Answer for Homework 7: Modern Macroeconomics I*

1. Suppose that s fraction of employed workers are separated. Assume that m proportion of unemployment workers meets such a job. Let E and U denote the number of employed workers and unemployed workers, respectively.

   (a) Derive the dynamics of the number of unemployed workers.

   Answer Note that the number of unemployed workers at period $t+1$ who are employed workers at period t is sE_t and that the number of unemployed workers at period $t+1$ who are unemployed workers at period t is $(1-m)\,U_t$. Then, the total number of unemployed workers at period $t+1$ is

   $$U_{t+1} = sE_t + (1-m)\,U_t.$$

   Therefore, the dynamics of unemployment is

   $$U_{t+1} = U_t + sE_t - mU_t.$$

   (b) Suppose that $U_{t+1} = U_t$ on the steady state. The natural rate of unemployment is defined as the unemployment on the steady state. Derive the natural rate of unemployment as a function of s and m.

   Answer In the steady state, we have

   $$sE_t = mU_t.$$

   Since unemployed workers and employed workers consists of labor force:

   $$N_t = E_t + U_t$$

   where N_t denotes labor force. Then, we have

   $$E_t = N_t - U_t.$$

   Therefore, we have

   $$\begin{aligned} s\,(N_t - U_t) &= mU_t \\ s\left(1 - \frac{U_t}{N_t}\right) &= m\frac{U_t}{N_t} \end{aligned}$$

*I thank to Wataru Tamura who made these sample answers.

By solving this equation with respect to U_t/N_t, we have natural rate of unempolyment in the steady state:

$$\begin{aligned}\frac{U_t}{N_t} &= \frac{s}{s+m} \\ &= \frac{1}{1+\frac{m}{s}}.\end{aligned}$$

(c) What influences the natural rate of unemployment through changes in s and m. Discuss it.

Answer People may change their taste and it may reduce demand for an industry. Emergence of new technology may destroy job opportunities for old skill. Thus, a change in economic environment may cause mismatch in the labor market and may increase s. However, when a new business demands more people, it will increase m and reduce frictional unemployment. Hence, the net effect is ambiguous.

2. Suppose that Bank of Japan wants to minimize the following loss function by choosing the inflation rate at date t, g_{pt},

$$L(u,\pi) = u_t + \frac{\gamma}{2} g_{pt}^2,$$

subject to the following Phillips curve,

$$u_t = u^n - \alpha\left[g_{pt} - g_{pt}^e\right]$$

where u_t is the unemployment rate at date t, u^n is the natural rate of unemployment g_{pt} is the inflation rate, g_{pt}^e is the expected inflation rate held by Japanese people, and α and γ are parameters.

(a) Suppose that the Bank of Japan announces that it will choose 0 inflation rate, but they can not commit their announcement. What would be the equilibrium unemployment rate and inflation rate?

Answer Timing of this model is as follows. First, Bank of Japan announces the inflation rate. Second, given this announcement, Japanese people expected the inflation rate, g_{pt}^e. Third, given Japanese people's expectation, Bank of Japan determines the inflation rate, g_{pt}. Finally, equilibrium outcome is observed. We first examine Bank of Japan's optimal choice of inflation rate, given the expected inflation rate held by Japanese people. The minimization problem for Bank of Japan given g_{pt}^e is

$$\min_{g_{pt}} = u^n - \alpha\left[g_{pt} - g_{pt}^e\right] + \frac{\gamma}{2} g_{pt}^2$$

FOC is

$$\alpha = \gamma g_{pt}^*$$

Therefore, the optimal choice of inflation rate is

$$g_{pt}^* = \frac{\alpha}{\gamma} > 0$$

This implies that at the decision stage of interest rate, Bank of Japan has an incentive to choose the positive inflation rate, α/γ, regardless of Japanese people's expectation. Therefore, if Bank of Japan can not commit it will choose 0, then Japanese people expect that the inflation rate becomes α/γ. Therefore, the equilibrium unemployment rate and the inflation rate become

$$u_t^{**} = u^n$$
$$g_{pt}^{**} = \frac{\alpha}{\gamma}$$

This implies that unemployment does not change but the inflation occurs. The social loss is $u^n + \alpha^2/2\gamma$.

(b) Suppose that the Bank of Japan announces that it will choose 0 inflation rate and builds up a commitment device (for example, the Bank of Japan writes the chosen inflation rate in Japanese constitution). What would be the equilibrium unemployment rate and inflation rate?

Answer Since Bank of Japan builds up a commitment device, Japanese people believe and expect that the inflation rate is zero. Then, Bank of Japan chooses zero inflation rate. The equilibrium unemployment rate and the inflation rate become

$$u_t^c = u^n$$
$$g_{pt}^c = 0$$

In this case, the social loss is u^n.

(c) What can you learn from your answers in question (a) to (b)?

Answer By comparing the results, social loss when Bank of Japan builds up commitment device is smaller. This implies that establishment of commitment device leads to socially better equilibrium outcome. This result follows from time inconsistency problem. Note that social loss is minimized[1] when $g_{pt}^e = 0$ and $g_{pt} = \alpha/\gamma$. Before announces the inflation rate, Bank of Japan wants to set zero inflation rate. However, after announces the inflation rate, it has an incentive to break the announcement and to choose positive inflation rate in order to reduce unemployment. Because of time inconsistency problem, public can not fully trust government announcement unless they actually commit the policy. In this case, more government's discretionary power makes worse situation.

3. Answer following questions.

[1] Assume that the $\pi_t^e \geq 0, \pi_t \geq 0$.

(a) The permanent income hypothesis predicts that a temporal change in current income should not influence a change in consumption. However, data shows that a change in consumption is partially influenced by the movement of current income. What is two possible explanations?

Answer There are two possible reasons.

Serially correlated income When today's income is highly correlated with the future income, consumers can easily predict the steam of the future income based on the current income. In the labor market, when consumers are promoted, they typically expect not only a high income today, but also a high income in the future. Hence, a change in current income can be seen as the change in the permanent income.

Liquidity constraint The permanent income hypothesis implicitly assumes that every consumer can borrow money as long as they can return it in the future. But if some consumers can not borrow enough money, they can not buy consumption goods more than their income. Therefore, current disposable income limits consumption:

$$C_t \leq Y_t.$$

If many consumers are constrained by current income, and if current income increases, then obviously consumers will increase their consumption. In this way, a change in current income may affect a change in consumption.

(b) What is Ricardian equivalence? Explain its logic.

Answer The general principle is that government debt is equivalent to futures taxes, and if consumers are sufficiently forward looking, future taxes are equivalent to current taxes. Hence, financing the government by debt is equivalent to current taxes. The implication of Ricardian equivalence is that a debt-financed tax cut leaves consumption unaffected. When consumers care about their permanent income, they are worried not only today's income but also tomorrow's income. Since today's government debt can be seen as the future tax burden, it does not change their permanent income. Therefore, consumers do not change their consumption decision.

(c) Data shows that Ricardian equivalence does not literally hold. What are three possible reasons?

Answer There are three possible reasons.

Transfer among generations: If parents do not care about their children, parents can enjoy low tax today and enforce their children to pay for them. Hence, the issue is how much parents care about their children. If they care their children like themselves, Ricardian Equivalence can still hold. However, if they do not care their children, an increase in government debt induce their demand.

Liquidity constraint: If there is liquidity constraints, the reduction of tax can increase disposable income and raise consumption.

Distortional tax: If a tax change the marginal benefit or cost of consumption, it affects consumers' decision and affect consumption. In particular, if government raises capital income tax, consumers are discouraged to save and increase consumption.

4. Suppose that a consumer lives two periods, dates t and $t+1$. In each period, he earns the same labor income w and initially does not have any asset. When he receive his income at date t, he can consume c_t or leave it for tomorrow, c_{t+1}. Assume that he consume every income at date $t+1$ and does not leave asset for future. When he saves for $t+1$, he can earn the interest rate ρ at date $t+1$. Assume that the consumer has an instantaneous utility function $U(c)$ and discount factor, β. Suppose that there is no tax at date t. Government will levy a lump sum tax, τ at date $t+1$.

   (a) Formulate the consumer's decision problem.

   Answer The consumer's utility function takes the following form

$$U(c_t) + \beta U(c_{t+1})$$

   The budget constraint in period t is

$$s_t + c_t = w$$

   where s_t is saving for t. Similarly, the budget constraint in period $t+1$ is

$$c_{t+1} = w + (1+\rho) s_t - \tau.$$

   Combining these budget constraints, we have the lifetime budget constraint:

$$c_t + \frac{c_{t+1}}{1+\rho} = w + \frac{w-\tau}{1+\rho}.$$

   Thus, the consumer's decision problem is to choose the consumption in period t as follows:

$$\max_{w \geq c_t \geq 0} U(c_t) + \beta U\left((1+\rho)\left(x^P - c_t\right)\right)$$

   where $x^P = w + \frac{w-\tau}{1+\rho}$ denotes the permanent income.

   (b) Assume that the consumer has a quadratic instantaneous utility function: $U(c_t) = ac_t - \frac{b}{2}c_t^2$. Derive the first order condition of the problem above.

   Answer When the consumer has a quadratic instantaneous utility function, the consumer's decision problem above can be rewritten as

$$\max_{c_t \geq 0} \left(ac_t - \frac{b}{2}c_t^2\right) + \beta\left\{a\left[(1+\rho)(x^P - c_t)\right] - \frac{b}{2}\left[(1+\rho)(x^P - c_t)\right]^2\right\}$$

   Using the Chain rule. For $U(c_{t+1}) = ac_{t+1} - \frac{b}{2}c_{t+1}^2$ and $c_{t+1} = g(c_t) =$

$(1+\rho)(x^P - c_t)$, the chain rule gives

$$\begin{aligned}\frac{dU(g(c_t))}{dc_t} =& \frac{dU(c_{t+1})}{dc_{t+1}}\frac{dg(c_t)}{dc_t} \\ =& U'^P - c_t))\{-(1+\rho)\} \\ =& -(1+\rho)\left\{a - b(1+\rho)(x^P - c_t)\right\}.\end{aligned}$$

Then the first-order condition is given by

$$\begin{aligned}&U'(c_t) - \beta(1+\rho)U'(c_{t+1}) = 0, \text{ or} \\ &\{a - bc_t\} - \beta(1+\rho)\left\{a - b(1+\rho)(x^P - c_t)\right\} = 0.\end{aligned}$$

Without the chain rule. Since the instantaneous utility function is specified as a quadratic function, we can directly obtain the first-order condition. The objective function for the consumer is

$$\left(ac_t - \frac{b}{2}c_t^2\right) + \beta\left\{a\left[(1+\rho)(x^P - c_t)\right] - \frac{b}{2}(1+\rho)^2[(x^P)^2 - 2x^P c_t + c_t^2]\right\}.$$

Then the first-order condition is

$$a - bc_t - \beta a(1+\rho) + \beta b(1+\rho)^2(x^P - c_t) = 0.$$

(c) Assume that the consumer has a quadratic instantaneous utility function: $U(c_t) = ac_t - \frac{b}{2}c_t^2$. Derive the optimal c_t. How does a change in τ affect c_t? Why?

Answer Recall that the first-order condition implies

$$\begin{aligned}U'(c_t) =& \beta(1+\rho)U'(c_{t+1}), \text{ or} \\ \{a - bc_t\} =& \beta(1+\rho)\left\{a - b(1+\rho)(x^P - c_t)\right\}.\end{aligned} \tag{Euler eq.}$$

This is called Euler equation. By solving this equation, we have

$$c_t = \frac{[1 - \beta(1+\rho)]a + \beta(1+\rho)^2 bx^P}{[1 + \beta(1+\rho)^2]b}$$

where $x^P = w + (w - \tau)/(1+\rho)$. An increase in tax τ decreases the lifetime income x^P, and hence the consumer reduces the current consumption c_t (i.e., $\tau\uparrow \Longrightarrow x^P \downarrow \Longrightarrow c_t \downarrow$).

Euler equation represents the key trade-off between consumption and saving. The left-hand side of the equation is the marginal loss from an decrease in current consumption while the right-hand side is the marginal gain from an increase in saving. Therefore the optimal consumption in period t is determined as a cross point of the left- and right-hand sides. Figure 1 illustrates how the optimal consumption in period t is determined. To understand how a change in τ affects c_t, recall that the lifetime income $x^P = w + (w - \tau)/(1+\rho)$ is decreasing in τ and that $U'^P - c_t))$ is de-

Figure 1: HW7-3: Euler equation.

creasing in x^P. That is, $\tau \uparrow \implies x^P \downarrow \implies$ RHS $\uparrow$. Intuitively, an increase in τ decreases the consumer's lifetime income. Since the marginal benefit from consumption is decreasing, $U''(c_t) = -b < 0$, consumers prefer stable consumption to unstable consumption. In order to smooth consumption, he should decrease the current consumption.