Answer for Homework 6: Modern Macroeconomics I*

1. Assume that population is equal to 1. Consider the following IS-LM curve.

$$y = c(y - \tau) + \iota(\rho) + g \tag{2}$$

$$\frac{m^s}{P} = k(\rho)\, y \tag{3}$$

where price level P, tax revenue τ, government expenditure g and money supply m^s are given. Two equations determine GDP, y and the interest rate ρ. Suppose that

$$\begin{aligned} c(y - \tau) &= c_0 + c_1 \times (y - \tau) \\ k(\rho) &= \frac{k}{\rho} \\ \iota(\rho) &= -a\rho \end{aligned}$$

where c_0, c_1, k, and a are parameters.

(a) Assume that the interest rate ρ is constant. Derive GDP y, as a function of government expenditure and tax revenue. Show that $\frac{dy}{dg} > 1$ and explain its reason.

Answer Equation (2) can be rewritten as

$$\begin{aligned} y &= c(y - \tau) + \iota(\rho) + g \\ &= c_0 + c_1 \cdot (y - \tau) + -a\rho + g. \end{aligned}$$

By solving this equation with respect to y, we have

$$y = \frac{g + c_0 - \tau c_1 - a\rho}{1 - c_1}$$

GDP is a function of government expenditure and tax revenue. By differentiating this equation with respect to g,

$$\frac{dy}{dg} = \frac{1}{1 - c_1} > 1$$

*I thank to Wataru Tamura who made these sample answers.

Figure 1: GDP and interest rate in equilibrium.

Figure 2: Effect of an increase in tax revenue.

for all $c_1 \in (0,1)$. This implies that an increase in the government expenditure generates the multiplier effect. The increase in government expenditure rises not only the nation's income directly but also the consumption indirectly.

(b) Derive the GDP, y, and the interest rate, ρ, as the function of P, g, m^s, τ.

Answer Given P, g, m^s, τ, there are two equations (2) and (3) and two unknown variables, y and ρ. From (2),

$$y = -\frac{a}{1-c_1}\rho + \frac{g + c_0 - \tau c_1}{1-c_1}, \tag{IS}$$

and from (3), we have

$$\rho = \frac{kP}{m^s} y. \tag{LM}$$

Therefore, we can derive y and ρ as an intersection of these two lines (see Figure 1).

$$\begin{aligned} y(g,\tau,m^s,P) &= \frac{m^s(c_0 - c_1\tau + g)}{akP + m^s(1-c_1)} \\ \rho(g,\tau,m^s,P) &= \frac{Pk(c_0 - c_1\tau + g)}{akP + m^s(1-c_1)}. \end{aligned}$$

(c) What is the effect of an increase in tax revenue, τ, on GDP and the interest rate? Explain economic logic behind your answer.

Answer We analyze the effect of an increase in tax revenue, τ, on GDP and the interest rate. Note that IS curve depends on τ but LM curve does not depend on τ. When tax revenue increases, IS curve shifts down (See Figure 2). Both GDP and the interest rate decrease. The economic logic for this result is as follows. First, IS curve implies that an increase in τ directly decreases demand and GDP (see (IS)):

$$\tau \uparrow \Rightarrow y \downarrow$$

Second, the decrease in GDP reduces the demand for money (m^d). When the price level and money supply are constant, this actually decreases the interest rate in the money market (see (LM)):

$$\tau \uparrow \Rightarrow y \downarrow \Rightarrow m^d \downarrow \Rightarrow \rho \downarrow$$

Since investment $i = \iota(\rho) = -a\rho$, a decrease in ρ leads to an increase in the investment i :

$$\tau \uparrow \Rightarrow y \downarrow \Rightarrow m^d \downarrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow$$

Figure 3: Effect of an increase in government expenditure.

Figure 4: Effect of an increase in money supply.

This indirectly increases y

$$\tau \uparrow \Rightarrow y \downarrow \Rightarrow m^d \downarrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow \Rightarrow y \uparrow$$

In this model, the direct effect dominates the indirect effect. Therefore, the increase in τ decreases both GDP and the interest rate.

(d) What is the effect of an increase in government expenditure, g, on GDP and the interest rate? Explain economic logic behind your answer.

Answer Note that IS curve depends on g but LM curve does not depend on g. When government expenditure, g, increases, IS curve shifts up (see Figure 4). Both GDP and the interest rate increase. The economic logic for this result is as follows. First, the IS curve implies that an increase in g directly increases demand and GDP (see (IS)):

$$g \uparrow \Rightarrow y \uparrow$$

Second, the increase in GDP increases the demand for money. When the price level and money supply are constant, this actually increases the interest rate in the money market (see (LM)):

$$g \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow$$

Since $i = \iota(\rho) = -a\rho$, an increase in ρ leads to the decreases in the investment i :

$$g \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow$$

This indirectly decreases y

$$g \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow$$

In this model, the direct effect dominates the indirect effect. Therefore, the increase in g increases both GDP and the interest rate.

(e) What is the effect of increase in money supply, m^s, on GDP and the interest rate? Explain economic logic behind your answer.

Answer Note that IS curve does not depend on m^s but LM curve depends on m^s. When money supply, m^s, increases, LM curve shifts down (see Figure 4). GDP increases but the interest rate decreases.
The economic logic for this result is as follows. From the LM curve (LM), the increase in the money supply lowers the market value of money, and therefore directly lowers the interest rate

$$m^s \uparrow \Rightarrow \rho \downarrow$$

However, the IS curve (IS) implies that a decrease in the interest rate directly increases the investment and GDP

$$m^s \uparrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow \Rightarrow y \uparrow$$

The increase in GDP increases the demand for money. When the price level and money supply are constant, this actually increases the interest rate in the money market:

$$m^s \uparrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow$$

This indirectly decreases investment i and GDP y:

$$m^s \uparrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow$$

In this model, the direct effect dominates the indirect effect. Therefore, the increase in m^s increases GDP but decreases the interest rate.

(f) What is the limitation of IS-LM model?

Answer IS-LM model is static. In order to analyze long-term changes or effects by policies, we need a dynamic model. A static model cannot answer what is the temporal effect and what is the permanent effect for example.

2. Answer the following questions.

(a) A sticky nominal wage is considered as one of the source of an increasing aggregate supply curve. Explain its logic.

Answer Assume that the nominal wage is fixed in the short run: $W = \bar{W}$. Note that in the short run equilibrium,

$$w = \phi'(l_e)\, T,$$

where $\phi(l_e) \equiv F(1, l_e)$ and $\phi'(l_e) > 0$, $\phi''(l_e) < 0$. Since labor demand curve is a decreasing in the real wage, if the nominal wage is rigid, labor demand must be increasing in P:

$$P \uparrow \Rightarrow w = \frac{\bar{W}}{P} \downarrow \Rightarrow \phi'(l_e)\, T \downarrow \Rightarrow l_e \uparrow .$$

Therefore, the sticky nominal wage is one of the source of an increasing aggregate supply curve.

(b) An imperfect information can be considered as another reason for an increasing aggregate supply curve. Explain its logic.

Answer Assume that firms observe output price P, but workers cannot. Hence, workers must make information about price. Workers' expected price is de-

noted as P^e. Then workers respond to $\frac{W}{P^e}$:

$$\begin{aligned} h_e\left(\frac{W}{P^e}\right) &= h_e\left(\frac{W}{P}\frac{P}{P^e}\right) \\ &= h_e\left(w\frac{P}{P^e}\right). \end{aligned}$$

Suppose that the overall price level P goes up. However, workers do not know a change in aggregate price. Hence, P^e stays the same. Hence, $\frac{P}{P^e}$ goes up and supply curve shifts to right.

(c) How do two explanations change policy implications?

Answer Because there is unemployed workers under sticky nominal wage model, an increase in demand can increase GDP by employing more workers. However, if the imperfect information model is correct, there are no unemployed workers. If active stabilization policy itself brings the uncertain movement of price, it may increase workers' further misperception. It reduces the welfare of an economy.