

## Answer for Homework 4: Modern Macroeconomics I\*

1. Assume that

$$T_{t+1} = BN_t^T T_t + T_t \quad (1)$$

where  $B$  is the parameter,  $T_t$  is the level of knowledge at date  $t$  and  $N_t^T$  is population in knowledge accumulation sector at date  $t$ .

(a) Given this assumption, the growth rate of knowledge accumulation is proportional to population. What is an economic rational for this assumption?

**Answer** From equation (1), we have

$$\begin{aligned} T_{t+1} &= BN_t^T T_t + T_t \\ (1 + g_T) T_t &= BN_t^T T_t + T_t \\ g_T &= BN_t^T \end{aligned}$$

This implies that the model has a scale effect: If everybody has an equal probability to invent new idea, the larger the population of the knowledge accumulation sector, the higher the probability to find new invention.

(b) Jones (1995) criticizes equation (1). Summarize the essence of his arguments.

**Answer** Equation (1) implies that a knowledge accumulation is proportional to  $N_t$ . However, evidences in the OECD countries shows that the number of scientists engaged in R&D has dramatically increased, but the growth rate of TFP is quite stable. This evidence is against the above theory.

(c) Jones (1995) provides an alternative functional form:

$$T_{t+1} = BN_t^T T_t^\beta + T_t, \quad 0 < \beta < 1.$$

Assume that economy is on the steady state: that is  $g_T = g$ . Show that  $g$  is proportional to population growth in the knowledge accumulation sector,  $n$ .

**Answer** Note that

$$\begin{aligned} T_{t+1} &= BN_t^T T_t^\beta + T_t \\ (1 + g_T) T_t &= BN_t^T T_t^\beta + T_t \\ g_T &= \frac{B_t N_t^T}{T_t^{1-\beta}} \end{aligned}$$

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Then, we have

$$\begin{aligned}
g_{gT} &= \frac{g_{B_t N_t^T}}{T_t^{1-\beta}} \\
g_g &= g_{B_t} + g_{N_t^T} - (1-\beta) g_{gT} \\
0 &= g_{N_t^T} - (1-\beta) g \\
g &= \frac{g_{N_t^T}}{1-\beta}
\end{aligned}$$

Therefore,  $g$  is proportional to population growth in the knowledge sector,  $n$ .

2. Consider the following production function

$$Y = K^\alpha (TN)^{1-\alpha},$$

where  $Y$  is GDP,  $K$  is capital stock,  $N$  is the number of population and  $T$  is an unknown productivity measure. Suppose that productivity is the same across countries and  $\alpha = \frac{1}{3}$ .

(a) Suppose that  $T = Ah$ , where  $h$  is human capital and  $A$  is unknown productivity. Express GDP per capita as a function of capital output ratio, human capital and unknown productivity.

**Answer** Note that

$$\begin{aligned}
Y &= K^\alpha (TN)^{1-\alpha} \\
&= K^\alpha (AhN)^{1-\alpha} \\
1 &= \left(\frac{K}{Y}\right)^\alpha \left(\frac{AhN}{Y}\right)^{1-\alpha} \\
\left(\frac{Y}{N}\right)^{1-\alpha} &= \left(\frac{K}{Y}\right)^\alpha (Ah)^{1-\alpha} \\
y_t &= \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} Ah
\end{aligned}$$

(b) Economists find that the large proportion of income differences can be explained by unknown productivity differences. What would be the possible source of  $A$ ?

**Answer** Resource allocation is currently regarded as the potential source of  $A$ .

3. Any activities can be decomposed into productive activity and unproductive activity.

(a) What is the main difference in two activity?

**Answer** Productive activities are any kinds of activities which reallocate the output with making any additional value. On the other hand, unproductive activities are any kinds of activities which merely reallocate the output without making any additional value.

(b) Give us 5 examples of unproductive activity.

**Answer** The following activities are unproductive.

- i. Crime.
- ii. Lobbying for tax benefits.
- iii. Political corruption.
- iv. Frivolous lawsuits.
- v. Resistance to technology adoption

(c) What policies might the less-developed country pursue to raise its level of income?

**Answer** The government in the less-developed country might raise its level of income by consolidating property rights.