic Product (GDP) is the gross sum of value added of each product measured by market prices in a country during a period.
  - GDP can be viewed as the total income of the whole economy.
  - GDP can also be viewed as the total expenditure on the economy's outputs of goods and services.
  - For the economy as a whole, expenditure must equal income. Why?

Five main features of GDP

- *The use of market value:* GDP evaluates the value of goods and services by their market value since the prices of goods and services indicate how much consumers are willing to pay for them. Then GDP sums up the market value of goods and services in a country.

  - **Example:** Suppose that a country produces 5 apples and 10 bananas, and the price of an apple is 100 yen and the price of a banana is 30 yen. Then

$$GDP = 100yen \times 5 + 30yen \times 10 = 800yen$$

Five main features of GDP

- *The value added:* GDP is the sum of the value added of each product.
  - The value added of a firm equals the value of the firm's output minus the value of intermediate products that the firm has purchased.
  - **Example:** A firm purchases oranges from a farmer for 60 yen and sells an orange juice for 100 yen per cup. Then the value added of the orange juice is 40 yen. If a farmer does not buy any intermediate goods, then the value added of an orange is 60 yen. Therefore the value added of the two products equals

    $$40 \ yen + 60 \ yen = 100 \ yen.$$

    If the oranges are imported, GDP is

    $$60 \ yen$$

Five main features of GDP

- *GDP vs. GNP:*
    - GDP measures the total income in a country not by residents of the country.
    - Gross National Product (GNP) measures the total income earned by residents of the country.
    - The difference is factor payments (wages, profits and rents) from abroad and factor payments to abroad:

    $$
    \begin{aligned}
    GNP \quad = \quad & GDP \\
    & + factor\ payments\ from\ abroad \\
    & - factor\ payments\ to\ abroad.
    \end{aligned}
    $$

Five main features of GDP

- *Flow vs. Stock:*
  1. A flow is a quantity measured per unit of time.
  2. A stock is a quantity measured at a given time.

- **Example:**
  - Flow...Annual income, Saving...
  - Stock....Wealth, Asset

- Since GDP measures the total income earned during a period, such as a year, GDP is a flow variable.

Five main features of GDP

- *Gross vs. Net:*
  - GDP is the gross sum of value added. It does not subtract the depreciation of capital from the value added- the amount of capital (plants, equipment and residential structures) that wears out over a period of time.
  - Net National Product:

$$NNP = GNP - Depreciation$$

Some details for computing GDP

- *Used goods:* The sale of used goods does not increase the additional value in a country. Therefore, the sale of used good is not included in GDP.

- *Inventories:* National Income Accounting system treats inventory as the sale of goods to themselves during the current period.

    - Inventories are counted as part of GDP of the period that goods are produced..
    - Inventories are not counted as part of GDP of the period that goods are sold.

        - It is considered as used goods.

    - Because of this treatment of inventories, all goods produced are purchased by somebody. Therefore, total income always equals total expenditure of a country.

Some details for computing GDP

- *Imputations:* When some goods are not sold in a market, they do not have market prices. If GDP includes these goods and services, we must estimate their value. Such an estimated value is called imputed value.

  1. **The Value of Housing:** When you rent an apartment, the rent is a part of GDP. When you own a house, you do not pay the rent. GDP estimates the rent that house owners pay to themselves.
  2. **Home Production and Durable Goods:** The value of these rental service and home production is left out of the GDP.
  3. **Government Services:** The national income accounts estimate the value added of government services in the GDP at their cost.
  4. **Underground Economy:** no imputation is made for the value of goods and services sold in the underground economy.

Comparison across Time Periods

- Since GDP is measured by the market prices of goods and services, GDP increases both when prices increase and when the outputs increase. In order to exclude the impact of inflation, economists separate real GDP from nominal GDP.

    - **Nominal GDP** uses current prices to measure the value of goods and services.
    - **Real GDP** uses a constant set of prices to measure the value of goods and services. In order to compute real GDP, economists choose the base year.
    - **Example:** Consider a country in which people produce only apples and bananas during 2008 and 2009. Let me choose 2008 is the base year.

$$Real GDP\ in 2008 \quad = \quad P_{A2008} \times Q_{A2008} + P_{B2008} \times Q_{B2008}$$
$$Real GDP\ in 2009 \quad = \quad P_{A2008} \times Q_{A2009} + P_{B2008} \times Q_{B2009}$$

- *GDP deflator:* The ratio of nominal GDP to real GDP is called GDP deflator:

$$GDP\ Deflator = \frac{\text{Nominal GDP}}{\text{Real GDP}}$$

- The GDP deflator captures the movement of the overall price level in the economy.

- **International prices:**Different countries use different currencies. In order to compare income across countries, which prices should we use?
  - An international price of the goods...The weighted average of the price of the goods across countries by taking the country's share of expenditures as its weight.
- **Purchasing-Power Parity:** Purchasing-Power Parity (PPP) is the ratio of nominal GDP to real GDP measured by international prices

$$PPP = \frac{\text{Nominal GDP}}{\text{Real GDP measured by international prices}}$$

International Comparison

- **GDP per capita vs. GDP per worker:**

$$\text{GDP per capita} = \frac{\text{GDP}}{\text{total population}}$$

$$\text{GDP per worker} = \frac{\text{GDP}}{\text{the number of labor force}}.$$

- The production of goods normally made in the factory is mainly done in the household in developing countries. Since GDP cannot measure the value of home production, GDP per capita may underestimate well-being of developing countries. Since GDP does not value home production, it may be reasonable to divide it by labor force.

Components of Expenditure

- GDP or Y, can be divided into consumption of domestic goods and services, C^d, investment in domestic goods and services, I^d, government purchases of domestic goods and services, G^d, and exports of domestic goods and services, EX:

$$Y = C^d + I^d + G^d + EX$$

- Consumption, C, investment, I, and government expenditure, G can be divided into domestic goods or foreign goods:

$$C = C^d + C^f, \ I = I^d + I^f, \ G = G^d + G^f$$

where superscript f means foreign goods.
- GDP

$$
\begin{aligned}
Y &= C + I + G + EX - \left(C^f + I^f + G^f \right) \\
&= C + I + G + EX - IM \\
&= C + I + G + NX
\end{aligned}
$$

Consumer Price Index

In order to analyze the changes in the overall cost of living, we need a single index measuring the overall level of prices.

- **Consumer Price Index:** Economists compute the price of a basket of goods and services purchased by a typical consumer. CPI is the price of this basket of goods and services relative to the price of the same basket in some base year.

  - **Example**: Consider a country in which typical consumers buy 5 apples and 10 bananas in a year during 2008 and 2009. Then the basket of goods consists of 5 apples and 10 bananas. Let us set 2008 as the base year. The CPI can be defined as follows:

$$CPI \text{ in } 2008 = 1$$

$$CPI \text{ in } 2001 = \frac{5 \times P_{A2009} + 10 \times P_{B2009}}{5 \times P_{A2008} + 10 \times P_{B2008}}$$

Consumer Price Index

- **CPI vs. the GDP Deflator:** Both CPI and GDP deflator measure overall price level. But there are three main differences.

1. The GDP deflator measures the prices of all goods and services produced; CPI measures the prices of only the goods and services bought by consumers.
2. The GDP deflator includes only goods produced in a country. It excludes imported goods. CPI includes imports goods if the consumers buy such goods.
3. CPI assigns fixed weights to the prices of different goods, the GDP deflator allows the basket of goods to change over time as the composition of GDP changes.

Despite these differences, CPI and the GDP deflator show a similar behavior.

The Unemployment Rate

The unemployment measures the percentages of people who want to work but who do not have jobs.

- **Unemployed Workers:** People are called unemployed when

1. They do not have a paid job.
2. They conducted a job seeking activity
3. If there is a job, they can do it soon (They are available).

- **Employed Workers:** People are called employed if they do have a paid job.
- **Labor force:** The sum of employed workers and unemployed workers are called labor force.
- **The unemployment rate:**

$$unemployment\ rate = \frac{number\ of\ unemployed\ workers}{labor\ force} \times 100$$

- **Labor Force Participation rate:**

$$labor\ force\ participation\ rate = \frac{labor\ force}{adult\ population} \times 100$$

where adult population is the number of people 16 years old or more.

- If a person is 16 years old or more and he is neither employed not unemployed, he is not in the labor force.

Assignment

- Students must hand assignment 1 in at the next lecture.

The Basic Framework of Macroeconomics

- This Chapter provides a basic framework of macroeconomics. This is an application of a general equilibrium analysis in the context of macroeconomics.

- Using this framework, I will construct the neoclassical growth model later and ask the following questions.
  1. Why are some countries rich; others poor?
  2. What is the source of long run growth?

- In the chapter 5, I applied this model to the analysis of stabilization policy.

The Basic Framework of Macroeconomics

- Representative Firm
  - Produce goods or services using Labor and Capital
  - Sell goods or services to household

- Representative Household
  - Buy goods or services
  - Provide firms with labor and capital

- Markets (Labor Market, Capital Market and Goods Market.)

Firm

- Firms is assumed to maximize its profits given an aggregate production function:

$$\Pi = \max_{K,L} \{PY - WL - RK\}$$
$$Y \leq F(K, L)$$

where P is a price, Y is output, W is a nominal wage rate, L is labor, R is a nominal rental price and K is capital stock.

Firm

- The property of the aggregate production function: $F(K, L)$ is constant returns to scale in K and L:

$$tF(K, L) = F(tK, tL), \text{ for } \forall t > 0. \tag{1}$$

- Why should the aggregate production function be constant returns to scale?
  - CRS means that whatever an individual production function is, if we use the same production technology twice, the output will be doubled. This might be a reasonable assumption for the aggregate production function. For example, assume that an individual plant has a production function $y = \phi(l)$. Assume that a manager establishes the same K plants. Then the aggregate output Y is

$$Y = yK = \phi(l)K$$

Define an aggregate production function production function F such that

$$F(K, L) \equiv \phi(l)K, \forall K > 0$$

where $L = lK$. Clearly this is constant return to scale in K and L.

Firm

- Constant returns to scale and production possibility set

$$
\begin{aligned}
Y &\leq F(K, L) \\
&= F\left(\frac{K}{L}, 1\right) L \\
y &\leq F(k, 1)
\end{aligned}
$$

where $y = \frac{Y}{L}$ and $k = \frac{K}{L}$. Define f

$$f(k) \equiv F(k, 1)$$

Then

$$y \leq f(k)$$

Firm

- Profit maximization Problem

$$
\begin{aligned}
\Pi &= \max_{K,L} \left\{ PY - WL - RK \right\} \\
&= \max_{k,L} \left\{ yL - \frac{W}{P}L - \frac{R}{P}kL \right\} P \\
&= \max_{L} \pi PL \\
\text{where } \pi &= \max_{k} \left\{ y - w - rk \right\}
\end{aligned}
$$

where $w = \frac{W}{P}$ and $r = \frac{R}{P}$. Therefore

$$
\begin{aligned}
\Pi &= \max_{L} \pi PL \\
\pi &= \max_{k} \left\{ y - w - rk \right\} \\
y &\leq f(k)
\end{aligned}
$$

Firm

- Assumptions on the aggregate production function

  1. $f(0) = 0$
  2. $f'(k) = \frac{df(k)}{dk} > 0$.

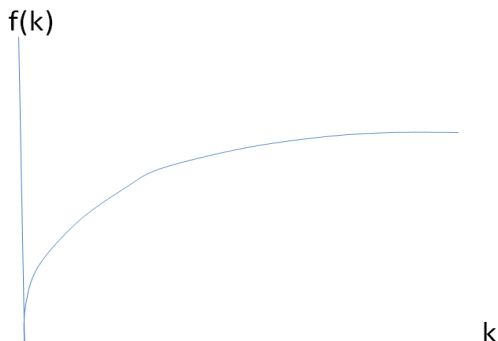   - When the firm employs more capital per workers, it increases output per workers.

  3. $f''(k) = \frac{d^2 f(k)}{dk^2} < 0$

     - This means that the marginal productivity of capital per workers is diminishing.

  4. Inada Conditions: technical conditions.

  $$\lim_{k \to 0} f'(k) = \infty, \lim_{l \to \infty} f'(k) = 0.$$

Aggregate Production Function

Firm

- Example: Cobb-Douglas Production Function: $f(k) = k^\alpha$

$$f'(k) = \alpha k^{\alpha-1} > 0 \Rightarrow \alpha > 0$$
$$f''(k) = \alpha(\alpha - 1) k^{\alpha-2} < 0 \Rightarrow \alpha < 1$$

Hence

$$\alpha \in (0, 1)$$

Firm

- Profit Maximization with respect to k

$$\pi = \max_k \{y - w - rk\}, \ s.t. \ y \leq f(k)$$
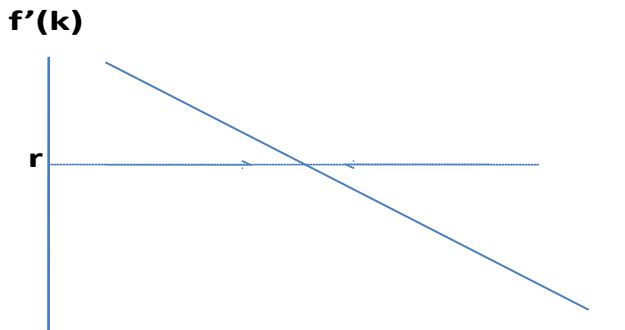$$\pi = \max_k \{f(k) - w - rk\}$$

- First Order Conditions

$$\frac{d\pi}{dk} = 0 \Rightarrow r = f'(k) \Rightarrow k \ is \ determined.$$

- π

$$\pi = f(k) - w - rk \Rightarrow \pi \ is \ determined.$$

Optimal Decision

Firm

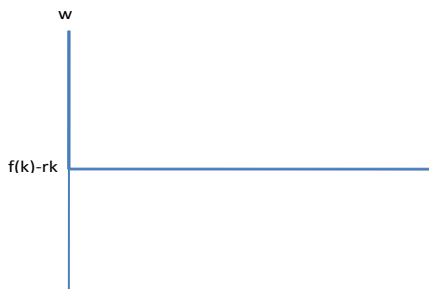- Profit maximization with respect to L

$$\Pi = \max_{L} \pi P L$$

- Labor Demand Function

$$
\begin{aligned}
L &= 0 \text{ if } \pi < 0, \ w > f(k) - rk \\
L &\in [0, \infty] \text{ if } \pi = 0, \ w = f(k) - rk \\
L &= \infty \text{ if } \pi > 0, \ w < f(k) - rk
\end{aligned}
$$

Labor Demand Function

Firm

- 0 Economic Profits

$$\Pi = \pi P L = 0$$

When the market is competitive, more entrepreneurs will enter as long as economic profits are positive. Hence, in the long run, economic profit is 0. Hence

$$\Pi = 0, L > 0 \Rightarrow \pi = 0 \Rightarrow w = f(k) - rk$$
$$Y = wL + rK$$

- Accounting Profits: The firm's revenue must be divided into wage payment, capital payment and economic profit:

$$Y = wL + rK + \Pi$$

where Π is economic profit. But in reality, a firm's owner owns capital also. Hence, we cannot distinguish economic profits from capital payment. It means

Accounting profit $= \Pi + rK$

Firm

- Example: Cobb-Douglas Production Function: $f(k) = k^\alpha$

$$
\begin{aligned}
r &= \alpha k^{\alpha - 1} \\
rk &= \alpha k^\alpha \\
\alpha &= \frac{rk}{k^\alpha} = \frac{rk}{y} = \frac{rK}{Y}
\end{aligned}
$$

$$
\begin{aligned}
w &= k^\alpha - rk \\
&= k^\alpha - \alpha k^\alpha \\
&= (1 - \alpha) k^\alpha \\
1 - \alpha &= \frac{w}{k^\alpha} = \frac{w}{y} = \frac{wL}{Y} \\
1 &= \frac{rK + wL}{Y}
\end{aligned}
$$

- Representative Households make decisions on
  1. How long they work,
  2. How much they consume today, and
  3. Where to invest.

Household

- How long do they work?: Assume that everybody works one unit of time no matter what wage rate is. Hence, the supply of labor is equal to total population.
- How much do they consume today?
  - If they do not consume today, they save for future. This is potentially a difficult question, because the decision depends not only on the current income but on the expected future income. I will leave the answer to this problem later and at this moment, I take the consumption per capita, c as given.
  - Budget Constraint: Nonetheless, chosen consumption has to be feasible. Hence, at least it must satisfy the following budget constraint.

$$a_{+1} + c = (1 + \rho^a) a + w$$

    where c is consumption per capita, ρ^a is the returns from investment, a is asset per capita and a_{+1} is an asset per capita at the next period.

Household

- Where do they invest?:There are several possibilities. They may save it in a bank. They can invest in firms' stock. They can also purchase investment goods such as house and rent it out.

    1. So far we assume that a firm rent capital. So the firm does not own capital. Moreover, economic profits of the firms are 0. Hence, there is no value on a firm.
    2. It means that household has two choices:
        1. to save it in a bank. Then they can earn a safe return, interest rate, ρ.
        2. to purchase investment goods and rent it out. Then it expects to earn r from a unit of investment. In addition, as they are the owner of capital, if the capital depreciates, the must bear the cost. Suppose that δ proportion of capital is depreciated. Then real return from purchasing investment goods is $r - \delta$.
    3. Assume that there are lots of investment opportunities so that household hedges idiosyncratic risk. Moreover, we assume that there is no adjustment cost of investment. Then the optimal condition is

$$\rho^a = \max\{r - \delta, \ \rho\}$$

Household

- Arbitrage Condition:
  1. If $r - \delta > \rho$, everybody buys investment goods. But then, nobody saves in a bank and the interest rate would become larger.
  2. If $r - \delta < \rho$, nobody buys investment goods. But then, everybody saves in a bank and the interest rate becomes smaller.
  3. In the equilibrium,
     $$r - \delta = \rho = \rho^a$$
     This is called an arbitrage condition.

Market Clearing Condition

- Labor Market Clearing Condition

$$L = N$$

- Capital Market Clearing Condition

$$K = aN$$

  - Note that even if banks collect assets from household, the banks must purchase investment goods. Hence, every assets in the economy is used for investment in capital goods.

- Hence capital market clearing condition and labor market clearing condition implies

$$k = \frac{K}{N} = a$$

Equilibrium

Given (c, a), a market equilibrium consists of $(y, k, a_{+1}, \rho, r, w)$ which satisfies

1. A Firm's Profit Maximization and the Production Function

$$
\begin{aligned}
y &= f(k) \\
r &= f'(k) \\
w &= f(k) - rk
\end{aligned}
$$

2. A Consumer's Budget Constraint

$$
a_{+1} + c = (1 + \rho^a) a + w
$$

3. An Arbitrage Condition

$$
r - \delta = \rho
$$

4. Capital and Labor market clearing conditions

$$
k = a
$$

- **What happens to a goods market?** A goods market is supposed to equate demand for output and supply of output and determine the price of the goods, P. However, note that $w = \frac{W}{P}$ and $r = \frac{R}{P}$. Hence, the wage rate and rental price are not measured by nominal term, but measured by the unit of output. Even if W, R and P double, w and r keep the same value. Hence, there is no change in our economy. In order to make our decisions, we care about the relative prices. We do not need information on the absolute prices. Hence, the price of a product can be set 1 without a loss of generality. In other words, the market that is supposed to determine the price is redundant.

Goods Market

- In order to prove the above statement, we derive goods market from equilibrium conditions. First, we derive a resource constraint. Second, we derive the flow expression of capital market. Combining two equations, we derive goods market.
- Budget clearing condition implies that

$$
\begin{aligned}
a_{+1} + c &= \left(1 + \rho^a\right) a + w \\
k_{+1} + c &= \left(1 + r - \delta\right) k + f\left(k\right) - rk \\
&= f\left(k\right) + \left(1 - \delta\right) k \\
y_t &= \left[k_{+1} - \left(1 - \delta\right) k\right] + c
\end{aligned}
$$

Define investment, i as

$$
i \equiv k_{+1} - \left(1 - \delta\right) k
$$

Hence,

$$
y_t = i_t + c_t \Rightarrow Y_t = I_t + C_t
$$

- Students must hand assignment 2 in at the next lecture.

Economic Growth and Nations' Income

1. Based on the previous framework, I first derive so called, Solow Model. This model is a useful starting point for the analysis of economic growth.

2. Solow model predicts that eventually economic growth converges to 0. This is not what we observe in data. In order to match the theory with data, we introduce an exogenous technology growth and population growth, and compare with the stylized facts about economic growth observed in many OECD countries.

3. Next, using the extended Solow model, we ask a question: can the model explain large differences in income across countries?

4. From the above exercises, we recognize the importance of productivity. Understanding productivity is the issue we discuss later.

Solow Model

- Capital accumulation equation: Assume that x_t means x at date t. Capital accumulation equation is

$$
\begin{aligned}
k_{t+1} &= i_t + (1 - \delta)\, k_t \\
&= y_t - c_t + (1 - \delta)\, k_t \\
&= f\left(k_t\right) - c_t + (1 - \delta)\, k_t
\end{aligned}
$$

Investment makes capital stock bigger, but in order to make large investment, households cannot enjoy consumption today very much. This is a basic trade-off in Solow model.

- Assumption on c_t:

$$
c_t = (1 - s)\, y_t
$$

Because $s_t = y_t - c_t$ is the definition of saving,

$$
s_t = s y_t
$$

The parameter s represents saving rate.

Solow Model

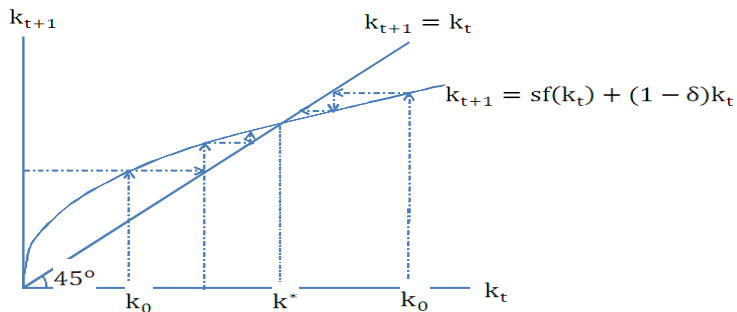- Substituting the consumption function into the capital accumulation equation, we have

$$
\begin{aligned}
k_{t+1} &= f(k_t) - (1-s) y_t + (1-\delta) k_t \\
&= f(k_t) - (1-s) f(k_t) + (1-\delta) k_t
\end{aligned}
$$

Hence,

$$
k_{t+1} = sf(k_t) + (1-\delta) k_t.
$$

This is Solow model.

Solow Model

Solow Model

Definition

The steady state is the points at which $\{(c_t^*, y_t^*, k_t^*)\}$ satisfies

$$c_{t+1}^* = c_t^*, \ y_{t+1}^* = y_t^*, \ k_{t+1}^* = k_t^*$$

- For any initial capital stock, economy eventually converges to the steady state. It means
    1. Economic growth rate eventually converges to 0.
    2. If the steady state is the same across countries, the poor countries eventually catch up.
- A key assumption to derive this result is $f''(k) < 0$.

Solow Model

- On the steady state,

$$
\begin{aligned}
k^* &= sf(k^*) + (1-\delta)k^* \\
sf(k^*) &= \delta k^* \\
\frac{k^*}{f(k^*)} &= \frac{s}{\delta}
\end{aligned}
$$

- Example: $f(k) = k^\alpha$

The Review of High School Mathematics

- Remember

$$
\begin{aligned}
k^{\alpha+\beta} &= k^{\alpha} k^{\beta} \\
(k^{\alpha})^{\beta} &= k^{\alpha\beta} \\
(kl)^{\alpha} &= k^{\alpha} l^{\alpha} \\
k^{-1} &= \frac{1}{k}
\end{aligned}
$$

- Therefore

$$
\begin{aligned}
k^{\alpha-\beta} &= k^{\alpha} k^{-\beta} = k^{\alpha} \left(k^{\beta}\right)^{-1} = \frac{k^{\alpha}}{k^{\beta}} \\
\left(\frac{k}{l}\right)^{\alpha} &= k^{\alpha} \left(\frac{1}{l}\right)^{\alpha} = k^{\alpha} l^{-\alpha} = k^{\alpha} \left(l^{\alpha}\right)^{-1} = \frac{k^{\alpha}}{l^{\alpha}}
\end{aligned}
$$

Solow Model

- Example: $f(k) = k^{\alpha}$

$$\frac{s}{\delta} = \frac{k^*}{f(k^*)} = \frac{k^*}{(k^*)^{\alpha}} = (k^*)^{1-\alpha}$$

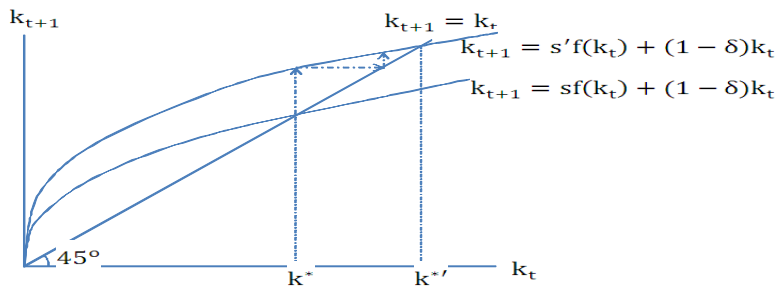$$k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = (k^*)^{\alpha} = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$c^* = (1-s)\, y^* = (1-s)\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$
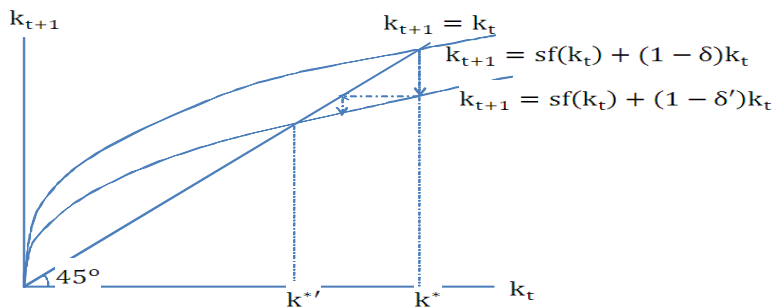
Solow Model

- The impact of s:

$$s' > s$$

Solow Model

- The impact of δ:

$$\delta' > \delta$$

k_{t+1}

$k_{t+1} = k_t$

$k_{t+1} = sf(k_t) + (1 - \delta)k_t$

$k_{t+1} = sf(k_t) + (1 - \delta')k_t$

45°

$k^{*'}$ k^* k_t

Solow Model

- An increase in s and a decrease in δ increase k^* and, therefore, y^*.
- Both changes cannot influence the long run growth rate.

The Extended Solow Model

- Solow model predicts that economic growth rate eventually converges to 0. This is not what we observe in data.
- In order to match the theory with data, we introduce technology growth and population growth.
  - Assume that

$$
\begin{aligned}
Y_t &= F\left(K_t, T_t N_t\right) \\
T_{t+1} &= \left(1 + g\right) T_t \\
N_{t+1} &= \left(1 + n\right) N_t
\end{aligned}
$$

  where T_t and N_t denote technology and population at date t, respectively. We assume labor augmenting technological progress.
  - Labor augmenting technological progress means that an increase in the technology has the same effect as an increase in population.
  - We assume the labor augmenting technological progress, because it leads the theory to data.

- GDP per unit of effective labor

$$
\begin{aligned}
Y_t &= F\left(K_t, T_t N_t\right) \\
&= F\left(\frac{K_t}{T_t N_t}, 1\right) T_t N_t \\
y_{et} &= f\left(k_{et}\right) \equiv F\left(k_{et}, 1\right)
\end{aligned}
$$

where $y_{et} = \frac{Y_t}{T_t N_t}$ and $k_{et} = \frac{K_t}{T_t N_t}$ are GDP per unit of effective labor and capital per unit of effective labor.
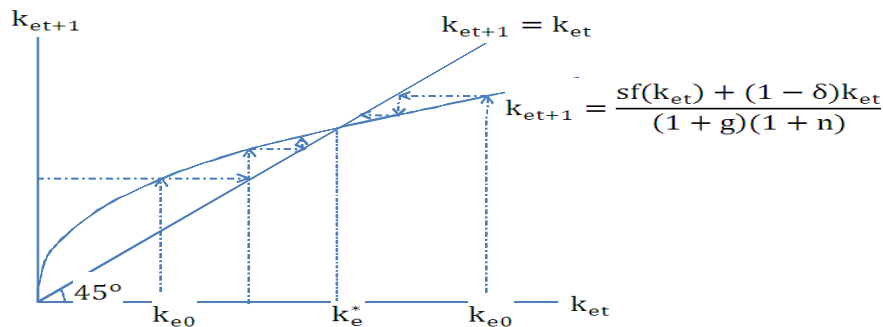
The Extended Solow Model

- Capital Accumulation

$$
\begin{aligned}
K_{t+1} &= I_t + (1-\delta) K_t \\
&= Y_t - C_t + (1-\delta) K_t \\
&= Y_t - (1-s) Y_t + (1-\delta) K_t \\
&= s Y_t + (1-\delta) K_t \\
&= s y_{et} T_t N_t + (1-\delta) k_{et} T_t N_t \\
\frac{K_{t+1}}{T_{t+1} N_{t+1}} \frac{T_{t+1} N_{t+1}}{T_t N_t} &= s y_{et} + (1-\delta) k_{et} \\
k_{et+1} (1+g)(1+n) &= s f(k_{et}) + (1-\delta) k_{et}
\end{aligned}
$$

- The extended Solow model

$$
k_{et+1} = \frac{s f(k_{et}) + (1-\delta) k_{et}}{(1+g)(1+n)}
$$

The Extended Solow Model

The Extended Solow Model

Definition

The steady state is the points at which $\{(c_e^*, y_e^*, k_e^*)\}$ satisfies

$$c_{et+1}^* = c_{et}^*, \ y_{et+1}^* = y_{et}^*, \ k_{et+1}^* = k_{et}^*$$

where $c_{et} = \frac{C_t}{T_t N_t}$, $y_{et} = \frac{Y_t}{T_t N_t}$, and $k_{et} = \frac{K_t}{T_t N_t}$.

- For any initial capital stock, economy eventually converges to the steady state.
- A key assumption to derive this result is also $f''(k) < 0$.

The Extended Solow Model

- On the steady state k^* must satisfy

$$k_e^* = \frac{sf\left(k_e^*\right) + (1-\delta)\, k_e^*}{(1+g)\,(1+n)}$$

$$(1+g)\,(1+n)\, k_e^* = sf\left(k_e^*\right) + (1-\delta)\, k_e^*$$

$$(1+g+n+gn)\, k_e^* = sf\left(k_e^*\right) + (1-\delta)\, k_e^*$$

$$sf\left(k_e^*\right) = (g+n+\delta+gn)\, k_e^*$$

The Extended Solow Model

- Example $f(k^e) = (k^e)^\alpha$

$$
\begin{aligned}
s(k_e^*)^\alpha &= (g + n + \delta + gn)\, k_e^* \\
\frac{k_e^*}{(k_e^*)^\alpha} &= \frac{s}{g + n + \delta + gn} \\
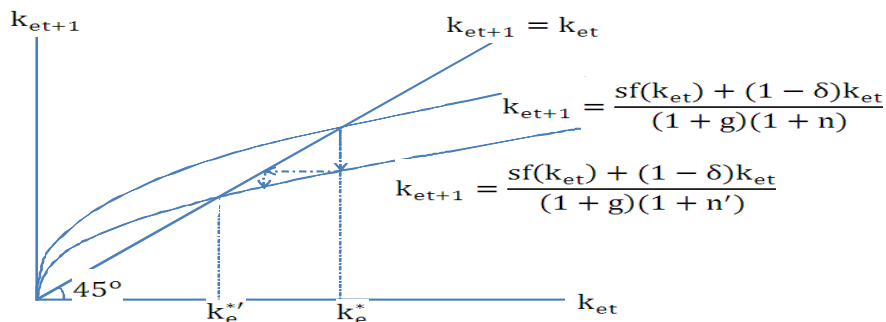(k_e^*)^{1-\alpha} &= \frac{s}{g + n + \delta + gn} \\
k_e^* &= \left[\frac{s}{g + n + \delta + gn}\right]^{\frac{1}{1-\alpha}}
\end{aligned}
$$

- The Impact of n :

$$n'>n$$

Figure showing k_{et+1} on the vertical axis and k_{et} on the horizontal axis, with a $45°$ line labeled $k_{et+1} = k_{et}$, and two curves:

$$k_{et+1} = \frac{sf(k_{et}) + (1-\delta)k_{et}}{(1+g)(1+n)}$$

$$k_{et+1} = \frac{sf(k_{et}) + (1-\delta)k_{et}}{(1+g)(1+n')}$$

with equilibrium points $k_e^{*'}$ and k_e^{*}.

The Extended Solow Model

- An increase in n lowers, k_e^* and therefore, y_e^*.

The Extended Solow Model

- Golden Rule Level of Capital Stock

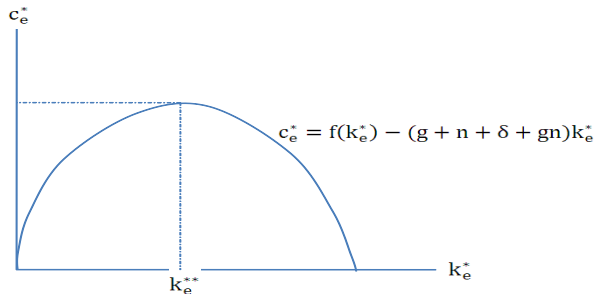$$sf\left(k_e^*\right) = \left(g + n + \delta + gn\right)k_e^*$$

$$
\begin{aligned}
c_e^* &= (1-s)\,f\left(k_e^*\right) \\
&= f\left(k_e^*\right) - \left(g + n + \delta + gn\right)k_e^*
\end{aligned}
$$

$$
\begin{aligned}
\frac{dc_e^*}{dk_e^*} &= f'\left(k_e^{**}\right) - \left(g + n + \delta + gn\right) = 0 \\
\frac{d^2 c_e^*}{d\left(k_e^*\right)^2} &= f''\left(k_e^*\right) < 0
\end{aligned}
$$

- k_e^{**} is the capital stock that maximizes consumption on the steady state.

The Golden Rule Level of Capital Stock

$$c_e^* = f(k_e^*) - (g + n + \delta + gn)k_e^*$$

The Extended Solow Model

- Example: $f(k_e) = (k_e)^\alpha$

$$\alpha (k_e^{**})^{\alpha-1} = g + n + \delta + gn$$
$$(k_e^{**})^{1-\alpha} = \frac{\alpha}{g + n + \delta + gn}$$
$$k_e^{**} = \left[\frac{\alpha}{g + n + \delta + gn}\right]^{\frac{1}{1-\alpha}}$$

Note that

$$k_e^* = \left[\frac{s}{g + n + \delta + gn}\right]^{\frac{1}{1-\alpha}}$$

Hence if $s = \alpha$, the golden rule level of capital stock is attained.

Kaldor's Stylized Facts (1963)

- Kaldor (1963) pointed out 6 stylized facts of economic growth. These facts are repeatedly observed by aggregate data of OECD countries.
- I would like to examine how the extended Solow growth model explains these stylized facts.
- As the extended Solow model predicts that economy eventually converges to the steady state, we expect that the behavior of real economy can be approximated by the behavior on the steady state. Hence, we compare the prediction of the theory on the steady state with Kaldor's facts.

Mathematical Preparation for the Analysis of Growth Rate

- Definition
$$\ln k \equiv \log_e k$$
where $e = \lim_{m \to \infty} \left(1 + \frac{1}{m}\right)^m = 2.71828...$ It is useful to use e as a base. The useful properties of e and $\ln$ are

$$\frac{de^t}{dt} = e^t, \ \frac{d \ln k}{dk} = \frac{1}{k}$$
$$If \ g \approx 0, \ \ln(1 + g) \approx g$$

- The review of high school mathematics

$$\ln kl = \ln k + \ln l$$
$$\ln k^\alpha = \alpha \ln k$$
$$\ln e = 1, \ln 1 = 0$$

Therefore

$$\ln \frac{k}{l} = \ln kl^{-1} = \ln k + \ln l^{-1} = \ln k - \ln l$$

Mathematical Preparation for the Analysis of Growth Rate

Lemma

Suppose that the growth rate of x, $g_x \approx 0$, then g_x is nearly expressed as follows

$$g_x \approx \ln x_{t+1} - \ln x_t$$

Proof.

Suppose that $x_{t+1} = (1 + g_x) x_t$,

$$
\begin{aligned}
\ln x_{t+1} - \ln x_t &= \ln (1 + g_x) x_t - \ln x_t \\
&= \ln (1 + g_x) + \ln x_t - \ln x_t \\
&= \ln (1 + g_x) \\
&\approx g_x \text{ if } g_x \approx 0
\end{aligned}
$$

$\square$

- **Remark:** If the time interval is not 1 but is almost 0, we can prove that the relationship is exact for any g_x:

  - Define g_x so that $x_{t+\Delta} = (1 + g_x\Delta)\, x_t$. Then, from the previous lemma, for any g_x there is a small Δ so that $g_x\Delta \approx \ln x_{t+\Delta} - \ln x_t$. It is shown that for any g_x,

$$\lim_{\Delta \to 0} \frac{\ln x_{t+\Delta} - \ln x_t}{\Delta} = \frac{d\ln x_t}{dt} = g_x, \; where \; g_x \equiv \frac{\frac{dx_t}{dt}}{x_t}$$

    Hence, we treat $g_x \approx \ln x_{t+1} - \ln x_t$ as if $g_x = \ln x_{t+1} - \ln x_t$ below.

Lemma

$$g_{xy} \approx g_x + g_y$$
$$g_{\frac{x}{y}} \approx g_x - g_y$$
$$g_{x^\alpha} \approx \alpha g_x$$

Proof.

$$
\begin{aligned}
g_{xy} &\approx \ln x_{t+1} y_{t+1} - \ln x_t y_t \\
&= \ln x_{t+1} + \ln y_{t+1} - \left[\ln x_t + \ln y_t \right] \\
&= \ln x_{t+1} - \ln x_t + \ln y_{t+1} - \ln y_t \\
&\approx g_x + g_y
\end{aligned}
$$

□

Proof.

$$
\begin{aligned}
g_{x^\alpha} &\approx \ln x_{t+1}^\alpha - \ln x_t^\alpha \\
&= \alpha \ln x_{t+1} - \alpha \ln x_t \\
&= \alpha \left[\ln x_{t+1} - \ln x_t \right] \\
&\approx \alpha g_x
\end{aligned}
$$

$$
\begin{aligned}
g_{\frac{x}{y}} &= g_{xy^{-1}} \\
&\approx g_x + g_{y^{-1}} \\
&\approx g_x - g_y
\end{aligned}
$$

$\square$

Kaldor's Stylized Facts (1963)

- *The growth rate of GDP per capita is nearly constant:*

$$g_y = g_{y_e^* T} \approx g_{y_e^*} + g_T = g_T = g$$

- *The growth rate of capital per capita is nearly constant:.*

$$g_k = g_{k_e^* T} \approx g_{k_e^*} + g_T = g_T = g$$

- *The growth rate of output per worker differs substantially across countries.*

Hence in order to fit the theory to data, g must be constant and differs among countries.

Kaldor's Stylized Facts (1963)

- *The rate of return to capital is nearly constant:*

$$r_t = f'\left(k_e^*\right) = const$$

- *The ratio of physical capital to output is nearly constant:*

$$\frac{K_t}{Y_t} = \frac{k_e^* T_t N_t}{y_e^* T_t N_t} = \frac{k_e^*}{y_e^*} = const$$

- *The shares of labor and physical capital are nearly constant:* Note that the marginal productivity of capital and labor are

$$\frac{r_t K_t}{Y_t} = r_t \frac{K_t}{Y_t} = const$$

Note that

$$1 = \frac{r_t K_t + w_t L_t}{Y_t}$$

Hence

$$\frac{w_t L_t}{Y_t} = 1 - \frac{r_t K_t}{Y_t} = const$$

Growth Accounting

- Assume that

$$y_{et} = (k_{et})^{\alpha}.$$

Then

$$\frac{y_t}{T_t} = \left(\frac{k_t}{T_t}\right)^{\alpha}$$
$$y_t = T_t^{1-\alpha}(k_t)^{\alpha}$$

Hence

$$g_{y_t} = g_{T_t^{1-\alpha}k_t^{\alpha}} \approx (1-\alpha)\,g_{T_t} + \alpha g_{k_t}$$

$$g_{y_t} = \alpha g_{k_t} + R(t),\ \text{where } R(t) = (1-\alpha)\,g_{T_t}$$

Growth Accounting

- Note that α can be estimated as follows:

$$\alpha = \frac{\frac{r_t K_t}{Y_t} + \frac{r_{t+1} K_{t+1}}{Y_{t+1}}}{2} = 1 - \frac{\frac{w_t N_t}{Y_t} + \frac{w_{t+1} N_{t+1}}{Y_{t+1}}}{2}$$

So we can estimate

$$R\left(t\right) = g_{y_t} - \frac{\frac{r_t K_t}{Y_t} + \frac{r_{t+1} K_{t+1}}{Y_{t+1}}}{2} g_{k_t} = g_{y_t} - \left(1 - \frac{\frac{w_t N_t}{Y_t} + \frac{w_{t+1} N_{t+1}}{Y_{t+1}}}{2}\right) g_{k_t}$$

- $R\left(t\right)$ is called the Solow residual or the growth rate of the Total Factor Productivity (TFP). It reflects that all sources of growth other than the contribution of capital accumulation.

Growth Accounting

- Remarks
  1. The equation for Solow residual can be derived without assuming $f(k) = k^\alpha$. We don't need constant returns to scale as well.
  2. This is the simplest case. Usually, we can include other inputs as well.
  3. Growth accounting can be applied for several analysis.
     1. Young (1994) finds that the high growth rate of Hong Kong, Singapore, South Korea, and Taiwan over the past three decades is almost entirely due to rising investment, increase in labor force participation, and increase in the level of education, but not to rapid technological progress.
     2. Productivity slow down puzzle. $R(t)$ became small after 1970s.

Solow Model and Income Differences

- In order for the Solow model on the steady state to explain the long run behavior of developed countries, we need to assume that g is constant and differs across countries.
- In this section, we ask a question whether the Solow model can explain the development facts.
  - First, I show what the new stylized facts are.
  - The next, I would like to ask if the Solow model can explain these facts.

Development Facts

- Parente and Prescott (1993) pointed out four main stylized facts.

  1. Income difference across countries is large.
  2. Wealth distribution has shifted up.
  3. Relative Income distribution does not show convergence.
  4. There have been development miracles and disasters.

- Durlauf and Quah (1998) also pointed out that

  1. Relative Income distribution across countries shows two peaks.

Development Facts

- Acemoglu (2009) points out that
  1. The relative ranking of countries has changed little between 1960 and 2000.
  2. The origins of the current cross-country differences in income per capita occur between the late eighteen century and early twentieth centuries.

- Barro and Sala–i–Martin (1995) shows that
  1. There is no evidence of absolute convergence.
  2. There is evidences of conditional convergence.

Can the Solow model explain large income differences?

- Let me first examine whether the Solow model explains the first stylized fact: large income differences.

- Let me start with simple exercises.

  1. Suppose that T is the same across countries.
  2. Ask whether differences in capital stock per capita alone can explain differences in income per capita.

Can the Solow model explain large income differences?

- *Calibration Exercises 1 (Lucas (1990))*:

$$y_e = (k_e)^\alpha \Rightarrow k_e = (y_e)^{\frac{1}{\alpha}}$$

Choose two arbitrary countries, say US and India.

$$\frac{k_{us}\frac{1}{T}}{k_{India}\frac{1}{T}} = \frac{\left(y_{us}\frac{1}{T}\right)^{\frac{1}{\alpha}}}{\left(y_{india}\frac{1}{T}\right)^{\frac{1}{\alpha}}} \Rightarrow \frac{k_{us}}{k_{India}} = \left[\frac{y_{us}}{y_{india}}\right]^{\frac{1}{\alpha}}$$
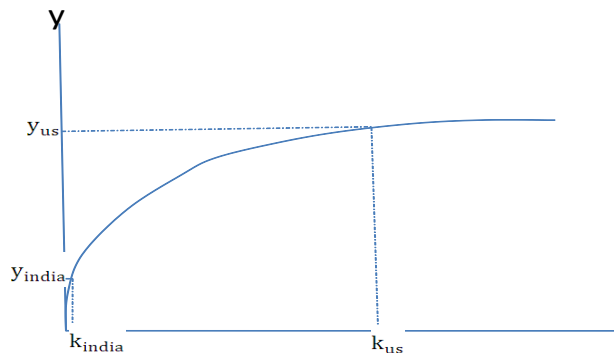
According to Penn World Table 6.3, $\frac{y_{us}}{y_{india}} \approx 10$ in year 2007 and $\alpha = \frac{1}{3}$,

$$\frac{k_{us}}{k_{India}} = 10^3 = 1000$$

In order to explain large income differences, required differences in capital are too large.

Why are Required Capital Differences so Large?

Can the Solow model explain large income differences?

- *Calibration Exercises 2 (Lucas (1990))*:

$$MPK = f'(k_e) = \alpha k_e^{a-1} = \alpha (y_e)^{\frac{\alpha-1}{\alpha}}$$

Choose again US and India

$$\frac{MPK_{us}}{MPK_{India}} = \frac{\alpha \left(y_{us} \frac{1}{T}\right)^{\frac{\alpha-1}{\alpha}}}{\alpha \left(y_{India} \frac{1}{T}\right)^{\frac{\alpha-1}{\alpha}}} = \left[\frac{y_{us}}{y_{india}}\right]^{\frac{\alpha-1}{\alpha}}$$
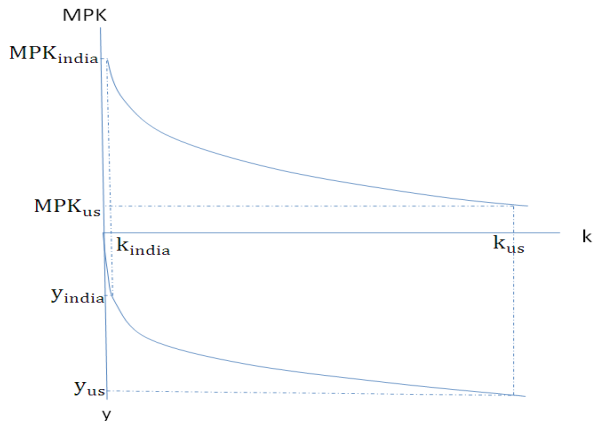
Since $y_{us}/y_{India} \approx 10$ and $\alpha = \frac{1}{3}$,

$$\frac{MPK_{us}}{MPK_{India}} = [10]^{\frac{\frac{1}{3}-1}{\frac{1}{3}}} = [10]^{\frac{-\frac{2}{3}}{\frac{1}{3}}} = [10]^{-2} = \frac{1}{100}$$

Attributing difference in output to difference in capital implies a huge variation in the rate of return on capital.

Can the Solow model explain large income differences?

Why are Required MPK Differences are so Large?

- Both examples indicate that
    - When $\alpha = \frac{1}{3}$, without productivity differences, the theory cannot explain large income differences.
    - If $\alpha \approx 1$, it may be possible to explain data. This may indicate the existence of unmeasured capital stock.

Can the Solow model explain large income differences?

- *Cross Country Regression (Mankiw, Romer and Weil (1992)):* Assume that every country has the same production function, $f(k_e) = k_e^\alpha$ and that all countries are on its steady state, then it is shown before that

$$
\begin{aligned}
k_e^* &= \left[\frac{s}{g + n + \delta + gn} \right]^{\frac{1}{1-\alpha}} \\
&\approx \left[\frac{s}{g + n + \delta} \right]^{\frac{1}{1-\alpha}}
\end{aligned}
$$

Hence

$$
\begin{aligned}
y_e &= (k_e^*)^\alpha \Rightarrow \frac{y_t}{T_t} = (k_e^*)^\alpha \Rightarrow y_t = T_t (k_e^*)^\alpha \\
y_t &= T_t \left[\frac{s}{g + n + \delta} \right]^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

Can the Solow model explain large income differences?

- Empirically Testable Equation

$$\ln y_t = \ln T_t + \frac{\alpha}{1-\alpha} \ln (s) - \frac{\alpha}{1-\alpha} \ln (g + n + \delta)$$

Suppose that $t = 0$ and

$$\ln T_0 = a + \varepsilon_i$$

where a is constant and ε_i is country specific shock.

$$\ln y_0 = a + \frac{\alpha}{1-\alpha} \ln (s_i) - \frac{\alpha}{1-\alpha} \ln (n_i + g + \delta) + \varepsilon_i$$

Assume that g and δ is constant, 0.05, across countries and s and n are independent of ε_i. This assumption implies that there are no productivity differences other than luck.

- *Data:* Penn World Table: Non-Oil countries, Non-Oil countries except for grade D countries and small population countries, OECD countries.
  1. n.. the average growth of working age population over 1960-1985, where working age is defined as 15 to 64.
  2. s...the average share of real investment in real GDP over 1960-1985.
  3. y...real GDP in 1985 divided by the working age population in that period.

Can the Solow model explain large income differences?

- The following 4 results are obtained by them.
    1. The coefficients on saving and population growth have predicted signs and 2 of 3 are significant.
    2. The restriction that the absolute values of the coefficients of $\ln(s)$ and $\ln(g + n + \delta)$ are the same cannot be rejected.
    3. High R^2.
    4. The estimated α is much higher than $1/3$.

Can the Solow model explain large income differences?

- **Implication from results:** Although the Solow model has qualitatively correct predictions, but quantitatively, α is too small to explain huge income differences. This indicates the existence of unmeasured capital.

- **The problem of the estimation:** If productivity differences are not random, ε_i is correlated with s and n. The estimated parameters are biased upward. It indicates that the true α may be lower than the estimated α.

- **Conclusion:** Including the unmeasured capital as a part of T, evidence suggests that T must differ across countries in order to explain the large income differences.

Assignment

- Students must hand assignment 3 in at the following lecture.

Knowledge Accumulation and the Source of the Long run Growth

- As I have shown, the long run growth rate is determined by g on the Solow growth model. It is natural to ask what determines g.

- This is the main question investigated by endogenous growth models. There are several different models: education, learning by doing, R&D and so on.

- But they roughly share the same spirits: the long run productivity growth is the results of the accumulation of knowledge.

- In this section, I review some arguments on the knowledge accumulation and the long run growth.

Knowledge Accumulation and the Source of the Long run Growth

- **The Nature of Knowledge:**
  1. Knowledge is nonrival. Once somebody invents new idea, others can imitate it.
  2. Knowledge is partially excludable. Using legal or possibly informal mechanisms, we can partially exclude someone to use the knowledge.
     1. Ex. Patent and Specific Capital

- **The Implication for Knowledge Accumulation**
  1. Without a reasonable institutional system to prevent others from copying a new idea, nobody has an incentive to invent new knowledge.
  2. Knowledge accumulation must have a scale effect.

Knowledge Accumulation and the Source of the Long run Growth

- Scale Effect: Many growth models share the following knowledge accumulation function

$$T_{t+1} = BN_t^T T_t + T_t$$

where N_t^T is the number of workers who work at a knowledge accumulation sector. This equation implicitly assumes that the larger the population at the knowledge accumulation sector, the higher the probability to find new invention. Because of externality, past knowledge T_t, has a positive impact on the creation of new knowledge.

- The above equation implies that

$$(1 + g_T) T_t = BN_t^T T_t + T_t \Rightarrow g_T = BN_t^T$$

The growth rate of knowledge is proportional to the level of population. Hence, if a policy increases the number of researchers or engineers, it can increase long run economic growth rate.

Knowledge Accumulation and the Source of the Long run Growth

- **Jones Critique (1995):** Jones (1995) criticizes that this prediction is against evidence in the OECD countries.
  - World war II, we observe the number of scientists engaged in R&D has dramatically increased, but the growth rate of TFP is quite stable.
  - Jones (1995) proposed different specification:

$$T_{t+1} = BN_t^T T_t^{\beta} + T_t$$

where $0 < \beta < 1$. The above model implies

$$(1 + g_T) T_t = BN_t^T T_t^{\beta} + T_t$$
$$g_T = \frac{BN_t^T}{T_t^{1-\beta}}.$$

The main difference is that we have $T_t^{1-\beta}$ in the denominator. This makes a big difference. Note that

$$N_t^T \uparrow \Rightarrow g_{T_t} \uparrow \Rightarrow T_{t+1}^{1-\beta} \uparrow \Rightarrow g_{T_{t+1}} \downarrow$$

Knowledge Accumulation and the Source of the Long run Growth

- **Jones Critique (1995):** We investigate the impacts of Jones specification on the steady state. On the steady state $g_T = g$.

$$g_{g_T} = g_{\frac{B N_t^T}{T_t^{1-\beta}}}$$

$$g_g = g_B + g_{N^T} - (1 - \beta) g_T$$

$$0 = g_{N^T} - (1 - \beta) g$$

$$g = \frac{n^T}{1 - \beta}, \text{ where } n^T = g_{N^T}.$$

- The new model implies that the growth rate is proportional to population growth at the knowledge accumulation sector. A it would be more difficult for a government to influence the growth rate of population, the model implies that the long run growth rate is more or less exogenous.

Knowledge Accumulation and the Source of the Long run Growth

- **Jones (2002):** Jones (2002) decomposes n^T into two parts. Because $N_t^T = h_t N_t$ where $h_t = \frac{N_t^T}{N_t}$,

$$n^T = g_{N^T} = g_{hN} = g_h + g_N = g_h + n$$

Jones (2002) documented that a rise in educational attainment and research intensity can explain 80 % of recent U.S. growth; population growth explains less than 20 percent. Note that an increase in h_t cannot continue indefinitely since it is bounded by 1. Hence, his model predicts that sooner or later, the world growth rate must decrease to the level of population growth. Of course, as the knowledge spillover goes beyond a country and many developing countries will increase the proportion of scientists and engineers in future, potentially g_h can continue to be positive in the middle run.

Investment Specific Technological Change

- We still don't know what the knowledge is. But, there is evidence that knowledge embodied in investment might be the important factor of knowledge accumulation.

    - The relative price of equipment falls by about 4 % in the U.S..

- Apparently, Solow (1957) model cannot explain this evidence, because the relative price of investment is always equal to 1 by Solow (1957) model.

$$Y = C + I$$

- There is another evidence that Solow (1957) model cannot explain.

    - Productivity growth slows down after 1970s. This evidence is odd because we observe more new technology after 1970s. It gives a question on what Solow residual captures.

- Motivated by data, researchers start to pay attention to the investment specific technological change.

Investment Specific Technological Change

- Suppose that $T = 1$. let us consider the following two sector model.
- Consumption Goods Sector

$$\max_{L_t^c} \pi_t L_t^c$$

$$\pi_t^c = \max_{k_t} \left\{ c_t - w_t - r_t k_t^c \right\}, \; c_t = f\left(k_t^c\right)$$

- First Order Conditions

$$r_t = f'\left(k_t^c\right)$$
$$w_t = f\left(k_t^c\right) - r_t k_t^c$$
$$= f\left(k_t^c\right) - f'\left(k_t^c\right) k_t^c$$

- $\frac{w_t}{r_t}$

$$\frac{w_t}{r_t} = \frac{f\left(k_t^c\right) - f'\left(k_t^c\right) k_t^c}{f'\left(k_t^c\right)}$$

Investment Specific Technological Change

- Investment Goods Sector

$$\max_{L_t^i} \pi_t^i L_t^i$$

$$\pi_t^i = \max_{k_t} \left\{ p_t i_t - w_t - r k_t^i \right\}, \ i_t = q_t f\left(k_t^i\right)$$

where q_t is the investment specific productivity.

- First Order Conditions

$$r_t = p_t q_t f'\left(k_t^i\right)$$

$$w_t = p_t q_t f\left(k_t^i\right) - r_t k_t^i$$

$$= p_t q_t f\left(k_t^i\right) - p_t q_t f'\left(k_t^i\right) k_t^i$$

$$= p_t q_t \left[f\left(k_t^i\right) - f'\left(k_t^i\right) k_t^i \right]$$

- $\frac{w_t}{r_t}$

$$\frac{w_t}{r_t} = \frac{f\left(k_t^i\right) - f'\left(k_t^i\right) k_t^i}{f'\left(k_t^i\right)}$$

Investment Specific Technological Change

- Equilibrium
  - k_t^*

$$\frac{f\left(k_t^c\right) - f'\left(k_t^c\right) k_t^c}{f'\left(k_t^c\right)} \quad = \quad \frac{w_t}{r_t} = \frac{f\left(k_t^i\right) - f'\left(k_t^i\right) k_t^i}{f'\left(k_t^i\right)}$$

$$\Rightarrow \quad k_t^* \equiv k_t^c = k_t^i$$

  - p_t

$$f'\left(k_t^*\right) = r = p_t q_t f'\left(k_t^*\right) \Rightarrow p_t q_t = 1 \Rightarrow p_t = \frac{1}{q_t}$$

- Relative Price and Investment Specific Technological Change:

$$g_q = g_{\frac{1}{p}} = -g_p$$

  This means that the declining price of equipment indicates an improvement in the productivity of equipment. Because the price drops by 4%, the estimated improvement in the productivity of equipment is 4%. Hence, there is no slowdown in improvement in q.

Investment Specific Technological Change

- There are several issues related to investment specific technological change

  1. Creative Destruction:
     1. New technology may replace old technology.
     2. If so, workers or firms that skillfully use old technology may also be replaced.
     3. This may cause several social problems: resistance, unemployment and so on.

  2. Learning
     - Because technology is new, there is some leaning period. Therefore, initially, productivity may slow down.

  3. Skill Premium
     - If new technology replaces unskilled workers and demands skilled workers, the wage inequality increases.

Can Human Capital Explain Income Differences?

- In order to explain income differences across countries, we need to assume that T differs across countries.
- If the knowledge accumulation is the source of increase in T, why the poor countries do not imitate the knowledge in the developed countries?
- One possible explanation is that the use of knowledge demands human capital. Let me investigate this possibility.

Can Human Capital Explain Income Differences?

- *Hall and Jones (1999):* Assume that a country i has the production function:

$$Y_i = K_i^{\alpha} \left(T_i N_i \right)^{(1-\alpha)}$$

where $T_i = A_i h_i$. The variable A_i is the unobserved productivity and h_i is the level of human capital. Then

$$1 = \left(\frac{K_i}{Y_i} \right)^{\alpha} \left(\frac{T_i N_i}{Y_i} \right)^{(1-\alpha)}$$

$$y_i^{(1-\alpha)} = \left(\frac{K_i}{Y_i} \right)^{\alpha} \left(T_i \right)^{(1-\alpha)}$$

$$y_i = \left(\frac{K_i}{Y_i} \right)^{\frac{\alpha}{1-\alpha}} T_i$$

Hence

$$y_i = \left(\frac{K_i}{Y_i} \right)^{\frac{\alpha}{1-\alpha}} A_i h_i$$

Can Human Capital Explain Income Differences?

- y_i...National income and labor force data are taken from Summers and Heston (1991).
- They assume that

$$
\begin{aligned}
h_i &= \exp\left(0.134 \times E\right), \text{ if } E \leq 4, \\
&= \exp\left(0.134 \times 4 + 0.101 \times (E - 4)\right), \text{ if } 4 < E \leq 8, \\
&= \exp\left(0.134 \times 4 + 0.101 \times 4 + 0.068 \times (E - 8)\right), \text{ if } E > 8,
\end{aligned}
$$

where E is average educational attainment. The coefficients, 13.4, 10.1 and 6.8, are taken from previous research. Average educational attainment is measured in 1985 for the population aged 25 and over, as reported in Barro and Lee (1993).

Can Human Capital Explain Income Differences?

- Capital stock is estimated by the perpetual inventory method:

$$K_{t+1} = I_t + (1 - \delta) K_t$$

where δ is assumed to be 0.06. We can recursively estimate capital stock when we know the initial value. Note that

$$
\begin{aligned}
K_{t+1} - K_t + \delta K_t &= I_t \\
g_K + \delta &= \frac{I_t}{K_t} \\
K_t &= \frac{I_t}{g_K + \delta}
\end{aligned}
$$

Using this relationship, the initial value at 1960 is estimated by

$$K_{1960} = \frac{I_{1960}}{g + \delta}$$

where g is the average geometric growth rate from 1960 to 1970.
- The parameter, α, is assumed to be $\frac{1}{3}$ and the variable, A_i, is estimated by the residual.

Can Human Capital Explain Income Differences?

- Results
    1. Output per worker in the five richest countries is 31.7 times higher than output per worker in the five lowest countries.
    2. Capital intensity, human capital, productivity in the five richest countries are 1.8, 2.2 and 8.3 times larger than those in the five lowest countries, respectively.
    3. It shows that including human capital does not help much explaining a huge income differences.

- Questions on *Hall and Jones (1999)*:Clearly, school quality, on the job training, child-rearing, and prenatal care vary across countries. How can we estimate these effects?

Can Human Capital Explain Income Difference?

- *Hendricks (2002):* Hendricks use the wage data of immigrants in the US to estimate relative human capital. Assume that production function is

$$y_e = (k_e)^\alpha \Rightarrow \frac{Y}{AhN} = \left(\frac{K}{AhN}\right)^\alpha \Rightarrow Y = (K)^\alpha (Ah)^{1-\alpha} N^{1-\alpha}$$

Then

$$w = MPL = (1-\alpha)(K)^\alpha (Ah)^{1-\alpha} N^{-\alpha}$$

where $k_e = \frac{K_{us}}{A_{us} h_i N_{us}}$. Let me choose two countries, US and India.

$$\frac{w_{us}}{w_{india}} = \frac{(1-\alpha)(K_{us})^\alpha (A_{us} h_{us})^{1-\alpha} N_{us}^{-\alpha}}{(1-\alpha)(K_{us})^\alpha (A_{us} h_{india})^{1-\alpha} N_{us}^{-\alpha}} = \left(\frac{h_{us}}{h_{india}}\right)^{(1-\alpha)}$$

$$\frac{h_{us}}{h_{india}} = \left[\frac{w_{us}}{w_{india}}\right]^{\frac{1}{1-\alpha}}$$

Can Human Capital Explain Income Difference?

- Using this estimate, Hendricks *(2002)* conduct the same calibration exercises as *Hall and Jones (1999) do.*
- He concludes that for countries below 40 percent of U.S. output per worker, less than half of the output gap relative to the United States is attributed to human and physical capital.
  - **Remark:** Discrimination and a country specific skill such as language strengthen his result because it means that the immigrants may be more productive in their home countries.
- **Issue:** the selection bias. If the average ability of immigrants are higher than the average ability of workers in their home countries, then the wage difference is lower than the average human capital difference between the US and their home country.
  - After considering the selection bias, he concluded that the selection bias cannot change his result.

Can Human Capital Explain Income Difference?

- *Can we dismiss human capital?* There are several reasons that human capital may be still important.
  - Externality: The exercises assume that there is no externality. But externality is important, the impact of human capital is larger.
  - Technology Adoption: Education may be important not only for production but also for stimulating the adoption of technology. This point is more rigorously analyzed below.
  - The Influence on Population: Even in the high productive society, if population grows faster than productivity does, per capita income cannot increase. As investment in human capital increases the cost of raising children, it reduces the number of children and reduces population growth.

- Nonetheless, it is less likely that the lacks of physical and human capital provide a whole story for large income differences. Note that the lacks of physical capital and human capital cannot explain development miracles and disasters. We need to investigate a potential source of A.

Resource Allocation and Aggregate Productivity

- Many macroeconomists currently examine resource allocation as the potential source of A. If we think that the aggregate production function must be the result of several micro activities such as

$$F\left(K, AhL\right) = \sum_{i}^{n} F_i\left(K_i, A_i h_i L_i\right)$$

where i implies ith plant, the allocation of resources across plants can influence the aggregate productivity.

- Hsieh and Klenow (2009) investigate how much the misallocation of resources across plants influence the aggregate TFP by using microdata on manufacturing establishments in China, India and the United States. (US: 1977, 1982, 1087, 1992 and 1997, China: 1998-2005, India: 1987-1995.)
    - **Results:** When capital and labor are hypothetically reallocated to equate marginal products to the extent observed in the United States, they calculate manufacturing TFP gains of 30 %~50% in China and 40%~60% in India.

Resource Allocation and Productivity

- In reality, misallocation would be everywhere. But, it is difficult for a government to dictate better allocation. If it could, many communist countries could have survived. That is, it is impossible to point out every inefficiency in a world. For a policy purpose, we need to focus a particular misallocation that would influence productivity very much.

- **What kinds of misallocation are important?** As we discussed before, if we believe that the transfer of knowledge is the main source of productivity differences, the factors that influence technology adoption are likely to be important. We can provide two possible factors which can prevent the reallocation of resources to the technology diffusion.

Misallocation of Resources and Productivity Differences

- *Misallocation of talent:* There is an indirect way to influence B. Baumol (1990) investigated historical evidence and provided the following three hypotheses:

  1. The social system, which determines the relative payoffs to different entrepreneurial activities, changes over time and across regions.
  2. Entrepreneurial behavior changes according to variations in the social system.
  3. The allocation of entrepreneurship between productive and unproductive activities has a large effect on the innovation of technology and dissemination of technological discoveries.
     - Productive activity...Activity to create new value.
     - Unproductive activity...Activity to transfer income from somebody to others.

- Murphy, Shleifer and Vishny (1991) provided a formal model that clarifies Baumol's hypotheses. It shows that 1) if talented people are misallocated to a declining industry or the sector that specializing transferring income, they may reduce the growth rate.

Misallocation of Resources and Productivity Differences

- *Resistance to Technology Adoption:* Parente and Prescott (1994) show that the barrier to the adoption of new technology have substantial impacts on income differences. Parente and Prescott (1999) argue that monopoly right can be one such barrier. When government protects a particular company or industry, new comers cannot enter the market with new technologies. It hampers the adoption of new technology.

- Why do the resistance to technology adoption prevail?

    - Innovation and technology adoption are accompanied with creative destruction.
    - Creative destruction demands the replacement of resources.
        - In particular, If the incumbent has a large specific skill for old technology, the adoption of new technology makes the skill obsolete.
    - The reallocation of resources brings the conflicts of interests.
        - If preventing new entrance is cheaper than adopting new technology, they may resist new technology.

Fundamental Determinants of Differences in Economic Performance

- Both arguments suggest that in order for the misallocation to prevail, there must have an institutional barrier to prevent market forces from reallocating resources.
- This point is similar to the arguments by Acemoglu (2009). He said that investment in physical and human capital, and the reallocation of resources are proximate causes of income differences. But it does not answer to the deeper questions? Why do some countries invest more than others? Why do some countries attain better allocation.
- Acemoglu (2009) listed four candidates of fundamental causes. 1) Luck, 2) Geography, 3) Culture and 4) Institution.

Fundamental Determinants of Differences in Economic Performance

- Acemoglu, Johnson and Robinson (2001) provide evidence that
  1. More than 75 % of the income gap between relatively rich and relatively poor countries are explained by differences in economic institution (as proxied by the security of property rights.)
  2. Once the effect of institutions is controlled, there appears to be no effect of geographic variables.
  3. Once economic institution is taken into account, cultural variables do not appear to have a direct effect on economic growth and income per capita.

Fundamental Determinants of Differences in Economic Performance

- If better institution is important, why do not they choose better arrangement?
- Coase Theorem: If utility is transferable, and there is no liquidity constraint and no transaction costs, negotiation reaches a Pareto optimal allocation among parties who join the negotiation.
  - Example: A company introduces IT system to enhance productivity of the firm. It may substitute many clerks, and reduce labor costs. To do so, managers must fire workers. It is costly for workers. But manager can negotiate retirement allowance with workers.
    1. Because utility is transferable, workers can agree if enough payment is offered.
    2. Because the introduction of IT system is assumed to be productive, a firm can offer enough payment to convince workers.
    3. Because there is no liquidity constraint, even though income is available in future, the firm can finance the payment.
    4. No transaction cost means that both parties cannot tell a lie, negotiation is costless and the contract is perfectly enforced.

Fundamental Determinants of Differences in Economic Performance

- The previous question can be rewritten as follows: Why cannot Coase Theorem be applied in a political world?

  1. Utility may not be transferable. Maybe, but in this case, enhancing economic growth is not a Pareto optimal policy. Economists cannot say much.
  2. There might be a liquidity constraint. Possibly, but there is a developed international financial system. So they can relatively easily borrow money if the return is huge.
  3. The existence of transaction cost. This is possible. But what kinds of transaction costs are important?
  4. Coase theorem guarantees Pareto optimal allocation only among parties who join the negotiation. There might have an external effect of the negotiation.

- We discuss the points 3 and 4.

Fundamental Determinants of Differences in Economic Performance

- The external effect of negotiation: If the results of negotiation influence the third party, it is possible that the parties who can join the negotiation may steal some income from the third party.
  - For example, if all policies are determined by the negotiation among political elite from landlord, they would prevent enhancing manufacturing sectors, because their peasants or slave may move to manufacturing sectors. Enhancing manufacturing sectors can be Pareto improving, but landlords have no incentive to do so because they can keep their rent at the expense of peasants.

- But why does not the third party make an offer to negotiate with them?
  - There is a cost of organizing an interest group to make a reasonable negotiation with them. One of difficulty in organizing an interest group is free rider in the group: if some body incurs the cost of organizing group, others do not need to pay.

Fundamental Determinants of Differences in Economic Performance

- What kinds of transaction cost are important?
  - **Hold Up Problem:** There is no way for politicians to commit their statements. Without commitment, Coase theorem does not work.
    1. For example, if North Korea introduces the property rights and commit that they maintain investment friendly environment, many firms may invest in North Korea and it can potentially grow. However, after many firms making investment, they have always an incentive to steal the return from investment by raising tax or so. Moreover, because Kim Jong-un and other politicians are making a law, they can easily change a law. They don't have any method to commit their statements. Because no firms can trust politicians, they don't invest.
    2. Similarly, suppose that if some politicians in North Korea can peacefully replace Kim Jong-un from North Korea, they can potentially restructure the economy. For this purpose, they must commit that they provide his families enough income to guarantee their wonderful life. But, new politicians are in power, they may not keep their promise. As new politicians don't have any commitment device, Kim Jong-il cannot trust them and they never step down.

- Students must hand assignment 4 in at the next lecture.

Stabilization Policy

- Every country experiences boom and recession.
- The main question is whether government should or can actively stabilize aggregate demand for economy.
- I would like to investigate how demand stabilization policy affects real economy.
- This was a central issue of policy discussions. A school of economists advocates that a government should actively stabilize demand for economy; others disagree with this opinion. I would like to provide the framework to understand the reasons behind these policy discussions.

Stabilization Policy

- My lecture in this section is organized as follows.

  1. I introduce money and government to the basic model. For this purpose, I first discuss money market.
  2. Incorporating money and government in the basic model, we discuss how monetary policy and fiscal policy influence GDP in the long run.
  3. I introduce a short run deviation of GDP and the possibility that demand stabilization policy has a real impact.
  4. I discuss several explanations that enforce the economy to deviate from the long run equilibrium.
  5. I discuss stabilization policy and unemployment. We emphasize the importance of distinguishing the source of unemployment in the short run and long run.
  6. Finally, I discuss a difficulty of implementing stabilization policy. I show that how lack of commitment causes undesirable outcome.

- **What is money?** Money is the stock of assets that can be readily used to make transactions.

    - Money has three functions
        1. a store of value
        2. a unit of account
        3. a medium of exchange.

Money Market

- **Money Supply:** Money supply includes both currency in the hands and deposits at banks that households can use on demand for transaction:

$$M^s = C + D \tag{2}$$

where M^s is money supply, C is currency and D is demand deposits such as ordinary deposits and checking accounts.

- **Banking System and Money Supply:** Banking System influences the amount of money supply.
  - Example: Banks are required to keep a proportion of deposit by law in that depositors can always withdraw money. The deposits that banks have received but have not lent out are called reserves, denoted by R. Suppose that you deposit 1000 yen, $R = rd \times 1000$ *yen* where rd is the reserve deposit ratio. Hence, the bank can use $(1 - rd) \times 1000 yen$ to make loans and $(1 - rd) \times 1000 yen$ goes to public as currency. Since money supply is sum of currency and deposits, money supply is $(1 - rd) \times 1000 yen + 1000 yen$. In this way, the banking system influences money supply.

Money Market

- **Interbank Lending Market:** If a bank cannot hold the required amount of liquidity asset as reserves, it will need to borrow money in the interbank market to cover the shortfall.

- **Financial Crisis:** During Financial Crisis, there are some difficulties in this interbank system. A bank defaults. Other banks that made loans to this bank may default too. All banks are worried about the risk of default and the transaction in the interbank lending market becomes low. It is said that low transaction volume in this market was a major factor to the financial crisis of 2007.

Money Market

- **A Model of Money Supply:** The total amount of YEN that is supplied by Bank of Japan is called the monetary base, denoted by B and the monetary base equals currency plus reserves

$$B = C + R \qquad (3)$$

- Using equation (2) and equation (3),

$$\frac{M^s}{B} = \frac{C + D}{C + R}$$

Hence

$$M^s = \frac{cd + 1}{cd + rd} B$$

where $cd = C/D$ and $rd = R/D$. The parameter $\frac{cd+1}{cd+rd}$ is called money multiplier, which is a decreasing function of rd.

- This equation implies that Bank of Japan can control money supply by changing monetary base and reserve-deposit ratio.

Money Market

- How does Bank of Japan control *B*? There are three instruments:

  1. **Open-market operations:** Bank of Japan can sell or buy government bonds. When it buys bonds in the market, it pays yen for the bonds. Therefore, monetary base goes up. On the other hand, when it sells, it receives yen. Hence monetary base goes down.
  2. **Operations at exchange market:** Bank of Japan can sell or buy dollars. When it buys dollars in the market, it pays yen for the dollars. Therefore, monetary base goes up. On the other hand, when it sells, it receives yen. Hence monetary base goes down.
  3. **Change in the Discount Rate:** When commercial banks find that they do not have enough reserves, they can ask Bank of Japan to discount their bills. When Bank of Japan reduces the discount rate, commercial banks can easily borrow money from Bank of Japan and it increases monetary base.

Money Market

- **Money Demand:** Why do people demand money? There is the benefits and costs of holding money.

  1. The cost of holding money is that you miss interest. Instead of holding money, you can buy stocks or bonds. If you buy bonds, you can earn interest. But it you hold money in your pockets, you miss this opportunity. Therefore, the higher the interest rate, the smaller the demand for money.

  2. Then what is the benefit of holding money? It makes our transactions smoother. When you made a deal with your business partners, you must pay money. Hence we expect that when we have more transactions, we must demand more money. Since transactions increase when real GDP is larger, we expect that the demand for money is an increasing function of real GDP.

Money Market

- The discussions suggest the following money demand function.

$$\frac{M^d}{P} = L\left(\rho, Y\right)$$

where M^d is the demand for money and P is the aggregate price. The above equation implies that a decrease in the interest rate and an increase in real GDP raise the demand for money.

- For a simple analysis, we assume that the money demand is proportional to real GDP:

$$\frac{M^d}{P} = k\left(\rho\right) Y, \ k'\left(\rho\right) = \frac{dk}{d\rho} < 0$$

$$\frac{m^d}{P} = k\left(\rho\right) y$$

where $m^d = \frac{M^d}{N}$ and $y = \frac{Y}{N}$, and $k\left(\rho\right)$ is called Marshall's k. Marshall's k measures how much money people want to hold for every income.

Money Market

- Money market is

$$\frac{m^s}{P} = \frac{m^d}{P} = k\left(\rho\right) y$$

  where $m^s = \frac{M^s}{N}$.

Government Sector

- **Government Sector:** Assume that government must finance government expenditure per capita g, and government bond per capita, b_t and transfer payments to households per capita, tr, such as welfare for the poor and Social Security payments for the elderly, by a lump sum tax, with which consumers must pay fixed cost, τ, to government.

$$b_{t+1} = (1+\rho)\, b_t + g + tr - \tau$$

- We assume that government keeps the same amount of debt. $b_{t+1} = b_t$.

$$
\begin{aligned}
b &= (1+\rho)\, b + g + tr - \tau \\
\tau^n &\equiv \tau - tr = \rho b + g
\end{aligned}
$$

where τ^n is the net tax.

Government Sector

- Following Ono's (2006) critique, we assume that θ proportion of g increases value added. That is, $(1 - \theta)$ proportion of g can be seen as a hidden income transfer, tr^h, to somebody:

$$tr^h = (1 - \theta)\, g.$$

Note that current national account presumes $\theta = 1$.

Firm

- Assumption: Constant returns to scale and the steady state.

  - Moreover, because we ignore capital accumulation from our following analysis, we simply assume $k^* = \frac{K}{TN} = 1 \Rightarrow k = \frac{K}{N} = T$.
  - Note that ignoring capital accumulation simplifies our explanation, but makes us impossible to analyze the long run impact of demand stabilization policy through capital accumulation.

$$Y \leq F\left(K, TlN\right) = F\left(1, \frac{TN}{K}l\right) K = F\left(1, l\right) K$$

Define ϕ

$$\phi\left(l\right) \equiv F\left(1, l\right)$$

Then

$$Y \leq \phi\left(l\right) K$$

Moreover,

$$y = \frac{Y}{N} \leq \phi\left(l\right) \frac{K}{N} = \phi\left(l\right) T$$

Firm

- Profit maximization Problem

$$
\begin{aligned}
\Pi &= \max_{K,L} \left\{ PY - WIN - RK \right\} \\
&= \max_{I,K} \left\{ P\phi\left(I\right)K - \frac{WI}{PT}\frac{TN}{K}PK - \frac{R}{P}PK \right\} \\
&= \max_{I,K} \left\{ P\phi\left(I\right)K - \frac{W}{PT}IPK - \frac{R}{P}PK \right\} \\
&= \max_{K} \pi_k PK \\
\text{where } \pi_k &= \max_{I} \left\{ \phi\left(I\right) - \frac{w}{T}I - r \right\}
\end{aligned}
$$

where $w = \frac{W}{P}$ and $r = \frac{R}{P}$.

Firm

- Assumptions on the aggregate production function

  1. $\phi(0) = 0$
  2. $\phi'(l) = \frac{d\phi(l)}{dl} > 0$.
     - When the firm employs more capital per workers, it increases output per workers.
  3. $\phi''(l) = \frac{d^2\phi(l)}{dl^2} < 0$
     - This means that the marginal productivity of capital per workers is diminishing.
  4. Inada Conditions: technical conditions.

$$\lim_{l \to 0} \phi'(l) = \infty, \lim_{l \to \infty} \phi'(l) = 0.$$

Firm

- Profit maximization with respect to l

$$\pi_k = \max_l \left\{ \phi(l) - \frac{w}{T}l - r \right\}$$

- First Order Conditions

$$\frac{w}{T} = \phi'(l) \Rightarrow w = \phi'(l) T \Rightarrow l \text{ is determined}.$$

- π_k

$$\pi_k = \phi(l) - \frac{w}{T}l - r = \phi(l) - \phi'(l) l - r \Rightarrow \pi_k \text{ is determined}.$$

Firm

- Profit maximization with respect to K

$$\Pi = \max_K \pi_k PK$$

- Capital Demand Function

$$
\begin{aligned}
K &= 0 \text{ if } \pi_k < 0, \ r > \phi(I) - \phi'(I) I \\
K &\in [0, \infty] \text{ if } \pi_k = 0, \ r = \phi(I) - \phi'(I) I \\
K &= \infty \text{ if } \pi_k > 0, \ r < \phi(I) - \phi'(I) I
\end{aligned}
$$

Firm

- 0 Economic Profit

$$\Pi = \pi_k PK = 0$$

When the market is competitive, more entrepreneurs will enter as long as economic profits are positive. Hence, in the long run, economic profit is 0.Hence

$$\Pi = 0, K > 0 \Rightarrow \pi_k = 0 \Rightarrow r = \phi(I) - \phi'(I) I$$

Household

- How long do they work?: Assume that everybody works $h(w)$ of time. Hence, the supply of labor is equal to $h(w) N$. We assume that $h'(\cdot) = \frac{dh(w)}{dw} > 0$.
- How much do they consume today?:
  - I take the consumption per capita, c as given.
  - Budget Constraint: Assume that $\left(\frac{m^s}{P}\right)_{+1} = \frac{m^s}{P}$. Then

$$
\begin{aligned}
a_{+1} + c + \left(\frac{m^s}{P}\right)_{+1} &= (1+\rho^a) a + \frac{m^s}{P} + wh(w) - \tau + tr + tr^h \\
a_{+1} + c &= (1+\rho^a) a + wh(w) - \tau^n + tr^h \\
\rho^a &= \max\{r - \delta, \rho\}
\end{aligned}
$$

where τ is tax rate and tr is income transfer.

- Arbitrage Condition implies,

$$r - \delta = \rho = \rho^a$$

- Labor Market Clearing Condition

$$lN = h(w)N \Rightarrow l = h(w)$$

- Capital Market Clearing Condition

$$\begin{aligned} kN + bN &= aN \\ b &= a - k \end{aligned}$$

Stabilization Policy in the Long Run

- Given $(a, c, k \, (=T), M^s, g, \theta, \delta)$, a market equilibrium with government consists of $(y, l, a_{+1}, \rho, r, w, \tau^n, P, tr^h, b)$ which satisfies
  - A Firm's Profit Maximization and the Production Function determine:

$$y = \phi(l) T$$

$$w = \phi'(l) T$$
$$r = \phi(l) - \phi'(l) l$$

  - A Consumer's Budget Constraint

$$a_{+1} + c = (1 + \rho) a + wh(w) - \tau^n + tr^h$$

  - An Arbitrage Condition

$$r - \delta = \rho$$

  - Labor market clearing conditions

$$l = h(w)$$

  - Capital market clearing conditions

$$b = a - k$$

Stabilization Policy in the Long Run

- Government's budget constraint determines τ^n and tr^h:

$$\begin{aligned} \tau^n &= \rho b + g \\ tr^h &= (1 - \theta) g \end{aligned}$$

- The money market cleaning condition determines P:

$$\frac{m^s}{P} = k(\rho) y$$

Stabilization Policy in the Long Run

- Supply Side of Equilibrium: Because $l = h(w)$,

$$
\begin{aligned}
y &= \phi(h(w)) T \\
w &= \phi'(h(w)) T \\
\rho &= \phi(h(w)) - \phi'(h(w)) h(w) - \delta
\end{aligned}
$$

Stabilization Policy in the Long Run

- Goods market

$$
\begin{aligned}
a_{+1} + c &= (1 + \rho)\,a + wh(w) - \tau^n + tr^h \\
k_{+1} + b + c &= (1 + \rho)\,a + wh(w) - (\rho b + g) + (1 - \theta)\,g \\
k_{+1} + c &= (1 + \rho)\,(a - b) + wh(w) - \theta g \\
k_{+1} + c &= (1 + r - \delta)\,k + wh(w) - \theta g \\
k_{+1} + c &= \left(\phi(l) - \phi'(l)\,l + (1 - \delta)\right)k + \phi'(l)\,Tl - \theta g \\
k_{+1} + c &= \left(\phi(l) - \phi'(l)\,l\right)T + (1 - \delta)\,k + \phi'(l)\,Tl - \theta g \\
k_{+1} + c &= \phi(l)\,T + (1 - \delta)\,k - \theta g
\end{aligned}
$$

$$
\begin{aligned}
y &= c + k_{+1} - (1 - \delta)\,k + \theta g \\
  &= c + i + \theta g
\end{aligned}
$$

Stabilization Policy in the Long Run

- Supply Side determines (y, w, ρ)

$$
\begin{aligned}
y &= \phi\left(h\left(w\right)\right) T \\
w &= \phi'\left(h\left(w\right)\right) T \\
\rho &= \phi\left(h\left(w\right)\right) - \phi'\left(h\left(w\right)\right) h\left(w\right) - \delta
\end{aligned}
$$

- Demand Side determines (i, P)

$$
\begin{aligned}
y &= c + i + \theta g \\
\frac{m^s}{P} &= k\left(\rho\right) y
\end{aligned}
$$

- Remark: Note that current national income accounting presume that $\theta = 1$. In facts,

$$
\text{the measured GDP per capita} = y + \left(1 - \theta\right) g = c + i + g
$$

Labor Market Equilibrium
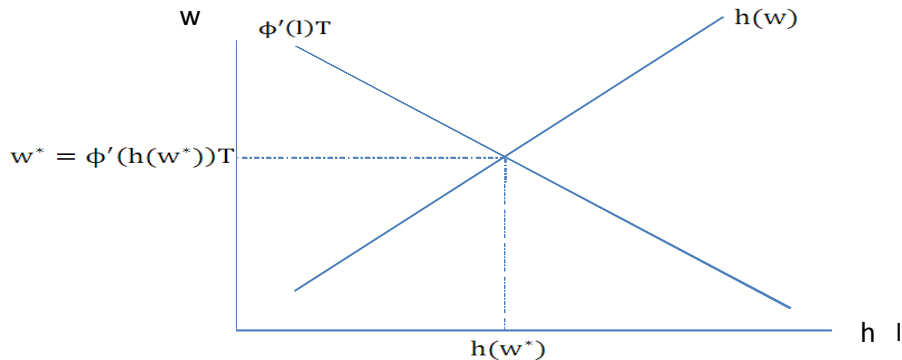
Stabilization Policy in the Long Run

- **Stabilization Policy vs. Supply Side Policy:** Both monetary supply, (a change in m^s), and a fiscal policy, (a change in g), cannot influence GDP per capita, y, the real interest rate, ρ, and the real wage rate w in the long run.

  1. Given a labor supply function $h(w)$, w is determined from $w = \phi'(h(w)) T$.
  2. Given w, $y = \phi(h(w)) T$ and $\rho = \phi(h(w)) - \phi'(h(w)) h(w) - \delta$

- If a government wishes to influence GDP in the long run, the policy must have an impact on supply side. I will give two examples.

Stabilization Policy in the Long Run

- Suppose that government levies a tax on labor income. In this case, households react to $(1 - \tau_l) w$, where τ_l is a flat labor income tax rate. Hence, as a reduction in labor income tax increases $(1 - \tau_l) w$, $h\left((1 - \tau_l) w\right)$ would be larger.

$$\tau_l{}' < \tau_l$$

- Suppose that government expenditure increases productivity T. Because $\phi'(h(w))T$ implies that the marginal product of labor is larger, firms have more incentive to employ labor. Hence, $y = \phi(h(w))T$ would be larger.

T'>T
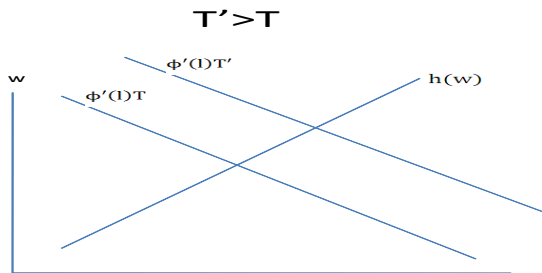
Stabilization Policy in the Long Run

- **Fiscal Policy and Crowding Out:** How does fiscal policy influence an economy in the long run? Consider

$$y = c + i + \theta g.$$

As y is already given supply side, an increase in g must result in reductions of $c + i$. It leads to the following theorem.

Theorem

A permanent increase in public expenditure (which only affect demand side) will be offset by a permanent reduction of private expenditure in the long run (crowding out).

Stabilization Policy in the Long Run

- **Monetary Policy and Inflation:** How does fiscal policy influence an economy in the long run? Consider

$$\frac{m^s}{P} = k(\rho) y,$$

Because ρ and y are already given by supply side, equation implies that an increase in money supply, m^s, causes inflation ($=$ an increase in the price index, P).

- All variables measured in physical units, such as output and relative prices, are called real variables. Variables expressed in terms of money are called nominal variables. This result implies that money supply affects nominal variables, but it does not affect real variables. This is called *the neutrality of money*.

Theorem

An increase in money supply causes inflation, but does not affect real variables in the long run.

Aggregate Demand and Aggregate Supply Curve

Aggregate Supply Curve

P

$p^{*\prime}$

P*

Aggregate Demand Curve

$\dfrac{m^{s\prime}}{k(\rho)y}$

$\dfrac{m^{s}}{k(\rho)y}$

$m^{s} < m^{s\prime}$

$\phi(h(w))T$
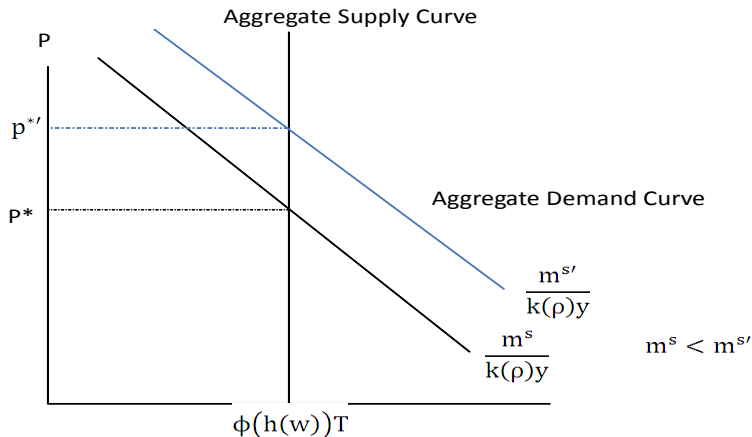
Money Market in the Long Run

- **The quantity theory of money:** *The quantity theory of money is derived from money market equilibrium.*

$$m^s V (\rho) = Py$$
$$M^s V (\rho) = PY$$

where $V (\rho) = \frac{1}{k(\rho)}$.

- The $V (\rho)$ is called the income velocity of money, which tells us the number of times money enters someone's' income during a given period of time. It measures the speed of transaction. Note that the income velocity of money is inverse of Marshall's k. If people wish to hold more money given income (= large k), money cannot move much (= small $V (\rho)$). If people hold little money in hand, money can frequently move around.

Money Market in the Long Run

- Note that

$$g_{M^s V} = g_{PY}$$
$$g_{M^s} + g_V = g_P + g_Y$$

Since the supply side determines ρ, ρ is given. If ρ is constant, V is constant. Hence, $g_V = 0$.

$$g_{M^s} = g_P + g_Y$$

If an economy is in a steady state, we know that $g_Y = g_{yN} = g_y + g_N = g_T + g_N$. we may be able to assume that it is near constant.

- Hence, the quantity theory of money suggests that the central bank can control inflation rate g_p by changing g_{M^s}.

Money Market in the Long Run

- **Nominal Interest Rate and Real Interest Rate:** Alternative difficulty in controlling inflation arises when we realize difference between nominal interest rate and real interest rate. The real interest rate is defined as nominal interest rate minus the expected inflation rate:

$$\rho^r = \rho^n - g_P^e \tag{4}$$

where ρ^r is the real interest rate, ρ^n is the nominal interest rate and g_P^e is the expected inflation rate.

  - ρ^r influences saving decisions and investment decisions.
  - ρ^n is considered to be opportunity costs of holding money. Hence, money demand depends on ρ^n.
  - Alternative expression:

$$
\begin{aligned}
1 + \rho^r &= \frac{1 + \rho^n}{1 + g_P^e} \\
\ln\left(1 + \rho^r\right) &\approx \ln\left(1 + \rho^n\right) - \ln\left(1 + g_P^e\right) \\
\rho^r &\approx \rho^n - g_P^e
\end{aligned}
$$

Money Market in the Long Run

- **Fisher equation:** Rearranging equation

$$\rho^n = \rho^r + g_P^e$$

This is called Fisher equation. As Marshall's k is a decreasing function of ρ^n, and therefore the V is an increasing function of ρ^n.

$$M^s V \left(\rho^r + g_P^e \right) = PY, \; V' \left(\cdot \right) > 0.$$

- This equation implies that expected future inflation rate can influence actual inflation rate.

$$g_P^e \uparrow \Rightarrow \rho^n \uparrow \Rightarrow k \left(\rho^n \right) \downarrow \Rightarrow V \left(\rho^n \right) \uparrow \Rightarrow P \uparrow$$

Hence, a careless expansionary monetary policy may induce a hyper inflation. So monetary policy must take into account how it influences people's expectation. But how?

Money Market in the Long Run

- Note that

$$\ln P_t + \ln Y = \ln M_t + \ln V\left(\rho^r + g^e_{P,t}\right)$$
$$\ln P_t = \ln M_t + \ln V\left(\rho^r + g^e_{P,t}\right) - \ln Y$$

- Assume that $\ln V\left(\rho^r + g^e_P\right) = \gamma\left(\rho^r + g^e_P\right)$, and ρ^r and $\ln Y$ are constant over time. Then

$$
\begin{aligned}
g^e_{P,t} &= E\left[\ln P_{t+1}\right] - \ln P_t \\
&= E\left[\ln M_{t+1} + \ln V\left(\rho^r + g^e_{P,t+1}\right) - \ln Y\right] \\
&\quad - \left[\ln M_t + \ln V\left(\rho^r + g^e_{P,t}\right) - \ln Y\right] \\
&= E\left[\ln M_{t+1} + \gamma\left(\rho^r + g^e_{P,t+1}\right)\right] \\
&\quad - \left[\ln M_t + \gamma\left(\rho^r + g^e_{P,t}\right)\right] \\
\left(1 + \gamma\right)g^e_{P,t} &= E\left[\ln M_{t+1}\right] - \ln M_t + \gamma g^e_{P,t+1}
\end{aligned}
$$

Money Market in the Long Run

- The expected Inflation rate

$$g_{P,t}^e = \frac{E\left[\ln M_{t+1}\right] - \ln M_t + \gamma g_{P,t+1}^e}{1 + \gamma}$$

  - It means that the current expected inflation rate depends on the expectation on the money supply in the next period and the expected inflation rate in the next period. We can also imagine that the expected inflation rate in the next period would depend on the expectation on the money supply in two period later and so on. It indicates that in order to control people's expectation, the commitment on the future monetary policy is necessary.

Discussion

- **Can Bank of Japan really control money supply?:** So far we assume that the Bank of Japan can perfectly control money supply. Is it true? During Xmas season, we can observe an increase in money supply and nominal GDP. However, it is difficult to believe that an increase in money supply during Xmas causes an increase in nominal GDP. Natural interpretation is opposite: since people transact more during Xmas seasons, people demand more money.
- Note that there is a relationship between B and M^s.

$$M^s = \frac{cd + 1}{cd + rd} B$$

where $cd = C/D$ and $rd = R/D$. We so far assume that cd and rd are constant. But in fact these numbers are endogenous variables. In Japan, money multiplier has decreased between 1992 and 2002. It means that although Although Bank of Japan has actively increased Monetary Base, its impacts on money supply has decline during the period.

Discussion

- **Interest rate as A Policy Target:** People in Bank of Japan typically believe that they rather accurately control a short term nominal interest rate by changing B. In this case, the quantity theory of money implies alternative effect in the long run:

  - Note that
    $$M^s V(\rho^n) = PY, \ V(\rho) = \frac{1}{k(\rho)}$$

  - If $B \uparrow \Rightarrow \rho^n \downarrow$,
    $$B \uparrow \Rightarrow \rho^n \downarrow \Rightarrow k(\rho^n) \uparrow \Rightarrow V(\rho^n) \downarrow \Rightarrow P \downarrow$$

  - This is different from
    $$B \uparrow \Rightarrow M^s \uparrow \Rightarrow P \uparrow$$

- **Zero interest rate policy:** In fact, in Japan, the nominal interest rate is set at 0, $\rho^n = 0$ between February 1999 and August 2000 and between March 2001 and July 2006. Since December 16, 2008, the United States conducts the same policy. When the nominal interest rate is 0, the opportunity cost of holding money is 0. There is evidence that $k(\rho^n) = \frac{M^s}{PY}$ has increased during the period.

Discussion

- **Zero interest rate policy in the long run:** Whatever variables that Bank of Japan can control, zero interest rate policy is likely to induce deflation in the long run. Because of Fisher equation,

$$\begin{aligned} 0 &= \rho^r + g_P^e \\ g_P^e &= -\rho^r < 0 \end{aligned}$$

  - Key observation is that the real interest rate is not influenced by monetary policy in the long run. As we discuss later, in the short run, lowering nominal interest rate lowers real interest rate, and, therefore, encourages investment. But, as real interest rate is constant in the long run, lowering nominal interest rate simply reduces the expected inflation rate.

Discussion

- Fukuda (2010) claims that the zero interest rate policy couldn't stop deflation during this period. On the other hand, Honda (2011) claims that the zero interest rate policy rather caused the depreciation of the exchange rate and succeeded to stimulate economy. But Saito (2010) claims that it was a just bubble and couldn't not enhance real economic growth in the long run.

Discussion

- **Friedman Rule:** Friedman argues that an optimal nominal interest rate is 0. In this case, real interest rate is equivalent to deflation and the money supply must be adjusted to satisfy $\frac{M^s}{P} = k\left(0\right) Y$. Because the opportunity cost of holding money is 0, as far as the price system works well, everybody can enjoy the benefits of money without cost. What is wrong with this?

- **Cost of Deflation:** There are two potential problems on Friedman rule.

    1. Friedman rule assumes that everybody accepts a reduction in nominal wage payment. If not, we should observe more unemployed workers or bankrupt companies.
    2. Because debt is based on nominal value, lower nominal wage means that debtors will face a difficulty in paying their interest.
        - In Japan, the largest debtor is government. Hence, government must suffer from the cost of deflation, which causes higher tax rate in future.

Assignment

- Students must hand assignment 5 in at the next lecture.

Adjustment Cost of Investment and Stabilization Policy

- So far I assumed that firms can instantaneously make any investment. This may be an innocuous assumption in the long run. However, the assumption may be questionable in a short run problem. This section introduces the adjustment cost of investment.

- When there is no adjustment cost of investment, one unit of output is transferred to produce one unit of investment. Hence the value of capital is also equal to 1.

- When the cost of investment exists, the value of capital deviates from one. Assume that q is the marginal value of capital and $C\left(I^f\right)$ is the adjustment cost of investment. The firm's investment problem can be written as follows.

$$\max_{I^f} qK' - I, \ I = I^f + C\left(I^f\right)$$

$$K' = I^f + (1-\delta) K$$

where $C\left(0\right) = C'\left(0\right) = 0$, $C'\left(I^f\right) > 0$ and $C''\left(I^f\right) > 0$.

Adjustment Cost of Investment
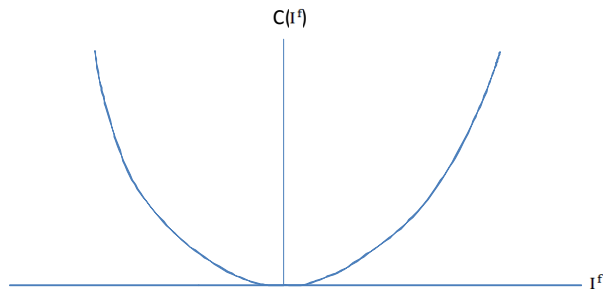
Adjustment Cost of Investment and Stabilization Policy

- The first order condition is

$$q = C'\left(I^f\right) + 1$$

Note that this equation implies that if there is no adjustment cost, the value of capital, q, is equal to 1. The marginal value capital, q, is called the marginal q.

- Because $C'\left(I^f\right)$ is large when I^f is large,

$$(q-1)\uparrow \Rightarrow C'\left(I^f\right)\uparrow \Rightarrow I^f\uparrow \Rightarrow I\uparrow$$

- Hence investment can be written as an increasing function of $q-1$.

$$I = \psi\left(q-1\right), \ \psi'\left(\cdot\right) > 0$$

Detour to the Q theory of Investment

- *Q* **Theory of Investment:** With an additional technical condition on the adjustment cost ($C\left(I^f\right) = C^*\left(I^f, K\right)$ and C^* is CRSs in I^f and K), it is known that the marginal q can be estimated by the Tobin's Q where

$$q = Tobin's\ Q \equiv \frac{Market\ Value\ of\ a\ Firm}{Replacement\ Cost\ of\ Capital\ Stock}$$

and there is $\psi^*\left(q-1\right)$ such that

$$I = \psi\left(q-1\right) = \psi^*\left(q-1\right)K.$$

  - It means that if the market value of firm is greater than replacement cost of capital, then the firm should invest, if not, it should sell some assets. In this way, Tobin's Q can be considered as the proxies of the importance of the expected investment opportunities. This is called the Q theory of investment.

Adjustment Cost of Investment and Stabilization Policy

- **What influences q?** Let us maintain the current assumption ($C\left(I^f\right)$ does not depend on K) and discuss what influences the expected investment opportunities. If you have one unit of capital stock in your hand, you can sell it and earn q_t. If you save q_t in your bank, your account at the next year would be $(1+\rho)\,q_t$. On the other hand, if you keep the capital, you expect to receive rental price r_{t+1}^e at date $t+1$ and you expect to sell the capital in the next year by the price q_{t+1}^e. Because δ portion of capital is assumed to be depreciated, you will expect to earn $q_{t+1}(1-\delta)$ by selling the capital. Hence, an arbitrage condition implies

$$
\begin{aligned}
(1+\rho)\,q_t &= r_{t+1}^e + q_{t+1}^e\,(1-\delta) \\
r_{t+1}^e &= E\left[\phi\left(h\left(w_{t+1}\right)\right) - \phi'\left(h\left(w_{t+1}\right)\right) h\left(w_{t+1}\right)\right]
\end{aligned}
$$

  - Hence, the expectation on the future marginal productivity of capital must influence the expected investment opportunities.

- Assume that there is no capital gain and loss for simple analysis and economy is stationary:

$$q_{t+1} = q_t = q, \ r_{t+1}^e = r^e = E\left[\phi\left(h\left(w\right)\right) - \phi'\left(h\left(w\right)\right) h\left(w\right)\right]$$

Then

$$
\begin{aligned}
(1+\rho)\, q &= r^e - \delta q + q \\
q &= \frac{r^e}{\rho + \delta} \\
r^e &= E\left[\phi\left(h\left(w\right)\right) - \phi'\left(h\left(w\right)\right) h\left(w\right)\right]
\end{aligned}
$$

- Hence

$$I = \psi \left(\frac{r^e}{\rho + \delta} - 1 \right), \ r^e = E \left[\phi \left(h \left(w \right) \right) - \phi' \left(h \left(w \right) \right) h \left(w \right) \right]$$

- Note that

$$
\begin{aligned}
r^e - \delta \ &= \ \rho \Rightarrow \frac{r^e}{\rho + \delta} = 1 \Rightarrow I = 0 \\
r^e - \delta \ &> \ \rho \Rightarrow \frac{r^e}{\rho + \delta} > 1 \Rightarrow I > 0 \\
r^e - \delta \ &< \ \rho \Rightarrow \frac{r^e}{\rho + \delta} < 1 \Rightarrow I < 0
\end{aligned}
$$

Hence, the long run condition can be seen as the situation that any adjustment is finished.

- Define $\iota(\cdot)$ such that

$$i = \frac{\psi\left(\frac{r^e}{\rho+\delta}-1\right)}{N} \equiv \iota(\rho), \ \iota(r^e-\delta) = 0, \ \iota'(\rho) < 0.$$

  - **Remark:** This function ignores that the expected investment opportunities influence investment decision. Hence, the following analysis presume that the expected future returns on the investment $r^e = E\left[\phi(h(w)) - \phi'(h(w))h(w)\right]$, does not influence investment.

- Because $r^e - \delta \neq \rho$ in general, the marginal product of capital, $\phi(h(w)) - \phi'(h(w))h(w)$, does not determine the interest rate.

Adjustment Cost of Investment and Stabilization Policy

- Supply Side determines (y, w)

$$
\begin{aligned}
y &= \phi\left(h\left(w\right)\right) T \\
w &= \phi'\left(h\left(w\right)\right) T
\end{aligned}
$$

Demand Side determines (ρ, P)

$$
\begin{aligned}
y &= c + \iota\left(\rho\right) + \theta g \\
\frac{m^s}{P} &= k\left(\rho\right) y
\end{aligned}
$$

Adjustment Cost of Investment and Stabilization Policy

- Supply side conditions still determine GDP and the wage rate. Hence, fiscal policy and monetary policy has no impact on GDP and the wage rate.

  1. As an increase in government expenditure cannot change output, it reduces private expenditure.
  2. Money supply simply raises the price level. Hence, money is still neutral, too.

- The main difference is that fiscal policy can change the real interest rate. Because an increase in government expenditure reduces fund for the private investment. Hence, the real interest rate increases. To understand this, we can rewrite

$$
\begin{aligned}
\iota\left(\rho\right) &= y - c - \theta g \\
&= y - c - g + \left(1 - \theta\right) g \\
&= y - \tau^n + \rho b + tr^h - c = s^g
\end{aligned}
$$

$g'>g$

- Since a rise in government expenditure raises the interest rate, it raises the opportunity cost of money holding. Therefore, it reduces money demand. It means that money supply becomes larger than money demand. Hence, the price of money, $\frac{1}{P}$, goes down. In another word, we have inflation.

$$\rho \uparrow \Rightarrow k(\rho) \downarrow \Rightarrow k(\rho) y \downarrow \Rightarrow \frac{m^s}{P} \downarrow \Rightarrow P \uparrow$$

Adjustment Cost of Investment and Stabilization Policy
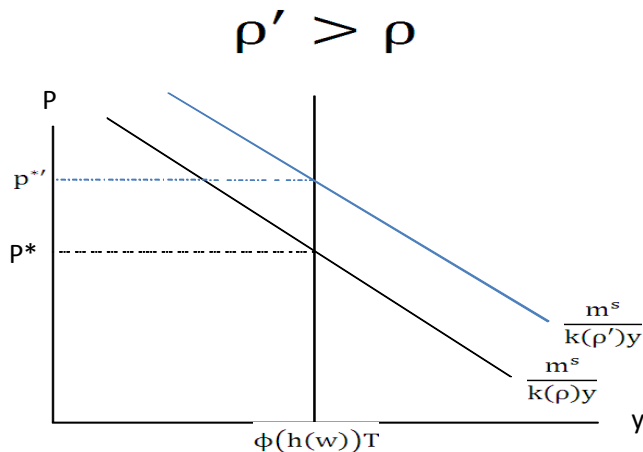
Theorem

A permanent increase in public expenditure (which only affect demand side) will be offset by a permanent reduction of private expenditure in the long run. When the adjustment of investment is slow, it also raises the real interest rate and causes inflation in the long run. An increase in money supply causes inflation, but does not affect real variables in the long run.

Stabilization Policy in the Short Run

- I define the short run as the period during which l differs from equilibrium working hours in the long run equilibrium ($h(w^*)$):

$$l \neq h(w^*)$$

where w^* is an equilibrium wage.

- We further assume that actually employed hours, e, are an increasing function of the price level:

$$l = e(P), \ e'(P) > 0$$

  - When an increase in demand raises output price, firms can make more profits by employing labor.

Stabilization Policy in the Short Run

- Supply Side

$$
\begin{aligned}
y &= \phi\left(e\left(P\right)\right) T \\
w &= \phi'\left(e\left(P\right)\right) T
\end{aligned}
$$

- Demand Side

$$
\begin{aligned}
y &= c + \iota\left(\rho\right) + \theta g \\
\frac{m^s}{P} &= k\left(\rho\right) y
\end{aligned}
$$

Aggregate Demand and Aggregate Supply in the Short Run
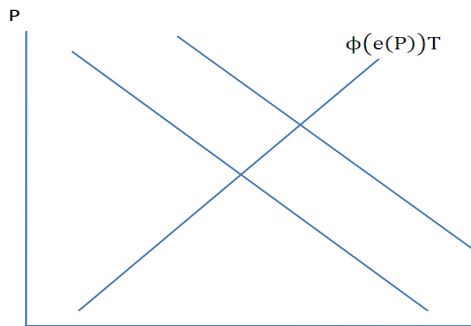
Stabilization Policy in the Short Run

- Suppose P is given. Then the demand side two equations determine y and ρ.

$$IS \quad : \quad y = c + \iota(\rho) + \theta g$$
$$LM \quad : \quad \frac{m^s}{P} = k(\rho) y$$

Two equations describe IS curve and LM curve. IS is the abbreviation of Investment and Saving. LM is the abbreviation of Liquidity and Money.

Aggregate Demand and Aggregate Supply when P is given

$\phi(e(P))T$
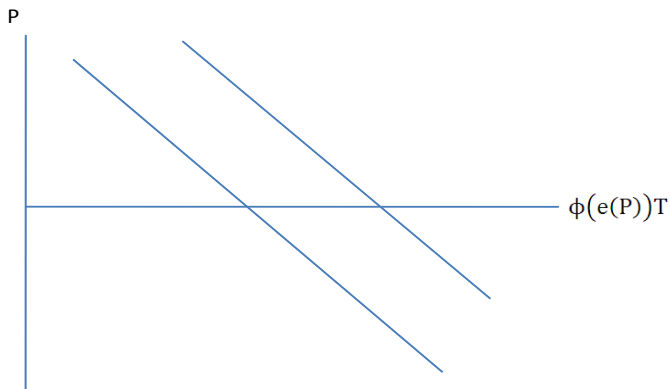
Keynesian Cross

- Assume that P and ρ are given. The IS equation determines y.

$$y = c + i + \theta g$$

where $i = \iota(\rho)$.

- Suppose that

$$c = c\left[y + \rho b + tr^h - \tau^n\right], \ c'[\cdot] \in (0.1)$$

where $tr^h = (1 - \theta)g$. This assumption implies that consumption depends only on the current disposable income.

- Define the planned expenditure, y^d as follows.

$$y^d = c\left[y + \rho b + (1 - \theta)g - \tau^n\right] + i + \theta g$$

Then IS equation is satisfied when $y = y^d$.

Keynesian Cross

y^d

$y^d = y$

$y^d = c[y + \rho b + (1 - \theta)g - \tau^n] + i + \theta g$

45°

y
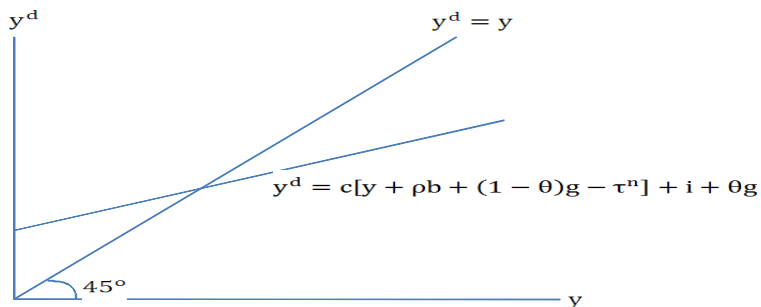
Keynesian Cross

Keynesian Cross
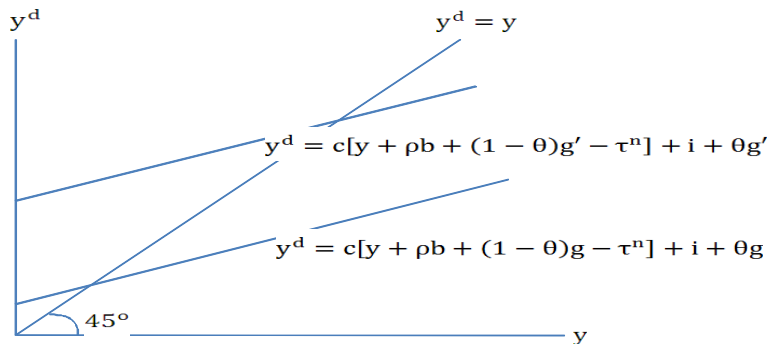
- Example,

$$c = c_0 + c_1 * (y + \rho b + (1 - \theta) g - \tau^n)$$

Then

$$
\begin{aligned}
y^d &= c_0 + c_1 * (y + \rho b + (1 - \theta) g - \tau^n) + i + \theta g \\
&= c_0 + c_1 * (y + \rho b - \tau) + i + [c_1 * (1 - \theta) + \theta] g
\end{aligned}
$$

$$
\begin{aligned}
y &= y^d = c_0 + c_1 * (y + \rho b - \tau^n) + i + [c_1 * (1 - \theta) + \theta] g \\
&= \frac{1}{1 - c_1} \left\{ c_0 + c_1 * (\rho b - \tau^n) + i + [c_1 + (1 - c_1) \theta] g \right\}
\end{aligned}
$$

Multiplier Effect

$$\frac{dy}{dg} = \frac{c_1}{1 - c_1} + \theta > \theta$$

- Note that we have assumed that government keeps the same amount of debt over time, $b_{t+1} = b_t$, which means $\tau^n = g + \rho b$. In this case,

$$
\begin{aligned}
y^d &= c\left[y + \rho b + (1 - \theta)\,g - (g + \rho b)\right] + i + \theta g \\
&= c\left[y - \theta g\right] + i + \theta g \\
x^d &= c\left[x\right] + i
\end{aligned}
$$

where $x = y - \theta g$ and $x^d = y^d - \theta g$, where x is the disposal income and x^d is the planed private expenditure. It means that goods market clearing condition is satisfied when $x = x^d$.

Keynesian Cross
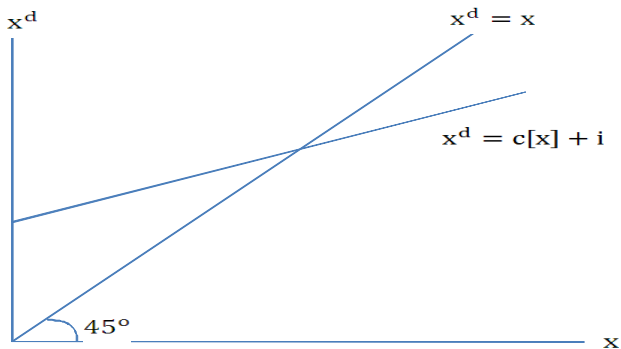
- Because $x = y - \theta g$,

$$y = \theta g + x$$

- Multiplier Effect: (because x is independent of g)

$$\frac{dy}{dg} = \theta$$

IS Curve

- Suppose that P is given. Then IS equation determines the relationship between y and ρ.

$$y = c + \iota(\rho) + \theta g$$

- In order to understand the relationship, note that Keynsian Cross predicts that an increase in i results in an increase in y.

IS Curve
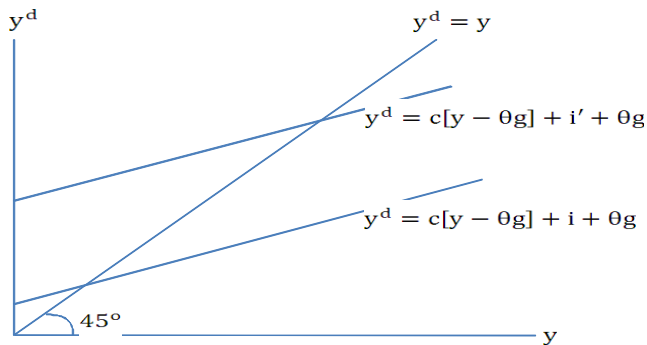
IS Curve

- Note also that
$$i = \iota'(\rho) < 0, \ \rho \uparrow \Rightarrow i \downarrow$$

- Hence
$$\rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow$$

This relationship can be depicted by IS curve.

IS Curve
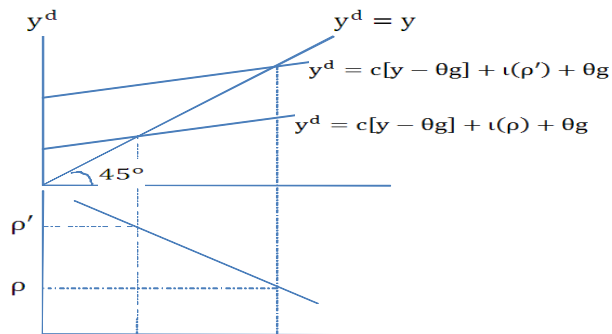
IS Curve

- A shift in IS curve. Note that for any given ρ, $\frac{dy}{dg} > 0$. Hence, an increase in g shifts the IS curve to the left.
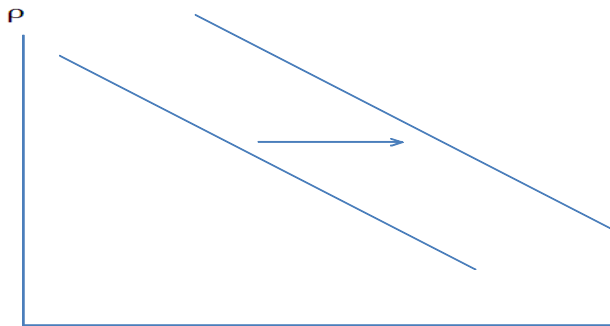
Money Market

- When the price P is given, money market determines interest rate ρ.

$$LM : \frac{m^s}{P} = \frac{m^d}{P} = k(\rho) y$$

Money Market in the Short Run

- In the short run, an increase in money supply lowers interest rate.

$$m^s < m^{s\prime}$$

LM Curve
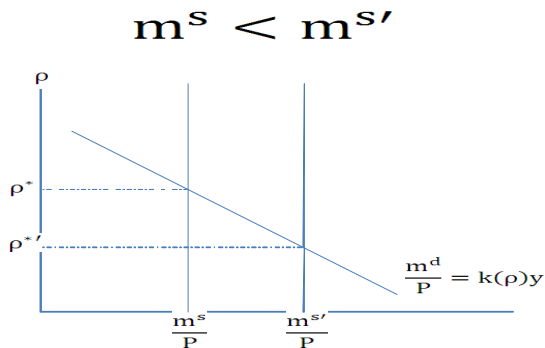
- For example, suppose that $k(\rho) = \frac{k}{\rho}$. Then

$$
\begin{aligned}
\frac{m^s}{P} &= k(\rho) y = \frac{ky}{\rho} \\
\rho &= \frac{kPy}{m^s}
\end{aligned}
$$

Hence, there is a negative relationship between m^s and ρ.

- Note that this relationship is true only if P is given. As we discuss before, if an increase in money supply results in an increase in P, the negative relationship may not hold. Moreover, if the expected inflation, g_P^e, goes up, because $\rho^n = \rho^r + g_P^e$, the nominal interest rate may increase.

LM Curve

- When P is given, LM equation also suggest a positive relationship between y and ρ. This relationship is described by *LM* curve.

LM Curve

$$\frac{m^d}{P} = k(\rho)y'$$

$$\frac{m^d}{P} = k(\rho)y$$

$$\frac{m^s}{P}$$
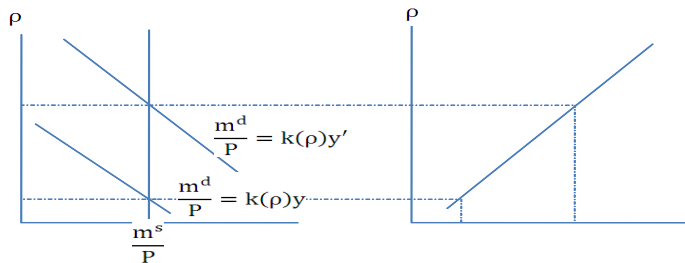
LM Curve

- A shift in LM curve: Because $\frac{d\rho}{dm^s} < 0$, an increase in m^s shifts LM curve down.
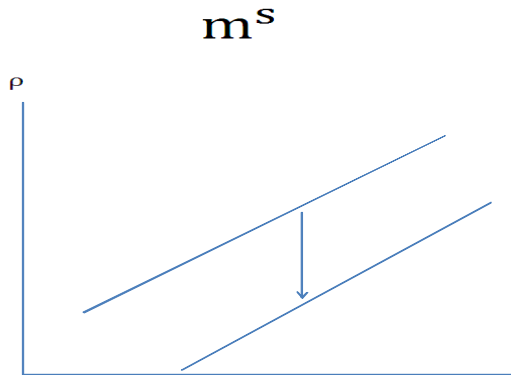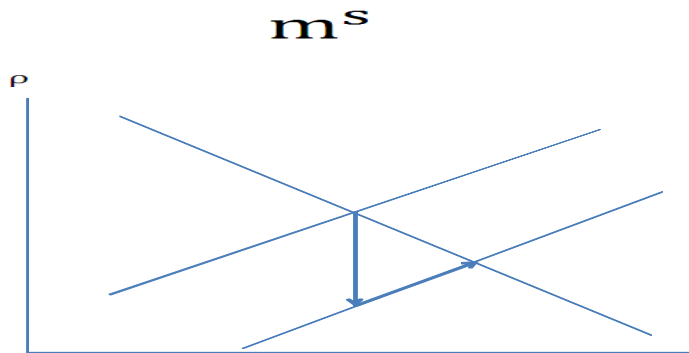
IS-LM analysis

- The impacts of a fiscal policy

$$g \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow \ldots$$

IS-LM analysis
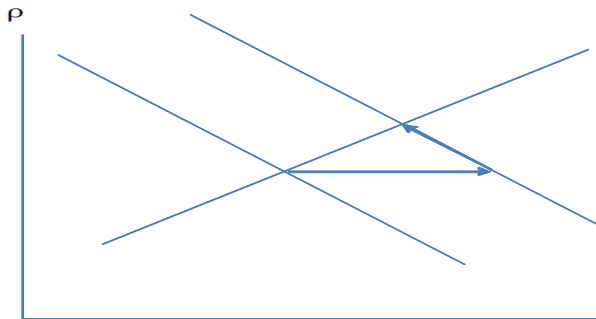
- The impacts of a monetary policy

$$m^s \uparrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow \dots$$
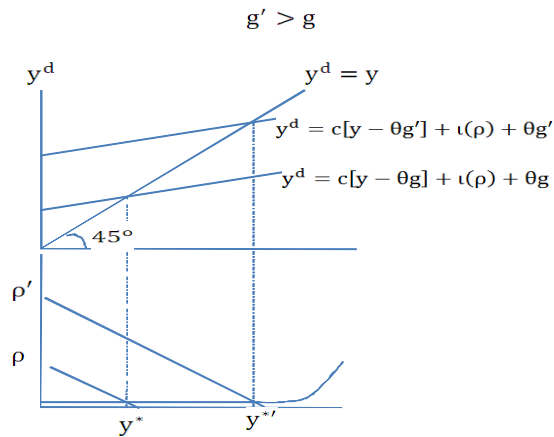
- **Liquidity Trap:** When nominal interest rate is near 0, money demand can be infinite, $k(0) = \infty$. In this case, an increase in money supply cannot reduce nominal interest rate furthermore. Hence, LM curve becomes flat. This situation is called liquidity trap.
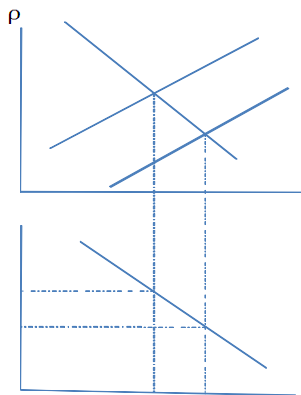
Liquidity Trap and Monetary Policy

IS-LM analysis

$g' > g$

y^d

$y^d = y$

$y^d = c[y - \theta g'] + \iota(\rho) + \theta g'$

$y^d = c[y - \theta g] + \iota(\rho) + \theta g$

$45°$

ρ'

ρ

y^*

$y^{*'}$
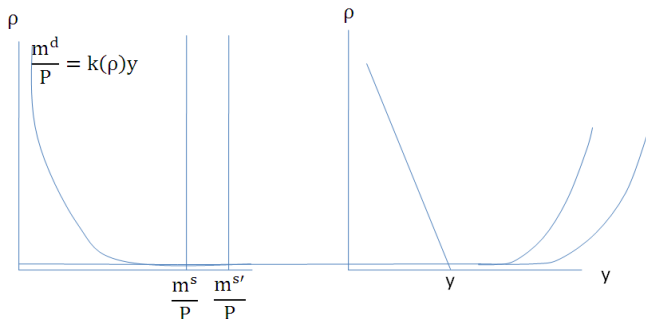
- Note that

$$\rho = \frac{kPy}{m^s} \Rightarrow \frac{d\rho}{dP} > 0.$$

- Hence,

$$P \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow \Rightarrow m^d \downarrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow \Rightarrow y \uparrow \dots$$

It shows that the overall relationship between P and y is negative. This relationship is depicted by AD curve.
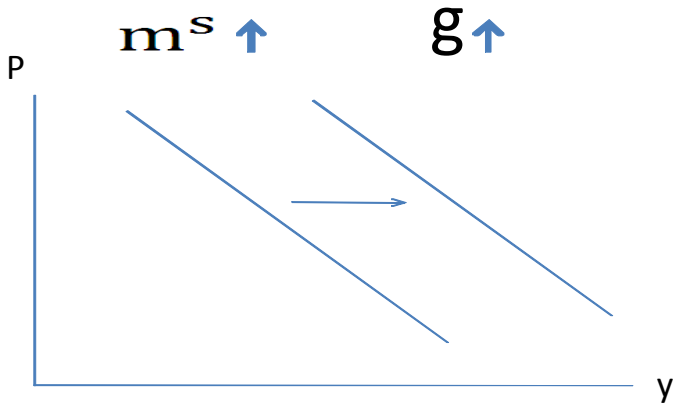
- A shift in AD curve. Note that for any given P, IS-LM analysis suggests that

$$g \uparrow \Rightarrow y \uparrow, \ m^s \uparrow \Rightarrow y \uparrow$$

Hence, increases in government expenditure or/and in money supply shifts AD curve to the right.

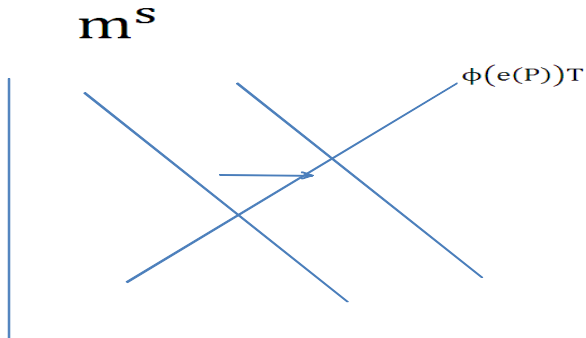# AS-AD analysis

Theorem

An increase in government expenditure or money supply raises aggregate demand in general.

AS-AD analysis

Theorem

An increase in government expenditure or money supply causes inflation and raises GDP in the short run.

The Short Run Aggregate Supply Curve

- As I defined before, the short run is the period the number of employed workers differs from its long run equilibrium level. I also assume that the number of employed workers is increasing in price level in the short run.

  1. Why does it differ from the equilibrium level in the short run?
  2. How can I derive a positive employment function?.

- I introduce two prototype models, which illustrate their main idea.

The Short Run Aggregate Supply Curve

- **Sticky Nominal Wage Model:**
- Let me assume that the nominal wage is fixed in the short run: $W = \bar{W}$. There might be several reasons for this assumption. For example, the bargaining process with labor union may prevent a reduction of nominal wage. For several contractual reasons, it may be difficult to change the nominal wage quickly.
- Since labor demand curve is a decreasing in the real wage, if the nominal wage is rigid, labor demand must be an increasing in P:

$$P \uparrow \Rightarrow w = \frac{\bar{W}}{P} \downarrow \Rightarrow \phi'(l)\, T \downarrow \Rightarrow l \uparrow$$

Note that $\phi'(l_e)$ is decreasing in l_e.

- Note that because lowering nominal wage is much more difficult than increasing it, firms have more difficulties in adapting to the deflation. This is one of a reason that economists prefer mild inflation to deflation and no inflation.
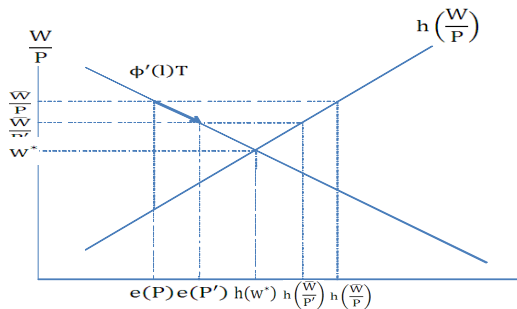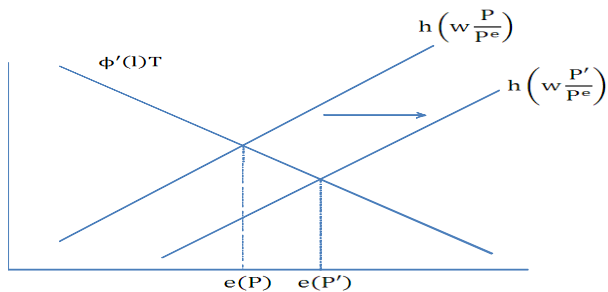
The Short Run Aggregate Supply Curve

- **Imperfect Information Model:** Another possible reason on an increasing aggregate supply curve is the imperfection of information. There are several variants of imperfect information model. I describe a worker-misperception model to convey its main intuition.

- Assume that firms observe output price P, but workers cannot. Hence, workers must make their inferences about price. Workers' expected price is denoted as P^e. Then workers respond to $\frac{W}{P^e}$.

$$h\left(\frac{W}{P^e}\right) = h\left(\frac{W}{P}\frac{P}{P^e}\right) = h\left(w\frac{P}{P^e}\right)$$

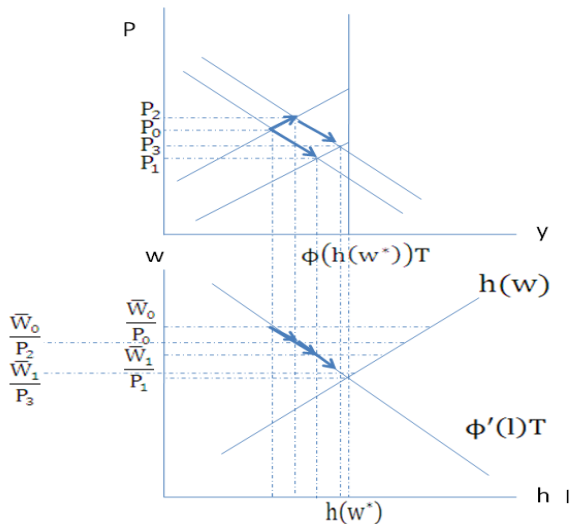- Suppose that the overall price level P goes up. However, workers do not know a change in aggregate price. Hence, P^e stays the same. Hence, $\frac{P}{P^e}$ goes up and supply curve shifts to right.
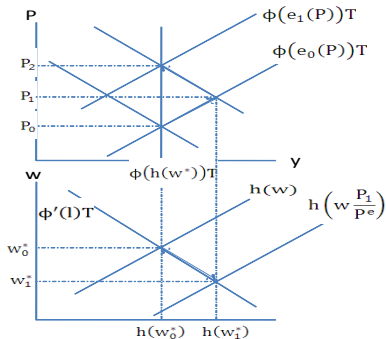
The Short Run Aggregate Supply Curve

Adjustment under a Sticky Nominal Wage Model

Adjustment under a Worker-Misperception Model

Discussions

- Both models derives the short run aggregate supply curve. However, policy implication may change.

  1. Sticky nominal wage model brings unemployment; the imperfect information model does not. When nominal wage is sticky, there are workers who are willing to work with lower wage. When information is imperfect, all workers and firms agree on the market price given their perception.

  2. Policy implication differs. Because there is unemployed workers under sticky nominal wage model, an increase in demand can increase GDP by employing more workers. However, if the imperfect information model is correct, there are no unemployed workers. If active stabilization policy itself brings the uncertain movement of price, it may increase workers' further misperception. It reduces the welfare of an economy.

Lost Decades

- Japanese economy stagnated more than 10 years. It is difficult to believe that only sticky price and noisy information can explain such a long term stagnation.

- There is one possibility that a short run deviation can be longer than one predicted by the above mechanism: DEFLATIONARY SPIRALS. Suppose that people start to expect a deflation, $g_P^e < 0$. Because investment decisions depend on the real interest rate, IS equation changes from IS_1 to IS_2.

$$IS_1 \quad : \quad y = c + \iota \left(\rho^n + \delta \right) + \theta g$$
$$IS_2 \quad : \quad y = c + \iota \left(\rho^n - g_P^e + \delta \right) + \theta g$$

Lost Decades
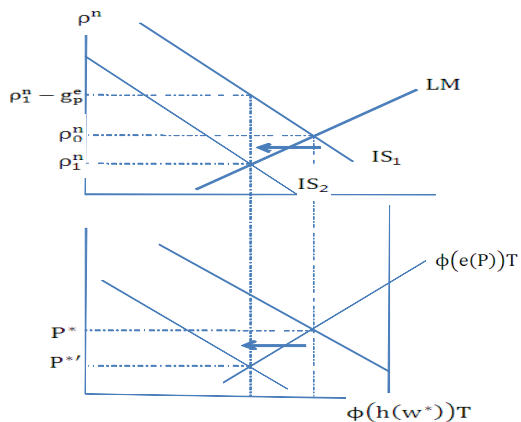
- Hayashi and Prescott (2002) provide an alternative explanation. Whatever the reason for the deviation, they believe that 10 years are too long to justify the deviation from long run equilibrium. They point out that the growth rate of TFP declines during 90s. That is, they argue that a demand stagnation cannot explain Japanese problem, but supply side problem can. Following their arguments, several researchers point out several misallocation of resources in Japanese economy during 90s.

Lost Decades
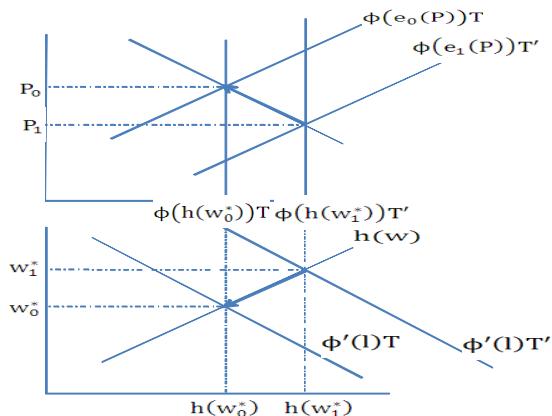
- Economists still do not have a consensus on 90s. But, because Japanese economy is still stagnated after recovery, more economists feel that it is difficult to deny that Japanese economy has structural problems.

- Nonetheless, we continuously observe the statement which demands more expansionary monetary and fisical policy in this 20 years. This is understandable because macroeconomics suggests that as far as there is no inflation, expansionary fiscal and monetary policies can enhance GDP.

- Does this mean that there is no cost of expansionary fiscal and monetary policies when there is no inflation?

    - No. Remember that our current analysis ignore the interaction between now and future.

        1. An increase in I_t does not increase K_{t+1}.
        2. $E[MPK_{t+1}]$ does not influence I_t.
        3. The future income, Y_{t+1} does not influence C_t

Lost Decades

- There are several potential mechanisms that an expansionary fiscal and monetary policy can influence the long run growth.

  1. Expansionary monetary policy lowers real interest rate and stimulate investment, even if there are no large investment opportunities. Because MPK declines as more capital is accumulated, the returns from investment would be lower in the long run. In fact we observe an increase in $\frac{K}{Y}$ and the declining MPK after 1990 (e.g., Fukao (2012)). Ando (2002), Hayashi (2006) and Saito (2008) claim that there are the indications of over-investment.

  2. Similar token, although our current model does not take into account government investment, if a part of government expenditure is used as investment and the marginal productivity of government capital declines, expansionary fiscal policy reduces the return from expansionary fiscal policy. This effect is likely to have a similar impact as a reduction in θ.

Lost Decades

- Because continuous investment without promising investment opportunities enforce us to misallocate resources to unproductive use, it is likely to reduce aggregate productivity. In facts, Nishimura et al. (2005), Cabarello et al. (2008) and Kwon et all. (2010) found that the indication of misallocation.

- Note that our long run model suggests that the growth through investment eventually ends and only productivity growth enhances sustainable growth. Hence, this types of economic growth may not be desiable in the long run.

Lost Decades

- Moreover, this long run effects can also influence current demand. Note that an reduction in the expected future productivity reduces the expected future MPK and Y_{t+1}. But as Fukao (2012) suggested, reductions in MPK_{t+1} and Y_{t+1} can lower current demand.

  1. A reduction in $E[MPK_{t+1}]$ reduces I_t. (According to Fukao (2012), a reduction in TFP and labor are main sources of a reduction in MPK.)
  2. A reduction in Y_{t+1} reduces C_t.

- These mechanisms also indicate that we can think about different policies that can increase demand.

  1. If deregulation increases the investment opportunities in future, it increases $E[MPK_{t+1}]$ and I_t.
  2. If there is a secured pension scheme, household may feel that they can spend more today and increases C_t, (though the saving rate at household sector declines now.).

- So, even if there is no deflaton, we cannot say that there is no cost of expansionary monetary and fisical policies. These considerations suggest that the expansionary monetary and fisical policy (short run policies) must be accompanied with the policy that can enhance the future productivity (long run policies).

- Before a jump into an conclusion, because the existence of unemployed workers is the main reason for the demand stabilization policy, we need more careful examination of Japanese labor market.

- Students must hand assignment 6 in at the next lecture.

Stabilization Policy and Unemployment

- In order to understand the importance of stabilization policy, we need to understand why unemployment exists.

- There are several reasons to believe that unemployed workers exist in the long run. Economists call the unemployment rate in the long run *the natural rate of unemployment*. Unemployment in the short run can be seen the deviation of unemployment rate from the natural rate of unemployment.

- Unfortunately, the natural rate of unemployment may not be constant. Therefore, it is difficult to distinguish the short run deviation from the natural rate of unemployment.

The Natural Rate of Unemployment

- Let me provide a model to explain the natural rate of unemployment:
  - Suppose that s fraction of employed workers are separated every day.
  - Suppose that $\tilde{m}$ fraction of unemployment workers meets a suitable job and leave from an unemployment pool.

- Assume that $\tilde{m}$ is an increasing function of labor market tightness, Θ:

$$\tilde{m} = mq\left(\Theta\right), q'\left(\Theta\right) > 0, \Theta = \frac{v}{u}$$

where m is the productivity of matching, v is the vacancy rate and u is the unemployment rate.

The Natural Rate of Unemployment

- The dynamics of the number of unemployed workers is

$$U_{t+1} = U_t + sE_t - mq(\Theta_t) U_t,$$

where U_t is the number of unemployed workers at date t and E_t is the number of employed workers at date t.

- As unemployed workers and employed workers consists of labor force,

$$N = E_t + U_t$$

where N denotes labor force. Hence

$$
\begin{aligned}
U_{t+1} &= U_t + s(N - U_t) - mq(\Theta_t) U_t \\
&= U_t + sN - (s + mq(\Theta_t)) U_t
\end{aligned}
$$

The Natural Rate of Unemployment

- **Natural Rate of Unemployment:** I assume $U_{t+1} = U_t = U$ and $\Theta_t = \Theta$ in the steady state. Then

$$sN = (s + mq(\Theta)) U_t$$

Hence, the steady state unemployment rate is

$$
\begin{aligned}
u &\equiv \frac{U}{N} \\
&= \frac{s}{s + mq(\Theta)}
\end{aligned}
$$

- If matching probability is large, then unemployment rate is small: $mq(\Theta) \uparrow \Rightarrow u \downarrow$
- If the separation rate is large, then unemployment rate is large: $s \uparrow \Rightarrow u \uparrow$

The Natural Rate of Unemployment

- Evidence from Japanese labor market suggests that
  1. $mq(\Theta)$ and s has a negative correlation,
  2. while $mq(\Theta)$ declined over time, s increased over time.
- This evidence suggests that $mq(\Theta)$ and s are both influenced by a demand effect and a long run effect.
- In fact, an increase in s is coincided with a gradual increase in non-regular workers during the same period.

The Natural Rate of Unemployment

- What might explain this long run trend?
  - **Adaptation to Changes in Economic Environment:** Firms always face changes in economic environment. The arrival of new technology may force firm to fire unskilled workers, firms may need to relocate their plants in foreign countries under the pressure of global competition, or people, especially women, may prefer the flexibility more than before. Whatever the reasons, firms must adapt to new environment. Hence, the reallocation of labor is inevitable. They will potentially influence both s and $mq\left(\Theta\right)$. An increase in s, but not $mq\left(\Theta\right)$ suggests that Japanese firms might be struggling to adapt to new environment.

The Natural Rate of Unemployment

- Assuming that m and s are constant, our model predicts the relationship between Θ and u.

$$u = \frac{s}{s + mq\left(\Theta\right)}$$

  - An increasing in labor market tightness reduces the unemployment rate: $\Theta \uparrow \Rightarrow q\left(\Theta\right) \uparrow \Rightarrow u \downarrow$.

- This relationship brings theoretical predictions on the Beveridge curve on the steady state.

The Natural Rate of Unemployment

- Beveridge curve display the relationship between the vacancy rate and unemployment rate. Because u is a decrease in Θ, Beveridge curve is downward sloping.

Beveridge Curve
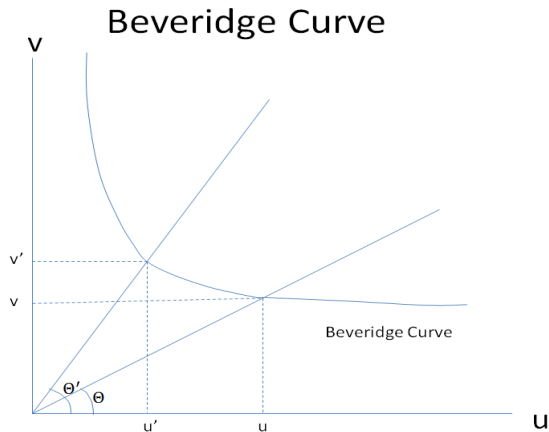
# The Natural Rate of Unemployment

- Obtaining data about u, $mq\,(\Theta)$, s and Θ, we can decompose the sources of the unemployment into three factors: m, s and Θ.

- For example, compare two years: 1997 and 2007.
  1. Unemployment rate u, matching probability $mq\,(\Theta)$, and separation rate s in 2007 are roughly the same as that in 1997.
  2. However, labor market tightness, Θ, in 2007 is much larger than that in 1997.
     - Because $mq\,(\Theta)$ is the same but Θ is higher in 2007 than 1997, m must be lower in 2007 than in 1997.

- It would be difficult, if not impossible, to justify a reduction of m in this period by a change in demand. It is likely that the mismatch has increased during this period. This means that a *frictional unemployment* has increased during this period.

- This reasoning is correct only if the data correctly captures Japanese economy. Abe (2014) questioned this point.

Frictional Unemployment

- **Frictional unemployment** occurs because finding a job takes time. It is not easy to find where a job offer is, what skill requirement of the job is and how much he expects to earn. It causes a temporal unemployment. If only a small number of jobs is open, m would be smaller and frictional unemployment is larger.

- Frictional unemployment causes a serious problem when changing jobs requires changes in skills or locations. This type of unemployment can be sometimes called **structural unemployment**.

Wait Unemployment

- Even if m and s are constant, if Θ increases, an unemployment rate can be reduced. Why do not the vacancies increase in the long run when there are many unemployed workers?

- Sticky real wage can be an alternative reason for unemployment in the long run. If real wage does not fall down to the equilibrium level, real wage cannot equate demand to supply. Therefore, we will observe excess supply of workers. Hence there is some rationing mechanism there. But what prevents real wage from falling down. This is called **wait unemployment**.

- Note that this is not sticky nominal wage. Hence, if a real wage is larger than an equilibrium wage, an increase in demand cannot help reducing unemployment.

Wait Unemployment

Wait Unemployment
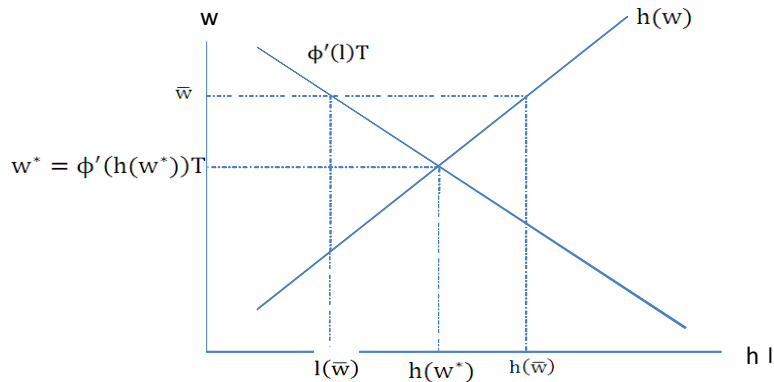
- There are several theories to explain why the real wage does not fall down.

  1. Several protection for workers..For example, if a minimum wage increases proportional to an increase in price, a real wage may not decline.
  2. Insider-Outsider Model...Incumbent workers resist their wage from falling down.
  3. Efficiency wage theory...If high wage induces a high productivity, a firm prefers to keep high wage.

Wait Unemployment

- There are several reasons why high wage brings high productivity.

  1. A high wage can increase workers' food consumption, which makes workers healthy and therefore productive.
  2. A higher wage can raise workers' incentive to work when managers cannot monitor it. When wage is higher than an equilibrium wage, workers in the firm receive rent. Since workers do not loose this rent, they will work harder in order to avoid the risk of being fired.
  3. If able person has high reservation wage because he can receive better offer from others, but if a firm does not know who is able person, offering a high wage raises the average quality of applicants.
  4. A high wage prevents skilled workers from quitting a job. When wage is higher than an equilibrium wage, skilled workers receives rent. Therefore, they are less likely to quit the job.
  5. A high wage may induce workers' effort because people are likely to take reciprocal actions.

Free Entry and Unemployment

- Even if incumbents try to keep wage high, new firms can offer lower wage. Hence, Insider-outsider model and efficiency wage theories may not explain unemployment if there is a free entry.

- Question: if there is a free entry, is it possible to support unemployment?

  - Answer: Yes, if firms must incur sunk costs (= the pre-investment specific to the employed workers) before production. Examples of such costs are search costs, training costs or pre-committed rental price.

- In other words, I claim that, *if there is no minimum wage and entry barriers, and if there are no sunk costs such as search costs and training costs, there is no unemployed workers in the long run and in the short run.*

Free Entry and Unemployment

- In addition, if we cannot write an explicit contract, observed unemployment is not social desirable because of hold up problems.
- **Hold Up Problems:** When there is the pre-investment, the lack of commitment on the returns to investment cause several problems. When a firm makes investment specific to a particular workers before production, a firm must expect to receive enough return from the investment. But if we cannot write an explicit contract on the investment, after the investment, workers can threaten firms to leave. Expecting this possibility, a firm hesitates to makes enough investment, which brings unemployment.
- There are two reasons that we cannot write an explicit contract on the investment.
    1. A firm must invest before meet workers: search costs.
    2. It is impossible or difficult to write a complete contract: training costs.

- **A Version of Caballero and Hammour (1996)**.
  - A firm searches and trains a worker by C_f and produces Ah output, where h is workers' relation specific human capital.
  - The rental cost, training cost and search cost is sunk. But, after paying this cost, the worker can walk away.

- Suppose that if the worker walks away, the worker expects to receive U, but a firm cannot receive anything today. Therefore, the surplus from this match is

$$S = Ah - U$$

Free Entry and Unemployment

- Assume that the firm cannot write and commit an wage before investment and that the wage is determined by the bargaining of two parties.
- On one hand, the worker's wage w from this bargaining can be written by

$$w = \beta S + U.$$

If the worker decides to leave, the worker expects to get U. This is the outside option of workers. The addition to this value, the worker can receive β part of surplus. We call β the bargaining power of workers. Hence, if $\beta S > 0$, workers always prefer being employed.

- On the other hand, If the worker leaves, a firm does not receive anything. Therefore, the firm expected profit from this match would be

$$J = (1 - \beta) S.$$

That is, the firm can receive $(1 - \beta)$ portion of surplus from this match.

Free Entry and Unemployment

- Before the production, the firm searches and trains workers. The free entry condition implies that

$$C_f = J$$

- Finally, we explain what determines the worker's outside option, U. The worker can find the similar job with probability $e = 1 - u$ where e is the employment rate and u is unemployment rate. On the other hand, with probability u, the worker cannot find a similar job and receives unemployment benefits or wage from the secondary market. This reservation value is denoted by $z < w$.

$$U = (1 - u) w + uz = w - u (w - z)$$

where is the wage paid by the similar job. It shows that the outside option of workers is lower than w due to the existence of unemployment.

Free Entry and Unemployment

- **Equilibrium:** equilibrium consists of $\{S, J, U, w, u\}$ that satisfies
  - Definition of Surplus

  $$S = Ah - U$$

  - Sharing Rule

  $$w = \beta S + U$$
  $$J = (1 - \beta) S$$

  - Free Entry Condition

  $$C_f = J$$

  - Worker's Outside Option

  $$U = w - u (w - z)$$

Free Entry and Unemployment

- Surplus: Substituting firm's profits into the free entry condition

$$C_f = (1 - \beta) S$$
$$S = \frac{C_f}{1 - \beta}$$

- The outside option of workers

$$S = Ah - U$$
$$U = Ah - S = Ah - \frac{C_f}{1 - \beta}$$

  - This means that the worker's outside option is smaller when the bargaining power of workers is large. If the bargaining power of workers is large, firms cannot expect much profits in the market and firms are reluctant to enter the market. This must lower the outside option of workers.

Free Entry and Unemployment

- Wage payment

$$
\begin{aligned}
w &= \beta S + U \\
&= \beta S + Ah - S \\
&= Ah - (1 - \beta) S \\
&= Ah - C_f
\end{aligned}
$$

- The equation implies that the wage is independent of the bargaining power. When the bargaining power is large, because the worker can receive the large share of surplus, the wage is large. On the other hand, when the bargaining power of workers is large, the worker's outside option is small. In this model, two opposite effects are always cancelled out and w is independent of β.

Free Entry and Unemployment

- Note that the outside option of workers is

$$U = w - u(w - z).$$

It means that the outside option of workers is small when the wage payment in the market is small or the number of unemployed workers is large. Because w is independent of β, but U is increase in β, an increase in β must increase the number of unemployed workers, u.

- Unemployment Rate

$$
\begin{aligned}
w - u(w - z) &= U \\
u(w - z) &= w - U \\
u &= \frac{\beta S}{w - z} = \frac{\beta \frac{C_f}{1 - \beta}}{w - z} \\
&= \frac{\beta C_f}{(1 - \beta)[Ah - C_f - z]}
\end{aligned}
$$

Free Entry and Unemployment

- Unemployment Rate

$$u = \frac{\beta C_f}{(1 - \beta)(Ah - C_f - z)}$$

- Unemployment is positive when a specific investment exists, C_f. Because of this specific investment, firms hesitate to enter the market. Therefore, job creation is smaller and unemployment is larger.

- Unemployment is larger when the bargaining power of worker, β, is larger. If $\beta = 0$, no unemployment. Similar to insider and outside model, a strong bargaining power of incumbent workers may increase unemployment rate.

Free Entry and Unemployment

- Question: If a firm can optimally choose β, does it choose 0?
  - Answer: No if the accumulation of relation specific investment requires the effort of workers.

- Assume that in order to accumulate h, a worker must pay cost $\frac{C_w}{2}h^2$, then

$$\max_h \left[w - \frac{C_w}{2}h^2 \right]$$

$$st. \; w = \beta S + U$$

$$S = Ah - U$$

- Hence

$$\max_h \left[\beta (Ah - U) + U - \frac{C_w}{2}h^2 \right]$$

- h: First order condition implies

$$\beta A = C_w h$$
$$h = \frac{\beta A}{C_w}$$

- If $\beta = 0$, $h = 0$. In this case, the profit of the firm $J = (1 - \beta) S = (1 - \beta) [Ah - U] = - (1 - \beta) U < 0$. Therefore, the firm must choose $\beta > 0$.

- Wage payment: Because the wage payment is expressed as

$$w = \beta S + U$$

and profit 0 condition implies

$$S = \frac{C_f}{1 - \beta} > 0.$$

the positive bargaining power $\beta > 0$ implies, the wage payment is larger than outside option, $w > U$. This is an essence of efficiency wage theory. Note that $C_f > 0$ is required for the positive surplus. That is, the efficiency wage theory also requires a firm's specific investment.

Unemployment in the Short Run

- In the short run the share of actually employed labor may deviate from the long run equilibrium level. Note that imperfect information model implies

$$e = h\left(\frac{W}{P^e}\right) = h\left(\frac{W}{P}\frac{P}{P^e}\right) = h\left(w\frac{P}{P^e}\right)$$

Because unemployment rate can be defined by $u = 1 - e$, the unemployment rate is an decreasing function of $\frac{P_t}{P_t^e}$.

$$u_t = u\left(\frac{P_t}{P_t^e}\right)$$

- Assume that

$$u\left(\frac{P_t}{P_t^e}\right) = u^n - \alpha \ln \frac{P_t}{P_t^e}$$

where u^n is the natural rate of unemployment.

- Then

$$
\begin{aligned}
u_t &= u^n - \alpha \left[\ln P_t - \ln P_{t-1} - \left(\ln P_t^e - \ln P_{t-1} \right) \right] \\
&= u^n - \alpha \left[g_{pt} - g_{pt}^e \right]
\end{aligned}
\tag{5}
$$

where $g_{pt} = \ln P_t - \ln P_{t-1}$ and $g_{pt}^e = \ln P_t^e - \ln P_{t-1}$.

Unemployment in the Short Run

- The equation, $u_t = u^n - \alpha \left[g_{pt} - g_{pt}^e \right]$, describes **Phillips curve**.

- Phillips curve shows a trade-off between high inflation and high unemployment rate in the short run. When government wishes to kill high inflation, government needs to accept a high unemployment rate. When government wants to reduce the unemployment rate, they must accept a high inflation rate.

- Assuming that Phillips curve is stable in the short run, government and Central Bank choose the optimal inflation rate and unemployment rate.

- Unfortunately, Phillips curve cannot be stable in the long run. The natural rate of unemployment and the expected inflation rate may change in the long run.

Unemployment in the Short Run

- Even if active stabilization policy is desired, economists doubt the ability of government to conduct timely stabilization policy. Politicians have always a pressure from their supporter. Politician may want to have boom right before their election.

- In order to avoid this difficulty, many countries give central bank a discretion of monetary policy. In this way, monetary policy can be conducted without political intervention. There is a clear negative relationship between the independence of central banks and inflation rates.

Unemployment in the Short Run

- Even if a central bank has a reasonable discretion. There is another difficult problem: **commitment**.

- Note that Phillips curve, $u_t = u^n - \alpha \left[g_{pt} - g_{pt}^e \right]$, shows that there is a trade-off between inflation rate and unemployment rate. It means that a central bank cannot reduce both the inflation rate and the unemployment rate at the same time.

- Suppose that Bank of Japan wants to minimize the following loss function:
$$L\left(u, \pi\right) = u_t + \frac{\gamma}{2} g_{pt}^2.$$

  Suppose that Bank of Japan directly controls inflation, but not the unemployment rate. Hence, it chooses g_{pt} in order to minimize the loss function given

$$u_t = u^n - \alpha \left[g_{pt} - g_{pt}^e \right]$$

Unemployment in the Short Run

- Assume that the central bank announces $g_{pt} = 0$ and assume that public believes this announcement. Then

$$u_t = u^n - \alpha g_{pt}.$$

Substituting this equation into the loss function,

$$L(u, \pi) = u^n - \alpha g_{pt} + \frac{\gamma}{2} g_{pt}^2$$

Therefore, the first order condition implies

$$\alpha = \gamma g_{pt}$$

and the optimal choice is

$$g_{pt} = \frac{\alpha}{\gamma} > 0.$$

Hence, after announcing $g_{pt} = 0$, Bank of Japan always have an incentive to break the promise and to choose a positive inflation in order to reduce unemployment. This is called **time inconsistency** problem.

Unemployment in the Short Run

- Once, public knows that Bank of Japan cannot commit their announcement and its best strategy is $g_{pt} = \frac{\alpha}{\gamma}$. Hence, $g_{pt}^e = \frac{\alpha}{\gamma}$.

$$u_t = u^n - \alpha \left[g_{pt} - \frac{\alpha}{\gamma} \right],$$

and

$$L(u, \pi) = u^n - \alpha \left[g_{pt} - \frac{\alpha}{\gamma} \right] + \frac{\gamma}{2} g_{pt}^2.$$

In this case the best strategy for Bank of Japan is again

$$g_{pt} = \frac{\alpha}{\gamma}.$$

Therefore, this is consistent with Public's expectation and we can sustain this equilibrium. In this case,

$$u_t = u^n.$$

Hence an active stabilization policy eventually brings a positive inflation and the natural rate of unemployment.

- Now consider passive stabilization policy. That is, Bank of Japan announces the rule of policy and commits the rule. Assume that this commitment is possible. In this example, assume that Bank of Japan announces $g_{pt} = 0$. Since there is a commitment device, public can believe that the inflation rate is 0: $g_{pt}^e = 0$. Then $u_t = u^n$.

- Note that a passive stabilization policy brings the better result for Bank of Japan. Because of time inconsistency problem, public cannot fully trust the bank's announcement unless they can actually commit the policy. Without commitment, large bank's discretionary power may make a worse situation.

Lucas's Critique and Micro Foundation

- So far, I assume a consumption function and a money demand function. Because these functions are the results of individual behaviors, changes in environment may influence the properties of these functions.

- Lucas (1976) argues that as people make decisions based on their expectation on the future economic environment, the expected future policy change is likely to influence their expectation, and, therefore, their decisions. It means that the estimated parameters on consumption functions and money functions are likely to change when a government changes its policy. So we cannot use the estimated parameters for policy simulations.

- Following Lucas's critique, many macroeconomists pay more attention to the micro foundation of the consumer's behavior and firm's behavior, and derives consumption function, investment function and money demand function. I would like to review these discussions.

Lucas's Critique and Consumption Function

- I would like to show the idea of Lucas's critique by using a simple consumer's decision problem. Let me consider the following consumption function

$$c = \phi_0 + \phi_1 x_t + \varepsilon$$

where x_t is disposable income.

- Once you obtain the parameters ϕ_1 and ϕ_2, one can conduct a policy simulation.

- Question: how much can we trust these estimated parameters? Lucas (1976) said that it might be good for a short run prediction. But it cannot be neither a basis of a policy evaluation nor a long run prediction.

- Let me demonstrate that Lucas's points here by using a simple example.

- Consider the following household's decision problem.

$$\max_{c_t, c_{t+1}} \left\{ u\left(c_t\right) + \beta u\left(c_{t+1}\right) \right\}$$

$$s.t.\ a_{t+1} + c_t = (1+\rho)\,a^* + w_t$$

$$a^* + c_{t+1} = (1+\rho)\,a_{t+1} + w_{t+1}$$

$$where\ s_t^n = a_{t+1} - a^*$$

Here we assume that household must leave the same amount of asset a^* as a bequest for the next generation and household can perfectly predict future.

Lucas's Critique and Consumption Function

- Permanent income

$$
\begin{aligned}
a^* + c_{t+1} &= (1+\rho)\left((1+\rho)\,a^* + w_t - c_t\right) + w_{t+1} \\
\frac{a^* + c_{t+1}}{1+\rho} &= (1+\rho)\,a^* + w_t - c_t + \frac{w_{t+1}}{1+\rho} \\
c_t + \frac{c_{t+1}}{1+\rho} &= \rho a^* + w_t + a^* - \frac{a^*}{1+\rho} + \frac{w_{t+1}}{1+\rho} \\
&= \rho a^* + w_t + \frac{\rho a^* + w_{t+1}}{1+\rho} \\
&= x_t + \frac{x_{t+1}}{1+\rho} \\
&= x^p
\end{aligned}
$$

where $x_t = \rho a^* + w_t$ and $x_t^p = x_t + \frac{x_{t+1}}{1+\rho}$. The parameter x_t^p is called a permanent income or a life time income.

- The budget constraint implies that

$$c_{t+1} = (1 + \rho)\left(x_t^p - c_t\right)$$

- Substituting the budget constraints into objective function, the original problem is rewritten as

$$\max_{c_t} \left\{ u\left(c_t\right) + \beta u\left[(1 + \rho)\left(x_t^p - c_t\right)\right] \right\}.$$

Lucas's Critique and Consumption Function

Lemma

Suppose that $y = g(x)$ and $z = f(y)$. Then

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx} = f'(y) g'(x)$$

Lucas's Critique and Consumption Function

- First Order Condition: Applying the lemma to $u(c_{t+1})$ and $c_{t+1} = (1+\rho)(x_t^p - c_t)$

$$0 = u'(c_t) - \beta(1+\rho)u'[c_{t+1}]$$

- For my simple analysis, assume that $u(c) = \gamma c - \frac{\eta}{2}c^2$

$$\gamma - \eta c_t = \beta(1+\rho)[\gamma - \eta c_{t+1}]$$

$$\gamma - \eta c_t = \beta(1+\rho)[\gamma - \eta(1+\rho)(x_t^p - c_t)]$$

$$\eta\left[\beta(1+\rho)^2 + 1\right]c_t = \gamma\beta(1+\rho) - \beta(1+\rho)[\gamma - \eta(1+\rho)x_t^p]$$

$$= (1 - \beta(1+\rho))\gamma + \beta\eta(1+\rho)^2 x_t^p$$

$$c_t = \frac{(1 - \beta(1+\rho))\gamma}{\eta\left[\beta(1+\rho)^2 + 1\right]} + \frac{\beta(1+\rho)^2}{\left[\beta(1+\rho)^2 + 1\right]}x_t^p$$

- Consumption Function:

$$c_t = A + B x_t^p$$

where

$$A = \frac{\gamma \left[1 - \beta \left(1 + \rho\right)\right]}{\eta \left[1 + \beta \left(1 + \rho\right)^2\right]}$$

$$B = \frac{\beta \left(1 + \rho\right)^2}{1 + \beta \left(1 + \rho\right)^2}$$

The Alternative Derivation of Consumption Function with High School Math

- For my simple analysis, assume that $u(c) = \gamma c - \frac{\eta}{2} c^2$.

$$\beta u \left[(1+\rho)(x_t^p - c_t) \right]$$

$$= \beta \gamma \left((1+\rho)(x_t^p - c_t) \right) - \frac{\beta \eta}{2} \left[(1+\rho)(x_t^p - c_t) \right]^2$$

$$= \beta \gamma (1+\rho)(x_t^p - c_t) - \frac{\beta \eta (1+\rho)^2}{2} (x_t^p - c_t)^2$$

$$= \beta \gamma (1+\rho)(x_t^p - c_t) - \frac{\eta (1+\rho)^2}{2} \left[(x_t^p)^2 - 2x_t^p c_t + c_t^2 \right]$$

$$= x^* + \beta \left[\eta (1+\rho)^2 x_t^p - \gamma (1+\rho) \right] c_t - \frac{\beta \eta (1+\rho)^2}{2} c_t^2$$

where $x^* = \beta \left[\gamma (1+\rho) x_t^p - \frac{\eta (1+\rho)^2}{2} (x_t^p)^2 \right]$.

The Alternative Derivation of Consumption Function with High School Math

- The maximization Problem

$$\max_{c_t} \left\{ u\left(c_t\right) + \beta u\left[\left(1+\rho\right)\left(x_t^p - c_t\right)\right]\right\}.$$

$$= \max_{c_t} \left\{ \begin{array}{c} \gamma c_t - \frac{\eta}{2} c_t^2 + x^* + \\ \beta \left[\eta\left(1+\rho\right)^2 x_t^p - \gamma\left(1+\rho\right)\right] c_t - \frac{\beta\eta\left(1+\rho\right)^2}{2} c_t^2 \end{array} \right\}.$$

- The first order condition is

$$0 = \gamma - \eta c_t + \beta \left[\eta\left(1+\rho\right)^2 x_t^p - \gamma\left(1+\rho\right)\right] - \beta\eta\left(1+\rho\right)^2 c_t$$

The Alternative Derivation of Consumption Function with High School Math

- Consumption Function: From the first order condition,

$$\eta \left[1 + \beta \left(1 + \rho \right)^2 \right] c_t$$
$$= \gamma \left[1 - \beta \left(1 + \rho \right) \right] + \beta \eta \left(1 + \rho \right)^2 x_t^p.$$

Hence

$$c_t = A + B x_t^p$$

where

$$A = \frac{\gamma \left[1 - \beta \left(1 + \rho \right) \right]}{\eta \left[1 + \beta \left(1 + \rho \right)^2 \right]}$$

$$B = \frac{\beta \left(1 + \rho \right)^2}{1 + \beta \left(1 + \rho \right)^2}$$

Lucas's Critique and Consumption Function

- Let me assume that $\rho a^* + w_{t+1} = (1 - \tau_x) * (\rho a^* + w_t)$.

$$
\begin{aligned}
x_t^p &= \rho a^* + w_t + \frac{\rho a^* + w_{t+1}}{1 + \rho} \\
&= \rho a^* + w_t + \frac{(1 - \tau_x) * (\rho a^* + w_t)}{1 + \rho} \\
&= \left[1 + \frac{1 - \tau_x}{1 + \rho} \right] [\rho a^* + w_t] \\
&= \left[\frac{2 + \rho - \tau_x}{1 + \rho} \right] x_t
\end{aligned}
$$

Lucas's Critique and Consumption Function

- The empirically testable equation is

$$
\begin{aligned}
c_t &= A + B x_t^p = A + B \left[\frac{2 + \rho - \tau_x}{1 + \rho} \right] x_t, \\
&= \phi_0 + \phi_1 x_t + \varepsilon_t
\end{aligned}
$$

where $\phi_0 + \varepsilon_t = A$ and $\phi_1 = B \left[\frac{2 + \rho - \tau_x}{1 + \rho} \right]$.

- Note that τ_x can change ϕ_1. Based on the estimated ϕ_0 and ϕ_1, a policy maker can simulate the impact of tax policy on aggregate consumption. But in fact, the future tax change affects ϕ_1 itself. Hence we cannot trust any estimations ϕ_1 for a basis of a policy evaluation. If we consider a general equilibrium effect, the results are more devastated. Since most likely policy changes will affect ρ, it changes the parameters ϕ_0 and ϕ_1. In other word, the parameters that are estimated from the reduced form estimation are not robust.

Lucas's Critique and Consumption Function

- After Lucas's critique, macroeconomists agree that it is important to examine the micro foundation of macroeconomics.

- Macro economists assume that changes in policy cannot influence preference and technology such as γ, β and η. Hence, once we identify γ, β and η, we may be able to conduct a policy simulation based on these parameters. For this purpose, macroeconomic model must be based on a reasonable micro foundation.

- Currently, most macro economists use the **dynamic stochastic general equilibrium model** as a foundation of macroeconomics.

Permanent Income Hypothesis

- The previous model derives current consumption as a function of a permanent income.

$$c_t = A + Bx_t^p$$

- The permanent income hypothesis has a strong implication: the temporal movement of income has little impact on consumption. Since the marginal benefit from consumption is decreasing, consumers prefer stable consumption to unstable consumption. Therefore, if it is possible, consumers have always incentive to smooth their consumption. When consumption depends on the permanent income, it is possible to smooth consumption by exchanging consumption today and tomorrow.

- Note that this implies that the even if a tax cut increases disposal income, it may fail to increase private consumption, and therefore, aggregate demand.

- To see this implication formally, note the first order condition of the previous problem implies

$$\beta \left(1 + \rho\right) \left[\gamma - \eta c_{t+1}\right] = \gamma - \eta c_t$$

Assume that $\rho_{t+1} = \rho$ is constant, which is satisfied by steady state. Then the equation implies that current consumption is the best predictor of the future consumption. Once we observe current consumption, we do not need any other information to predict c_t.

Permanent Income Hypothesis

- In particular, if $\beta = \frac{1}{1+\rho}$, then

$$c_{t+1} = c_t.$$

In this extreme case, the expected consumption is the same over time. That is, the predicted value of the future consumption is just current consumption, and a change in consumption is unpredictable.

$$\Delta c_t = 0$$

where $\Delta c_t = c_{t+1} - c_t$.

Permanent Income Hypothesis

- Evidence: Hall (1978) tested this observation. He cannot reject the hypothesis that lagged values of either income and consumption cannot predict a change in consumption. This result supports the implication of the permanent income hypothesis. After Hall (1978), much empirical research was conducted for this issue. For example, did the following regression: Campbell and Mankiw (1989)

$$\Delta c_t = \lambda \Delta x_t + v_t,$$
$$where\ v_t = (1 - \lambda)\,\varepsilon_t,$$

where ε_t is the change in consumers' prediction of their permanent income. Hence, if the permanent income hypothesis is right, λ is close to 0, and if the traditional theory is correct, λ is close to 1. They found $\lambda = 0.42\sim0.52$. This result indicates that consumption partially responds to disposable income.

Permanent Income Hypothesis

- If a change in current income has an impact on a change in consumption, what explains this behavior.

  1. *Serially Correlated Income*: When today's income is highly correlated with the future income, consumers can easily predict the steam of the future income based on the current income. Remember that assuming $\rho a^* + w_{t+1} = (1 - \tau_x) * (\rho a^* + w_t)$, I show that

  $$x_t^p = \left[\frac{2 + \rho - \tau_x}{1 + \rho} \right] x_t.$$

  It shows that a rise in current income increases the permanent income, and therefore consumption.

  2. *Liquidity Constraint:* If some consumers cannot borrow enough money, they cannot buy consumption goods more than their income. Therefore, current disposable income limits consumption:

  $$c_t \leq x_t.$$

  If the optimal consumptions of many household are constrained by current income, and if current income increases, then obviously, consumers will increase their consumption.

Ricardian Equivalence and Government Debt

- One of the interesting application of the permanent income hypothesis is government debt. When government finances its expenditure by taxes, we don't see any multiplier effect even in the short run. What if government issues its bond. In this way, it does not increase tax burden today.

- However, if consumers care about their permanent income, they are worried not only about today's income, but tomorrow's income. Since today's debt can be seen as the future tax burden, it does not change their permanent income. Therefore, consumers do not change their consumption decision. This is called **Ricardian Equivalence**.

Ricardian Equivalence and Government Debt

- Let me formally analyze Ricardian Equivalence. Assume that population is 1 for a simple analysis. In order to finance government expenditure, g_t, government either imposes a lump sum tax: τ_t or issues a bond, b_t. Assume that when government issues a bond at date t, it must pay back to consumers at date $t+1$. , the government's budget constraint is

$$
\begin{aligned}
b_{t+1} &= g_t - \tau_t, \\
g_{t+1} &= \tau_{t+1} - (1+\rho)\, b_{t+1}.
\end{aligned}
$$

- We can derive a government's intertemporal budget constraint.

$$
\begin{aligned}
g_{t+1} &= \tau_{t+1} - (1+\rho)\,(g_t - \tau_t) \\
\tau_t + \frac{\tau_{t+1}}{1+\rho} &= g_t + \frac{g_{t+1}}{1+\rho}.
\end{aligned}
$$

This equation shows that the present value of tax revenue must be equal to the present value of government expenditure.

Ricardian Equivalence and Government Debt

- Let $w_t^* = w_t - \tau_t$. Following the previous argument, it is shown that

$$
\begin{aligned}
c_t + \frac{c_{t+1}}{1+\rho} &= \rho_t^a a^* + w_t^* + \frac{\rho a^* + w_{t+1}^*}{1+\rho} \\
&= \rho a^* + w_t - \tau_t + \frac{\rho a^* + w_t - \tau_{t+1}}{1+\rho} \\
&= \rho_t a^* + w_t + \frac{\rho a^* + w_t}{1+\rho} - \left(\tau_t + \frac{\tau_{t+1}}{1+\rho} \right) \\
&= \rho_t a^* + w_t + \frac{\rho a^* + w_t}{1+\rho} - \left(g_t + \frac{g_{t+1}}{1+\rho} \right)
\end{aligned}
$$

- Note that the budget constraint does not depend on neither tax nor bond. That is, issuing bond does not change consumers' permanent income, therefore it does not change consumption.

Ricardian Equivalence and Government Debt

- *Deviation from Ricardian Equivalence:* In reality, Ricardian equivalence does not perfectly hold. There are several reasons that Ricardian Equivalence may not hold.

  1. *Liquidity Constraint:* If there is liquidity constraints, the reduction of tax can increase disposable income and raises consumption. More importantly, if a government faces liquidity constraint because of the fear of default, by definition, government cannot issue a bond to finance expenditure.

  2. *Transfer across generations:* If parents do not care about their children, parents can enjoy low tax today and enforce their children to pay for them. It means that if they care their children like themselves, Ricardian equivalence can still hold. However, if they do not care their children, an increase in government debt might induce their demand.

  3. *Distortional tax:* If a change in a tax influences the marginal benefit or cost of consumption, it affects consumer's decision and consumption. In particular, if government raises capital income tax, consumers are discouraged to save and increases consumption. In this case, obviously tax schedule matters.

- Students must hand assignment 7 in at the next lecture.

Final Words

- Congratulation. This the end of this lecture. You are now standing in front of the door of a graduate level macroeconomics. It uses the dynamic stochastic general equilibrium model as a basic tool. If you find it interesting, see you again somewhere.