Macroeconomics: Modern Macroeconomics 1

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OSIPP

Introduction

- **Purpose:** The Course is designed to help you understand the basic concepts and framework of modern macroeconomics. The theories are supplemented by relevant empirical evidence.
- **Office hour:** Room 602, 17:00-18:00 on Monday. Appointment is required for other time.
- **E-mail Address:** takii@osipp.osaka-u.ac.jp

Introduction

- **Grading Policy:** 42% on assignments and 58% on a final exam.

  1. Seven assignments will be given during the course. Each assignment must be submitted at the following lecture. A complete assignment with full answers submitted by the due date will be awarded 6 points. An assignment submitted late, or one that demonstrates insufficient effort (i.e., fewer than half of the questions are meaningfully attempted), will be awarded 4 points. An assignment submitted late and one that demonstrates insufficient effort (i.e., fewer than half of the questions are meaningfully attempted), will be awarded 2 points.
  2. The full score of final exam is 58 points. I guarantee that 30 points out of 58 points will come from the assignment. So, if you hand in all assignments by the due date and you perfectly answer the questions appeared in assignments, you can certainly receive B.

Introduction

- **Remarks:**
  1. I assume that students have already taken Intermediate Microeconomics (Microeconomic Analysis or Microeconomics) and the introduction to econometrics.
  2. This course is comparable to the intermediate Macroeconomics in the economics department.
  3. I will mainly teach this course in Japanese. However, I will not prevent students from asking questions in English. I can discuss your questions and comments in Japanese or English at my office hour.

- **Course Outline**
  1. The Data of Macroeconomics (2 lectures):
  2. The Framework of Macroeconomics (1 lecture):
  3. Economic Growth and Nation's income (4 lectures):
  4. Stabilization Policy and Unemployment (5 lectures):
  5. Gorvernment Debt (1 lecture):
  6. Lucas's Critique and Micro Foundation (1 lecture): Consumption.
  7. Final Exam (1 lecture).

What is Macroeconomics?

- Macroeconomics is a study to explain the behavior of aggregate data such as GDP per capita, inflation rate, and unemployment rate.

  1. Observing Statistics, macroeconomists examine the health of our economy and make policy suggestions.
  2. For this purpose, we must infer the structure of the economy that brings the observable data.
  3. Macroeconomics is the current consensus on the inferences about the economic structure.

The Data of Macroeconomics

- Three main statistics
  1. Gross Domestic Product...the measure of richness.
  2. Consumer Price Index...the measure of cost of living
  3. Unemployment Rate..the measure of joblessness

Gross Domestic Product

- Definition: Gross Domestic Product (GDP) is the gross sum of value added of each product measured by market prices in a country during a period.
    - GDP can be viewed as the total income of the whole economy.
    - GDP can also be viewed as the total expenditure on the economy's outputs of goods and services.
    - For the economy as a whole, expenditure must equal income. Why?

Five main features of GDP

- *The use of market value:* GDP evaluates the value of goods and services by their market value since the prices of goods and services indicate how much consumers are willing to pay for them. Then GDP sums up the market value of goods and services in a country.
  - **Example:** Suppose that a country produces 5 apples and 10 bananas, and the price of an apple is 100 yen and the price of a banana is 30 yen. Then

$$GDP = 100yen \times 5 + 30yen \times 10 = 800yen$$

Five main features of GDP

- *The value added:* GDP is the sum of the value added of each product.
  - The value added of a firm equals the value of the firm's output minus the value of intermediate products that the firm has purchased.
  - **Example:** A firm purchases oranges from a farmer for 60 yen and sells an orange juice for 100 yen per cup. Then the value added of the orange juice is 40 yen. If a farmer does not buy any intermediate goods, then the value added of an orange is 60 yen. Therefore the value added of the two products equals

    $$40 \ yen + 60 \ yen = 100 \ yen.$$

    If the oranges are imported, GDP is

    $$60 \ yen$$

Five main features of GDP

- *GDP vs. GNP:*
  - GDP measures the total income in a country not by residents of the country.
  - Gross National Product (GNP) measures the total income earned by residents of the country.
  - The difference is factor payments (wages, profits and rents) from abroad and factor payments to abroad:

$$GNP \quad = \quad GDP$$
$$+ \textit{factor payments from abroad}$$
$$- \textit{factor payments to abroad}.$$

Five main features of GDP

- *Flow vs. Stock:*
    1. A flow is a quantity measured per unit of time.
    2. A stock is a quantity measured at a given time.

- **Example:**
    - Flow...Annual income, Saving...
    - Stock....Wealth, Asset

- Since GDP measures the total income earned during a period, such as a year, GDP is a flow variable.

Five main features of GDP

- *Gross vs. Net:*
  - GDP is the gross sum of value added. It does not subtract the depreciation of capital from the value added- the amount of capital (plants, equipment and residential structures) that wears out over a period of time.
 - Net National Product:

$$NNP = GNP - Depreciation$$

Some details for computing GDP

- *Used goods:* The sale of used goods does not increase the additional value in a country. Therefore, the sale of used good is not included in GDP.
- *Inventories:* National Income Accounting system treats inventory as the sale of goods to themselves during the current period.
  - Inventories are counted as part of GDP of the period that goods are produced..
  - Inventories are not counted as part of GDP of the period that goods are sold.
    - It is considered as used goods.
  - Because of this treatment of inventories, all goods produced are purchased by somebody. Therefore, total income always equals total expenditure of a country.

Some details for computing GDP

- *Imputations:* When some goods are not sold in a market, they do not have market prices. If GDP includes these goods and services, we must estimate their value. Such an estimated value is called imputed value.

  1. **The Value of Housing:** When you rent an apartment, the rent is a part of GDP. When you own a house, you do not pay the rent. GDP estimates the rent that house owners pay to themselves.
  2. **Home Production and Durable Goods:** The value of these rental service and home production is left out of the GDP.
  3. **Government Services:** The national income accounts estimate the value added of government services in the GDP at their cost.
  4. **Underground Economy:** no imputation is made for the value of goods and services sold in the underground economy.

図5-1：名目GDPの長期傾向
（単位：十億円、出所：内閣府）

名目GDP（1990年基準）　　名目GDP（2000年基準）　　名目GDP（2005年基準）　　名目GDP（2015年基準）
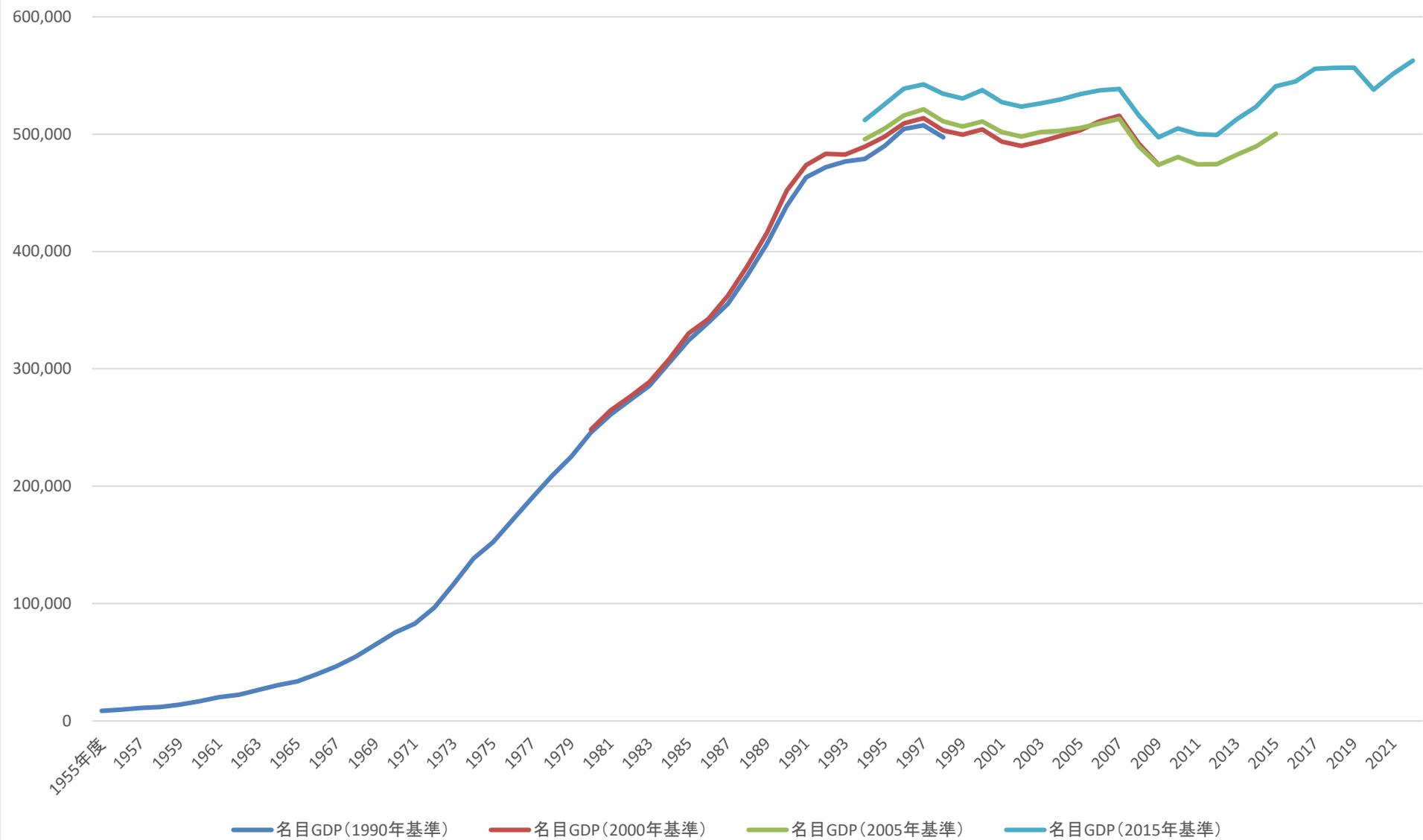
Comparison across Time Periods

- Since GDP is measured by the market prices of goods and services, GDP increases both when prices increase and when the outputs increase. In order to exclude the impact of inflation, economists separate real GDP from nominal GDP.

    - **Nominal GDP** uses current prices to measure the value of goods and services.
    - **Real GDP** uses a constant set of prices to measure the value of goods and services. In order to compute real GDP, economists choose the base year.
    - **Example:** Consider a country in which people produce only apples and bananas during 2008 and 2009. Let me choose 2008 is the base year.

$$RealGDP\ in\ 2008 = P_{A2008} \times Q_{A2008} + P_{B2008} \times Q_{B2008}$$
$$RealGDP\ in\ 2009 = P_{A2008} \times Q_{A2009} + P_{B2008} \times Q_{B2009}$$
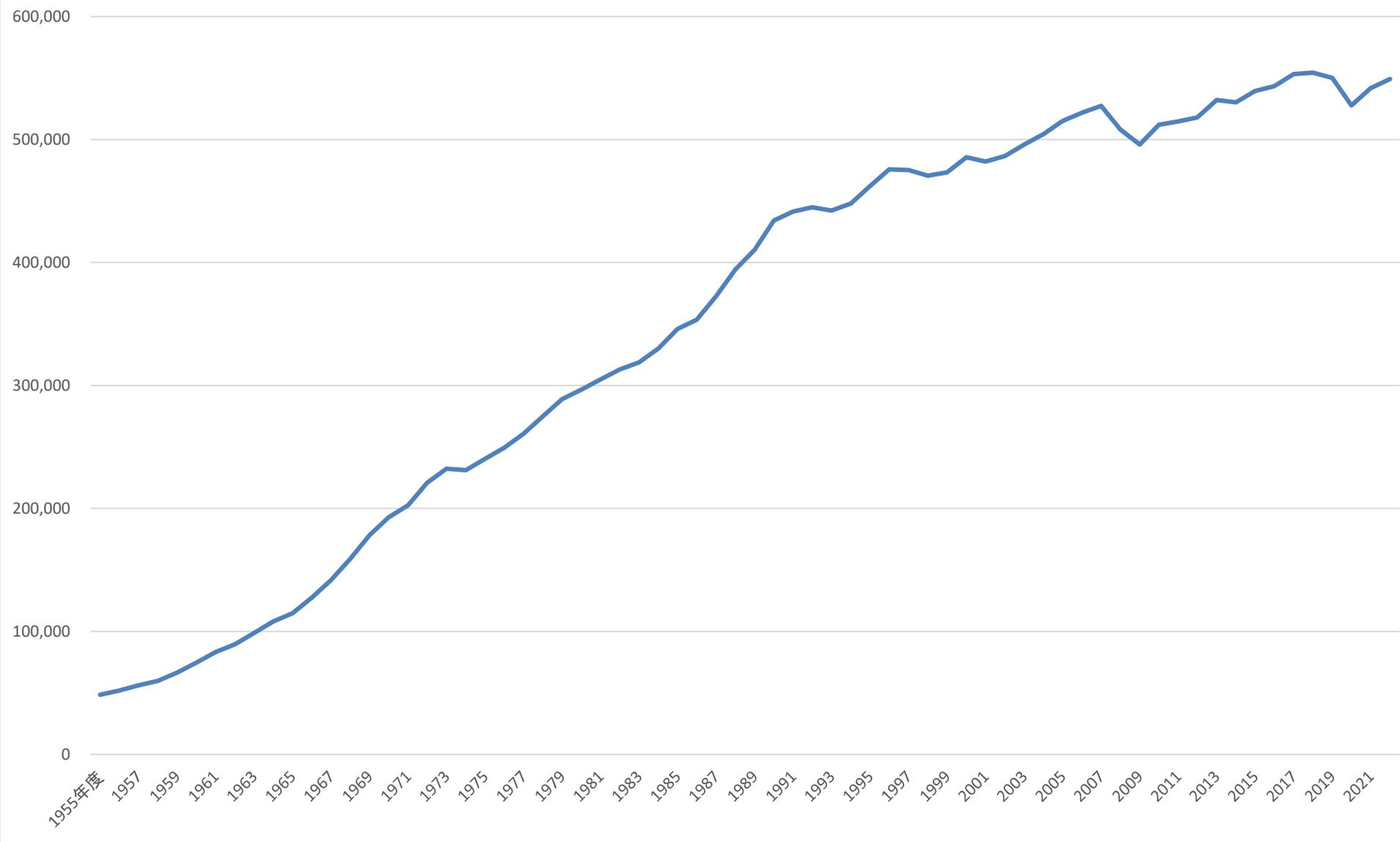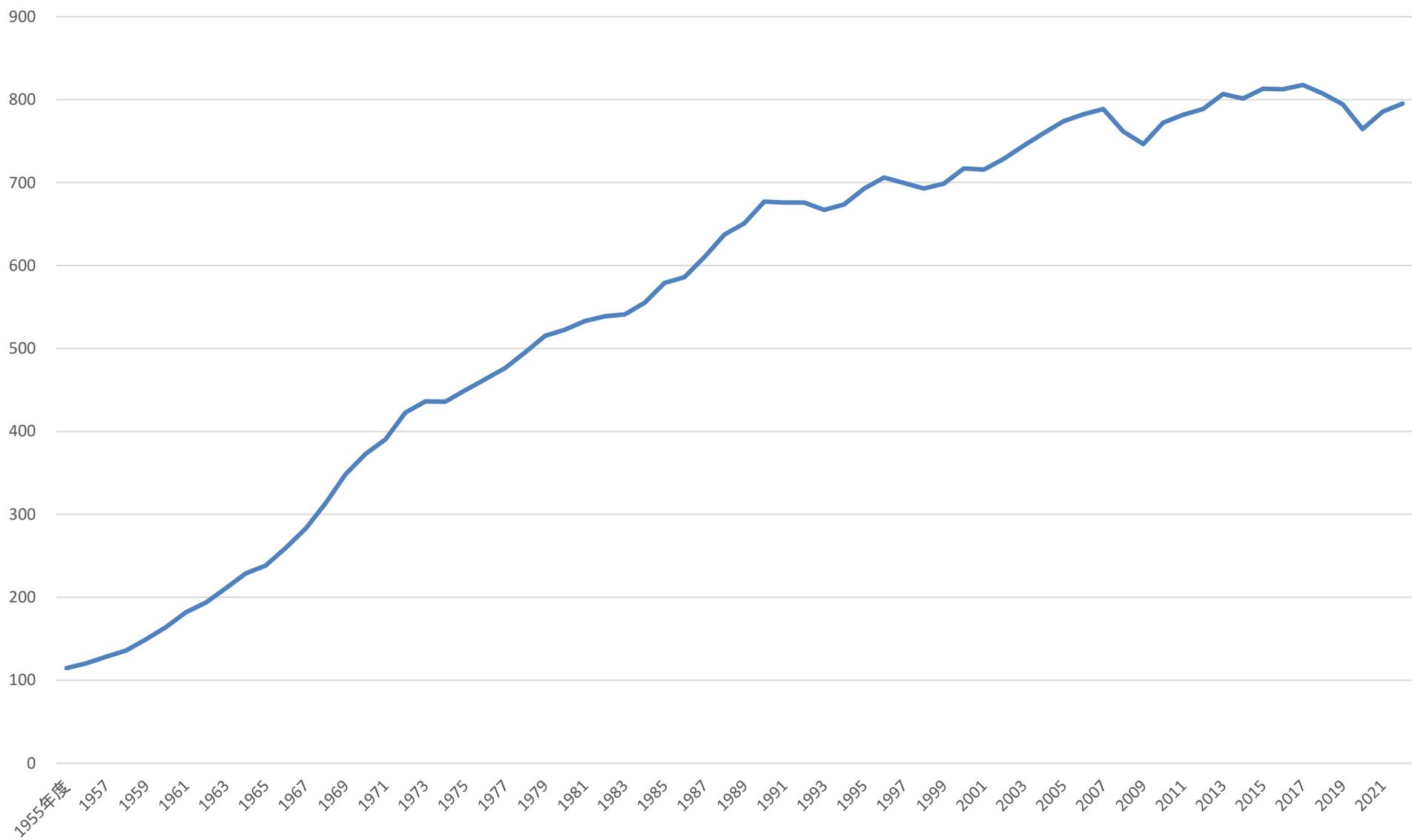
図5-2：実質GDPの長期動向
（2015年基準に合わせたもの，単位：10億円，出所：内閣府）
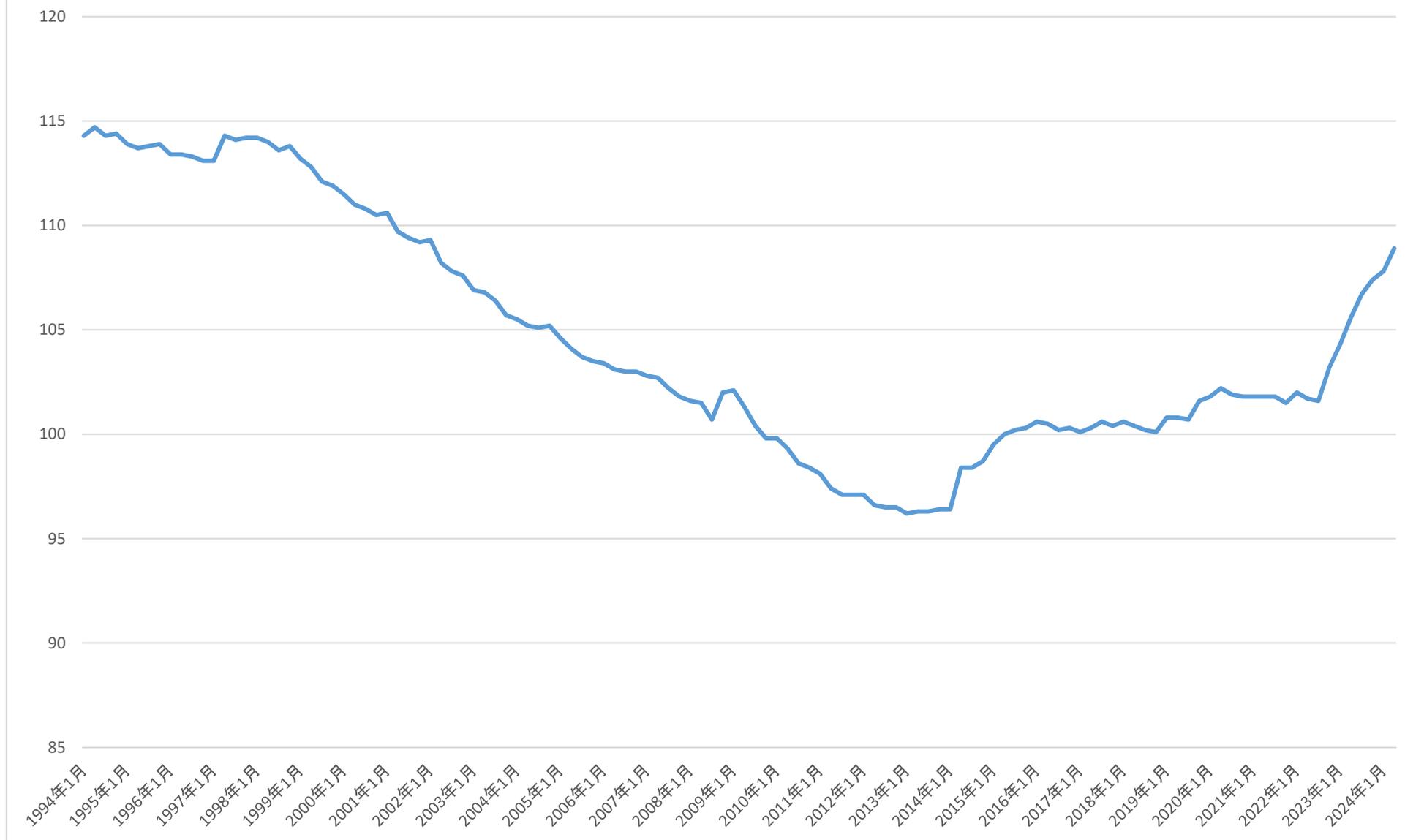
図5-11：労働力１人あたり実質GDPの推移
（2015年基準、単位：万円、出所：内閣府、総務省）

- *GDP deflator:* The ratio of nominal GDP to real GDP is called GDP deflator:

$$GDP\ Deflator = \frac{\text{Nominal GDP}}{\text{Real GDP}}$$

- The GDP deflator captures the movement of the overall price level in the economy.

図2-3：GDPデフレーター（四半期、季節調整済み，2015年基準）
（出所：内閣府）

- **International prices:** Different countries use different currencies. In order to compare income across countries, which prices should we use?
  - An international price of the goods...The weighted average of the price of the goods across countries by taking the country's share of expenditures as its weight.

- **Purchasing-Power Parity:** Purchasing-Power Parity (PPP) is the ratio of nominal GDP to real GDP measured by international prices

$$PPP = \frac{\text{Nominal GDP}}{\text{Real GDP measured by international prices}}$$

Japan: Exchange Rate vs PPP (Nominal GDP / Real GDP at intl prices), 1950–2019

International Comparison
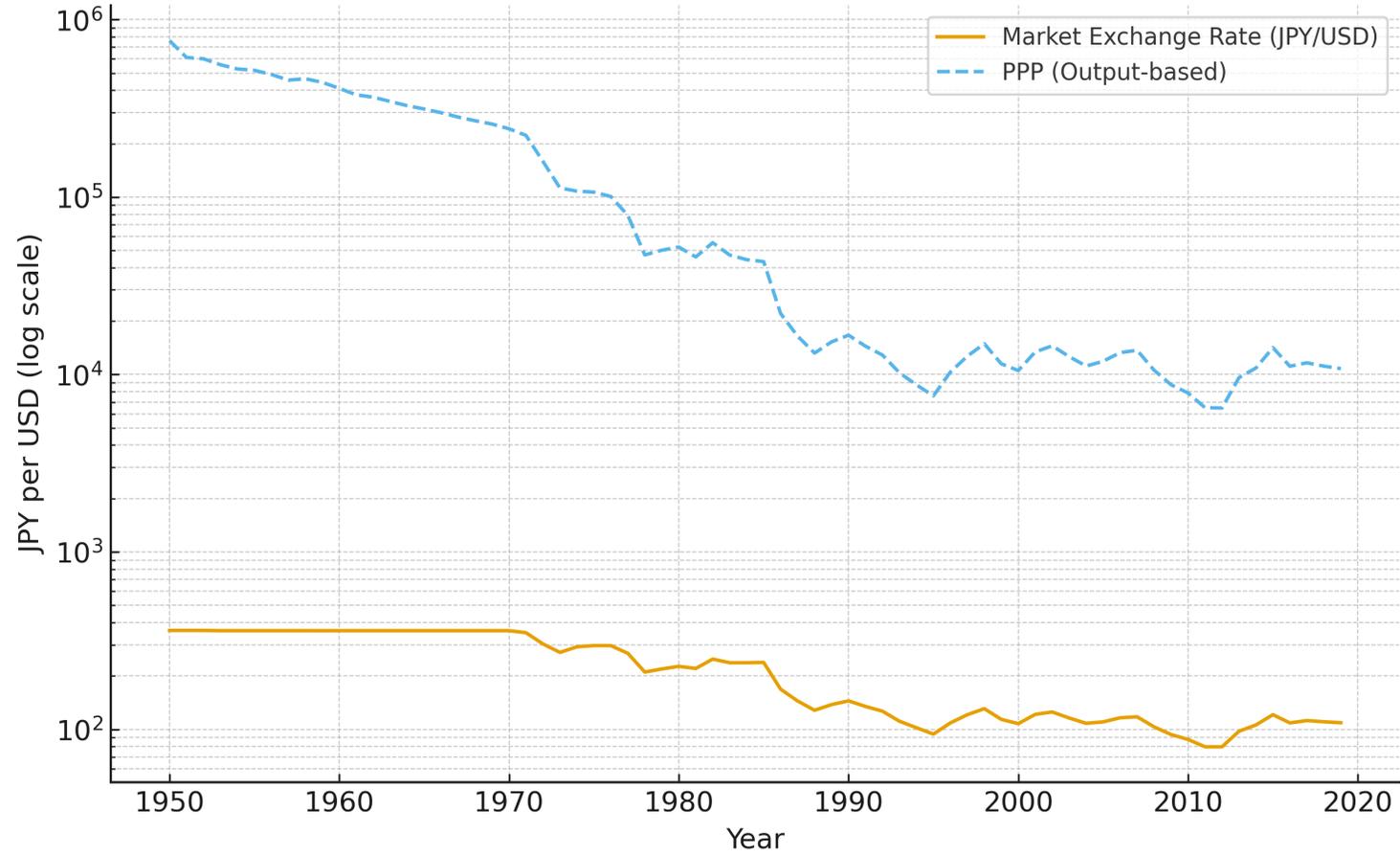
- **GDP per capita vs. GDP per worker:**

$$\text{GDP per capita} \quad = \quad \frac{\text{GDP}}{\text{total population}}$$

$$\text{GDP per worker} \quad = \quad \frac{\text{GDP}}{\text{the number of labor force}}.$$

  - The production of goods normally made in the factory is mainly done in the household in developing countries. Since GDP cannot measure the value of home production, GDP per capita may underestimate well-being of developing countries. Since GDP does not value home production, it may be reasonable to divide it by labor force.

RGDPo per capita (USD, 2017 PPP), 2019

Japan — Rank: 34 — $39,704 (2019)

Top 10	RGDPo per capita	Bottom 10	RGDPo per capita
Qatar	114,101	Venezuela (Bolivarian Republic of)	251
Ireland	102,622	Burundi	790
China, Macao SAR	93,488	Central African Republic	978
Luxembourg	90,479	D.R. of the Congo	1,022
Singapore	82,336	Malawi	1,161
Switzerland	75,299	Niger	1,211
Norway	73,669	Mozambique	1,229
Brunei Darussalam	73,249	Liberia	1,258
United Arab Emirates	66,113	Madagascar	1,539
United States	62,589	Haiti	1,554

RGDPo per worker (USD, 2017 PPP), 2019

Japan — Rank: 42 — 71,980 (2019)

Top 10	RGDPo per worker	Bottom 10	RGDPo per worker
Ireland	221,661	Venezuela (Bolivarian Republic of)	612
Qatar	155,062	Burundi	1,821
China, Macao SAR	154,394	Central African Republic	2,517
Brunei Darussalam	142,733	Malawi	2,721
Norway	138,858	Madagascar	2,977
United States	130,107	Niger	3,182
Saudi Arabia	129,212	Liberia	3,238
Switzerland	129,095	Mozambique	3,387
Singapore	127,117	D.R. of the Congo	3,527
Luxembourg	120,936	Haiti	3,860

Components of Expenditure

- GDP or Y, can be divided into consumption of domestic goods and services, C^d, investment in domestic goods and services, I^d, government purchases of domestic goods and services, G^d, and exports of domestic goods and services, EX:

$$Y = C^d + I^d + G^d + EX$$

- Consumption, C, investment, I, and government expenditure, G can be divided into domestic goods or foreign goods:

$$C = C^d + C^f, \ I = I^d + I^f, \ G = G^d + G^f$$

where superscript f means foreign goods.

- GDP

$$\begin{aligned} Y &= C + I + G + EX - \left(C^f + I^f + G^f \right) \\ &= C + I + G + EX - IM \\ &= C + I + G + NX \end{aligned}$$
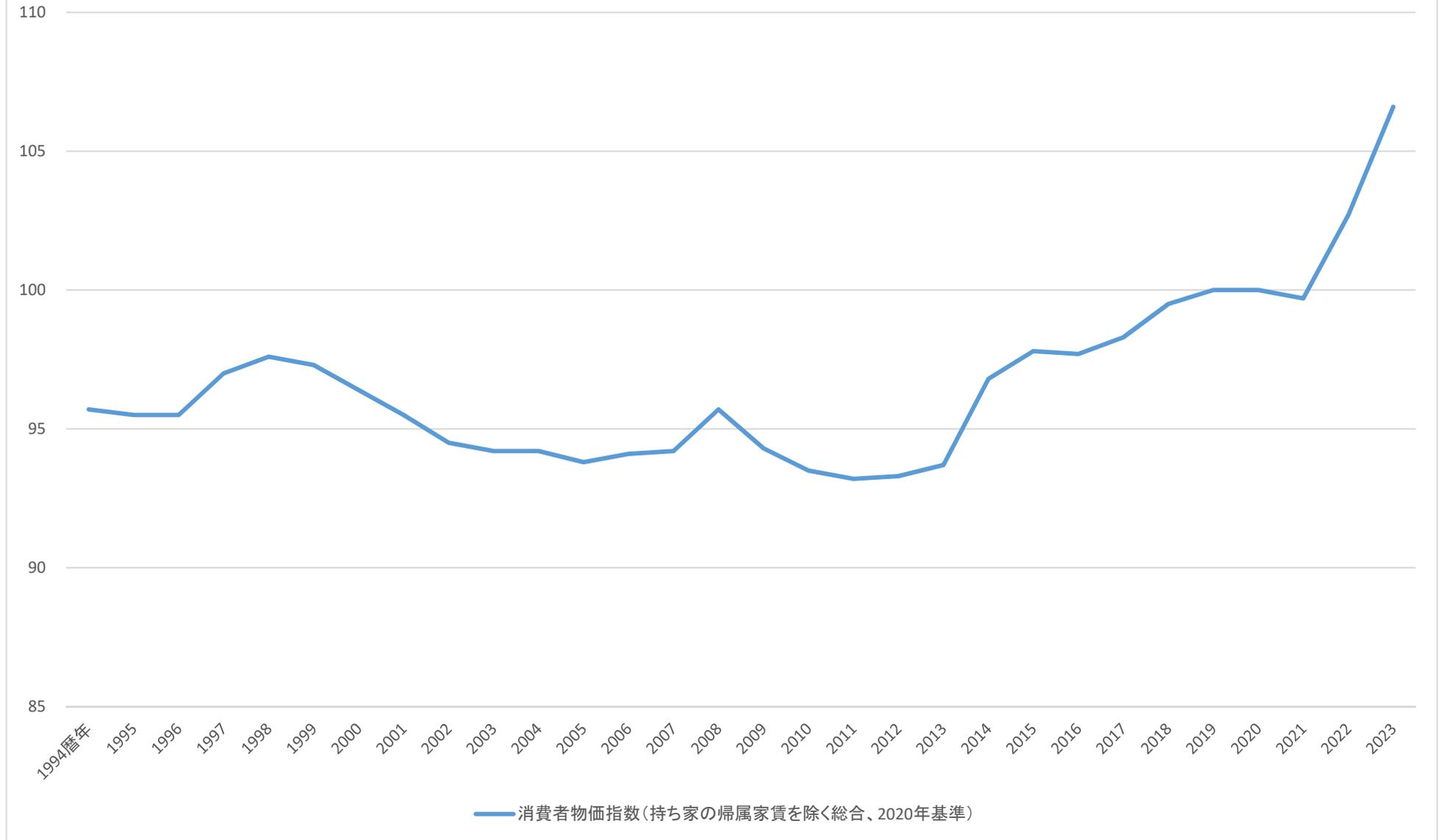
Consumer Price Index

In order to analyze the changes in the overall cost of living, we need a single index measuring the overall level of prices.

- **Consumer Price Index:** Economists compute the price of a basket of goods and services purchased by a typical consumer. CPI is the price of this basket of goods and services relative to the price of the same basket in some base year.

  - **Example**: Consider a country in which typical consumers buy 5 apples and 10 bananas in a year during 2008 and 2009. Then the basket of goods consists of 5 apples and 10 bananas. Let us set 2008 as the base year. The CPI can be defined as follows:
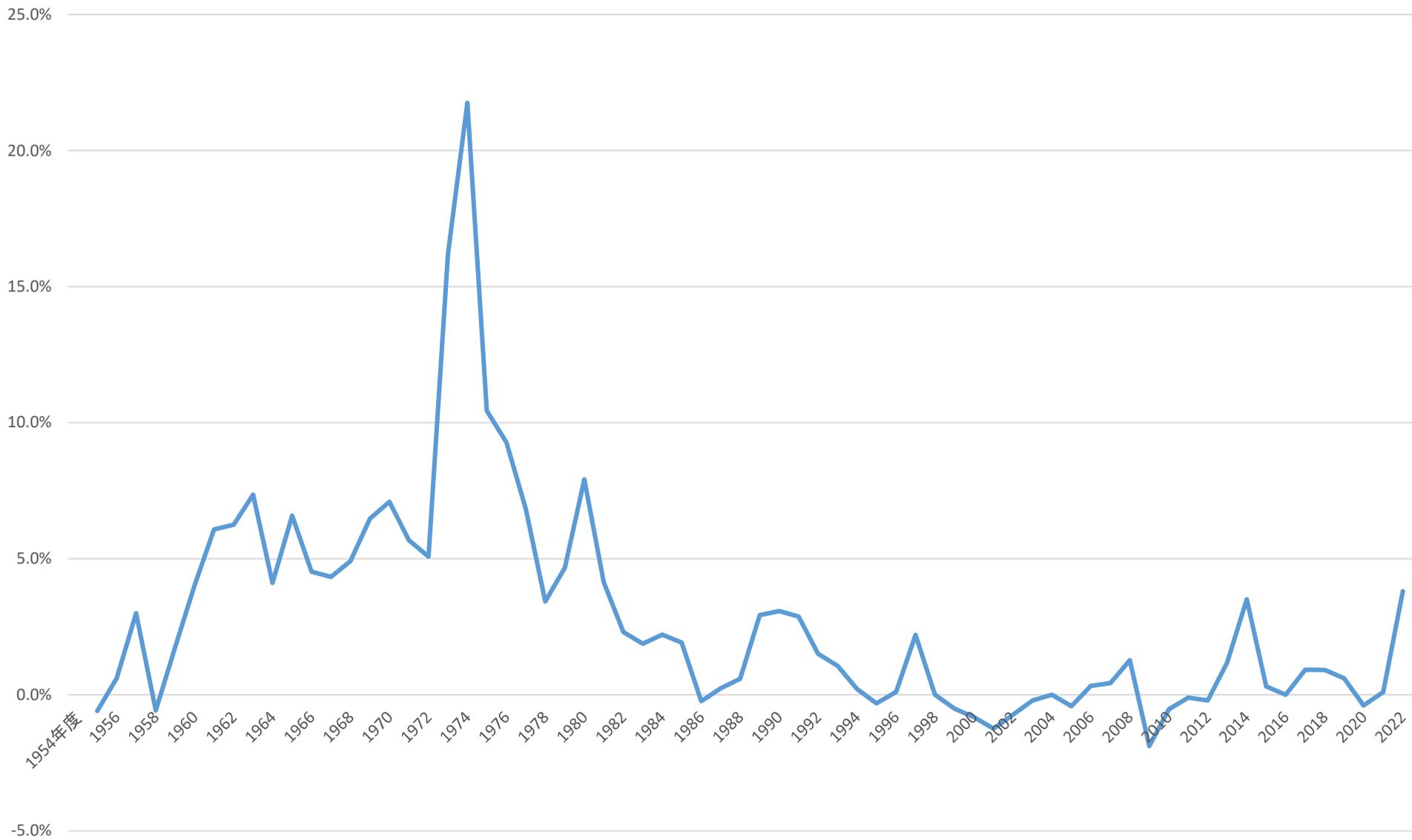
$$CPI \text{ in } 2008 = 1$$

$$CPI \text{ in } 2001 = \frac{5 \times P_{A2009} + 10 \times P_{B2009}}{5 \times P_{A2008} + 10 \times P_{B2008}}$$

図2-1：消費者物価指数
（出所：総務省、内閣府）

消費者物価指数（持ち家の帰属家賃を除く総合、2020年基準）

消費者物価上昇率の推移
（持家帰属家賃を除く総合指数、2020年基準、出所：総務省）

Consumer Price Index

- **CPI vs. the GDP Deflator:** Both CPI and GDP deflator measure overall price level. But there are three main differences.

1. The GDP deflator measures the prices of all goods and services produced; CPI measures the prices of only the goods and services bought by consumers.
2. The GDP deflator includes only goods produced in a country. It excludes imported goods. CPI includes imports goods if the consumers buy such goods.
3. CPI assigns fixed weights to the prices of different goods, the GDP deflator allows the basket of goods to change over time as the composition of GDP changes.

Despite these differences, CPI and the GDP deflator show a similar behavior.

The Unemployment Rate

The unemployment measures the percentages of people who want to work but who do not have jobs.

- **Unemployed Workers:** People are called unemployed when

1. They do not have a paid job.
2. They conducted a job seeking activity
3. If there is a job, they can do it soon (They are available).

- **Employed Workers:** People are called employed if they do have a paid job.
- **Labor force:** The sum of employed workers and unemployed workers are called labor force.
- **The unemployment rate:**

$$unemployment\ rate = \frac{number\ of\ unemployed\ workers}{labor\ force} \times 100$$

図4-2：完全失業率の年次推移
（単位：%、出所：総務省）

The Unemployment Rate
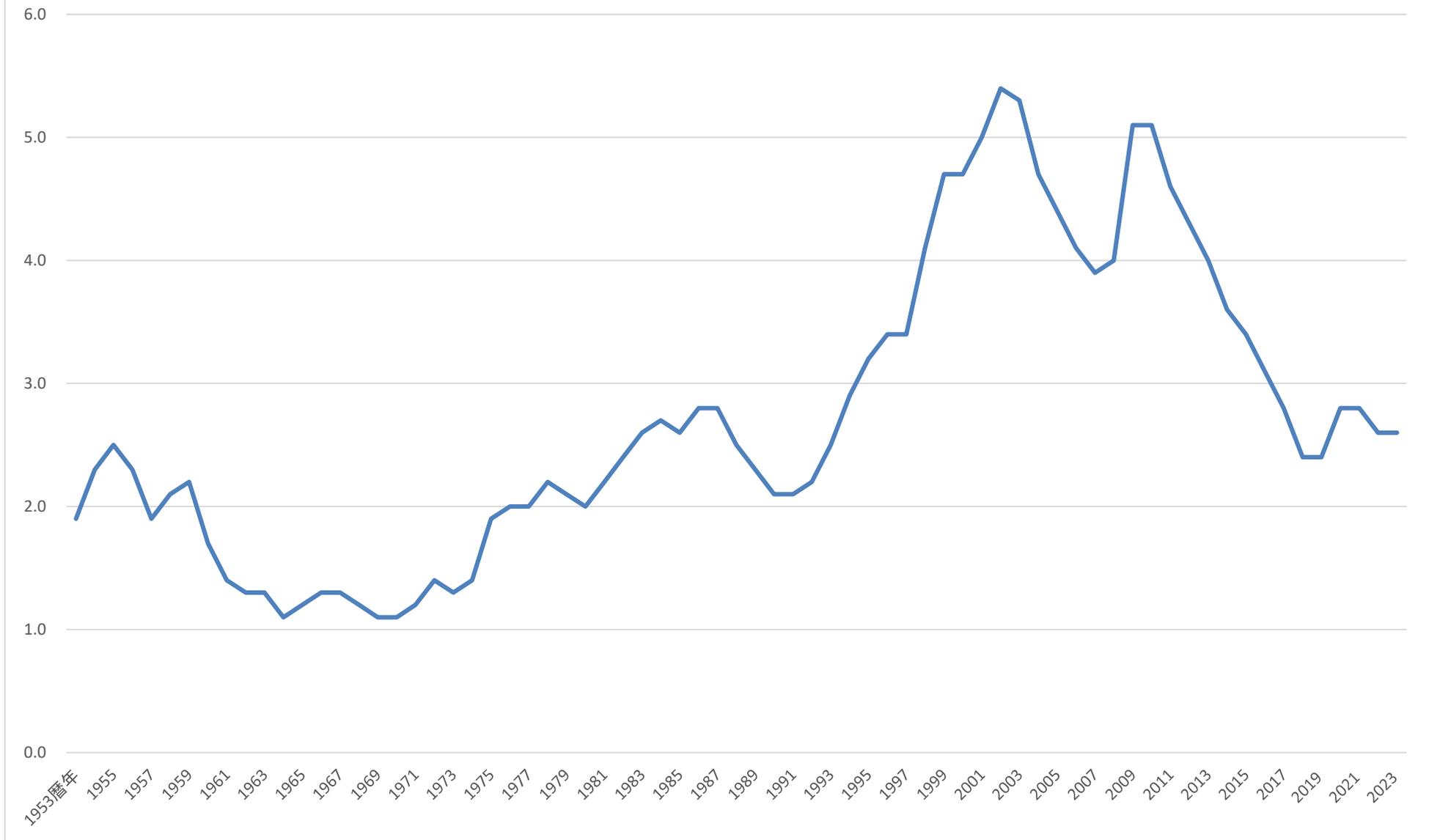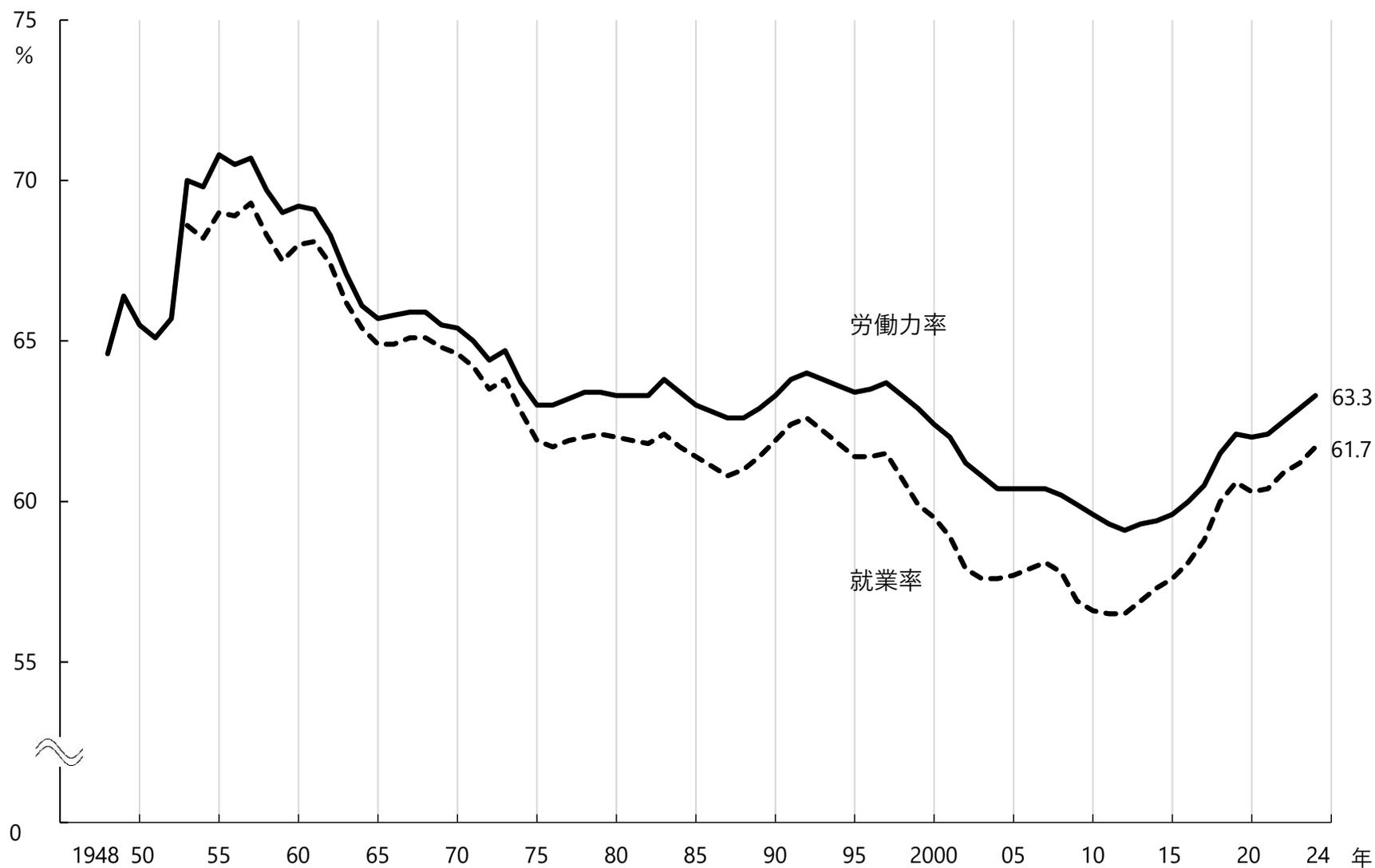
- **Labor Force Participation rate:**

$$labor\ force\ participation\ rate = \frac{labor\ force}{adult\ population} \times 100$$

where adult population is the number of people 15 years old or more.

- If a person is 15 years old or more and s/he is neither employed not unemployed, s/he is not in the labor force.

図3-1　労働力率、就業率　1948年～2024年 年平均

資料出所　総務省統計局「労働力調査」

JILPT

- Students must hand assignment 1 in at the next lecture.

The Basic Framework of Macroeconomics

- This Chapter provides a basic framework of macroeconomics. This is an application of a general equilibrium analysis in the context of macroeconomics.

- Using this framework, I will construct the neoclassical growth model later and ask the following questions.
  1. Why are some countries rich; others poor?
  2. What is the source of long run growth?

- Later, I applied this model to the analysis of stabilization policy.

The Basic Framework of Macroeconomics

- Representative Firm
    - Produce goods or services using Labor and Capital
    - Sell goods or services to household
- Representative Household
    - Buy goods or services
    - Provide firms with labor and capital
- Markets (Labor Market, Capital Market and Goods Market.)

Firm

- Firms is assumed to maximize its profits given an aggregate production function:

$$
\begin{aligned}
\Pi &= \max_{K,L} \{PY - WL - RK\} \\
Y &\leq F(K, L)
\end{aligned}
$$

where P is a price, Y is output, W is a nominal wage rate, L is labor, R is a nominal rental price and K is capital stock.

Firm

- The property of the aggregate production function: $F(K, L)$ is constant returns to scale in K and L:

$$tF(K, L) = F(tK, tL), \text{ for } \forall t > 0. \tag{1}$$

- Why should the aggregate production function be constant returns to scale?
  - CRS means that whatever an individual production function is, if we use the same production technology twice, the output will be doubled. This might be a reasonable assumption for the aggregate production function. For example, assume that an individual plant has a production function $y = \phi(l)$. Assume that a manager establishes the same K plants. Then the aggregate output Y is

$$Y = yK = \phi(l) K$$

Define an aggregate production function production function F such that

$$F(K, L) \equiv \phi(l) K, \ \forall K > 0$$

where $L = lK$. Clearly this is constant return to scale in K and L.

Firm

- Constant returns to scale and production possibility set

$$\begin{aligned}
Y &\leq F(K, L) \\
&= F\left(\frac{K}{L}, 1\right) L \\
y &\leq F(k, 1)
\end{aligned}$$

where $y = \frac{Y}{L}$ and $k = \frac{K}{L}$. Define f

$$f(k) \equiv F(k, 1)$$

Then

$$y \leq f(k)$$

Firm

- Profit maximization Problem

$$
\begin{aligned}
\Pi &= \max_{K,L} \left\{ PY - WL - RK \right\} \\
&= \max_{k,L} \left\{ yL - \frac{W}{P}L - \frac{R}{P}kL \right\} P \\
&= \max_{L} \pi PL \\
where \; \pi &= \max_{k} \left\{ y - w - rk \right\}
\end{aligned}
$$

where $w = \frac{W}{P}$ and $r = \frac{R}{P}$. Therefore

$$
\begin{aligned}
\Pi &= \max_{L} \pi PL \\
\pi &= \max_{k} \left\{ y - w - rk \right\} \\
y &\leq f(k)
\end{aligned}
$$

Firm

- Assumptions on the aggregate production function
  1. $f(0) = 0$
  2. $f'(k) = \frac{df(k)}{dk} > 0$.
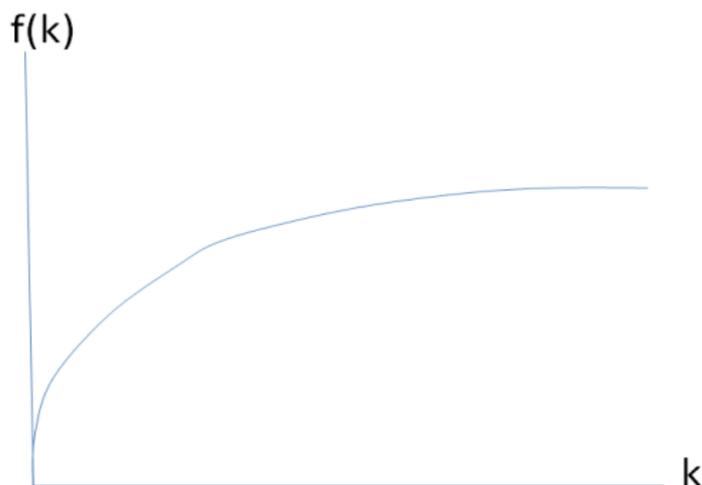   - When the firm employs more capital per workers, it increases output per workers.
  3. $f''(k) = \frac{d^2 f(k)}{dk^2} < 0$
     - This means that the marginal productivity of capital per workers is diminishing.
  4. Inada Conditions: technical conditions.

$$\lim_{k \to 0} f'(k) = \infty, \lim_{l \to \infty} f'(k) = 0.$$

Aggregate Production Function

Firm

- Example: Cobb-Douglas Production Function: $f(k) = k^{\alpha}$

$$
\begin{aligned}
f'(k) &= \alpha k^{\alpha-1} > 0 \Rightarrow \alpha > 0 \\
f''(k) &= \alpha(\alpha-1) k^{\alpha-2} < 0 \Rightarrow \alpha < 1
\end{aligned}
$$

Hence

$$
\alpha \in (0, 1)
$$

Firm

- Profit Maximization with respect to k

$$\pi = \max_k \{y - w - rk\}, \ s.t. \ y \leq f(k)$$
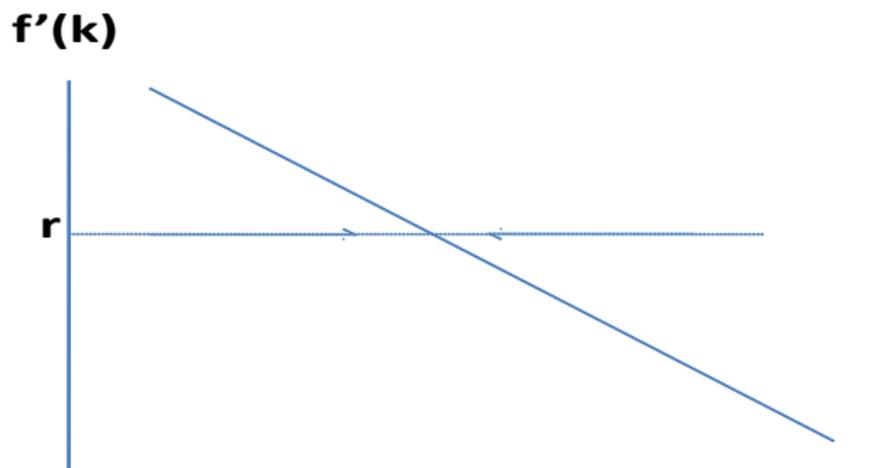$$\pi = \max_k \{f(k) - w - rk\}$$

- First Order Conditions

$$\frac{d\pi}{dk} = 0 \Rightarrow r = f'(k) \Rightarrow k \text{ is determined}.$$

- π

$$\pi = f(k) - w - rk \Rightarrow \pi \text{ is determined}.$$

Optimal Decision

Firm

- Profit maximization with respect to L

$$\Pi = \max_{L} \pi P L$$

- Labor Demand Function

$$
\begin{aligned}
L &= 0 \ \text{if} \ \pi < 0, \ w > f(k) - rk \\
L &\in [0, \infty] \ \text{if} \ \pi = 0, \ w = f(k) - rk \\
L &= \infty \ \text{if} \ \pi > 0, \ w < f(k) - rk
\end{aligned}
$$

Labor Demand Function

Firm

- 0 Economic Profits

$$\Pi = \pi PL = 0$$

When the market is competitive, more entrepreneurs will enter as long as economic profits are positive. Hence, in the long run, economic profit is 0.Hence

$$\begin{aligned}
\Pi &= 0, L > 0 \Rightarrow \pi = 0 \Rightarrow w = f(k) - rk \\
Y &= wL + rK
\end{aligned}$$

- Accounting Profits: The firm's revenue must be divided into wage payment, capital payment and economic profit:

$$Y = wL + rK + \Pi$$

where Π is economic profit. But in reality, a firm's owner owns capital also. Hence, we cannot distinguish economic profits from capital payment. It means

$$\textit{Accounting profit} = \Pi + rK$$

Firm

- Example: Cobb-Douglas Production Function: $f(k) = k^{\alpha}$

$$
\begin{aligned}
r &= \alpha k^{\alpha - 1} \\
rk &= \alpha k^{\alpha} \\
\alpha &= \frac{rk}{k^{\alpha}} = \frac{rk}{y} = \frac{rK}{Y}
\end{aligned}
$$

$$
\begin{aligned}
w &= k^{\alpha} - rk \\
&= k^{\alpha} - \alpha k^{\alpha} \\
&= (1 - \alpha) k^{\alpha} \\
1 - \alpha &= \frac{w}{k^{\alpha}} = \frac{w}{y} = \frac{wL}{Y} \\
1 &= \frac{rK + wL}{Y}
\end{aligned}
$$

- Representative Households make decisions on
  1. How long they work,
  2. How much they consume today, and
  3. Where to invest.

Household

- How long do they work?: Assume that everybody works one unit of time no matter what wage rate is. Hence, the supply of labor is equal to total population.

- How much do they consume today?
  - If they do not consume today, they save for future. This is potentially a difficult question, because the decision depends not only on the current income but on the expected future income. I will leave the answer to this problem later and at this moment, I take the consumption per capita, c as given.
  - Budget Constraint: Nonetheless, chosen consumption has to be feasible. Hence, at least it must satisfy the following budget constraint.
  $$a_{+1} + c = (1 + \rho^a) \, a + w$$
  where c is consumption per capita, ρ^a is the returns from investment, a is asset per capita and a_{+1} is an asset per capita at the next period.

Household

- Where do they invest?:There are several possibilities. They may save it in a bank. They can invest in firms' stock. They can also purchase investment goods such as house and rent it out.

  1. So far we assume that a firm rent capital. So the firm does not own capital. Moreover, economic profits of the firms are 0. Hence, there is no value on a firm.
  2. It means that household has two choices:
     1. to save it in a bank. Then they can earn a safe return, interest rate, ρ.
     2. to purchase investment goods and rent it out. Then it expects to earn r from a unit of investment. In addition, as they are the owner of capital, if the capital depreciates, the must bear the cost. Suppose that δ proportion of capital is depreciated. Then real return from purchasing investment goods is $r - \delta$.
  3. Assume that there are lots of investment opportunities so that household hedges idiosyncratic risk. Moreover, we assume that there is no adjustment cost of investment. Then the optimal condition is

  $$\rho^a = \max\{r - \delta, \ \rho\}$$

Household

- Arbitrage Condition:
  1. If $r - \delta > \rho$, everybody buys investment goods. But then, nobody saves in a bank and the interest rate would become larger.
  2. If $r - \delta < \rho$, nobody buys investment goods. But then, everybody saves in a bank and the interest rate becomes smaller.
  3. In the equilibrium,
  $$r - \delta = \rho = \rho^a$$
  This is called an arbitrage condition.

Market Clearing Condition

- Labor Market Clearing Condition

$$L = N$$

- Capital Market Clearing Condition

$$K = aN$$

  - Note that even if banks collect assets from household, the banks must purchase investment goods. Hence, every assets in the economy is used for investment in capital goods.

- Hence capital market clearing condition and labor market clearing condition implies

$$k = \frac{K}{N} = a$$

Equilibrium

Given (c, a), a market equilibrium consists of $(y, k, a_{+1}, \rho, r, w)$ which satisfies

1. A Firm's Profit Maximization and the Production Function

$$
\begin{aligned}
y &= f(k) \\
r &= f'(k) \\
w &= f(k) - rk
\end{aligned}
$$

2. A Consumer's Budget Constraint

$$a_{+1} + c = (1 + \rho^a) a + w$$

3. An Arbitrage Condition

$$r - \delta = \rho$$

4. Capital and Labor market clearing conditions

$$k = a$$

- **What happens to a goods market?** A goods market is supposed to equate demand for output and supply of output and determine the price of the goods, P. However, note that $w = \frac{W}{P}$ and $r = \frac{R}{P}$. Hence, the wage rate and rental price are not measured by nominal term, but measured by the unit of output. Even if W, R and P double, w and r keep the same value. Hence, there is no change in our economy. In order to make our decisions, we care about the relative prices. We do not need information on the absolute prices. Hence, the price of a product can be set 1 without a loss of generality. In other words, the market that is supposed to determine the price is redundant.

Goods Market

- In order to prove the above statement, we derive goods market from equilibrium conditions. First, we derive a resource constraint. Second, we derive the flow expression of capital market. Combining two equations, we derive goods market.
- Budget clearing condition implies that

$$
\begin{aligned}
a_{+1} + c &= (1 + \rho^a) a + w \\
k_{+1} + c &= (1 + r - \delta) k + f(k) - rk \\
&= f(k) + (1 - \delta) k \\
y_t &= [k_{+1} - (1 - \delta) k] + c
\end{aligned}
$$

Define investment, i as

$$
i \equiv k_{+1} - (1 - \delta) k
$$

Hence,

$$
y_t = i_t + c_t \Rightarrow Y_t = I_t + C_t
$$

- Students must hand assignment 2 in at the next lecture.

Economic Growth and Nations' Income

1. Based on the previous framework, I first derive so called, Solow Model. This model is a useful starting point for the analysis of economic growth.

2. Solow model predicts that eventually economic growth converges to 0. This is not what we observe in data. In order to match the theory with data, we introduce an exogenous technology growth and population growth, and compare with the stylized facts about economic growth observed in many OECD countries.

3. Next, using the extended Solow model, we ask a question: can the model explain large differences in income across countries?

4. From the above exercises, we recognize the importance of productivity. Understanding productivity is the issue we discuss later.

Solow Model

- Capital accumulation equation: Assume that x_t means x at date t. Capital accumulation equation is

$$
\begin{aligned}
k_{t+1} &= i_t + (1 - \delta)\, k_t \\
&= y_t - c_t + (1 - \delta)\, k_t \\
&= f\left(k_t\right) - c_t + (1 - \delta)\, k_t
\end{aligned}
$$

Investment makes capital stock bigger, but in order to make large investment, households cannot enjoy consumption today very much. This is a basic trade-off in Solow model.

- Assumption on c_t:

$$
c_t = (1 - s)\, y_t
$$

Because $s_t = y_t - c_t$ is the definition of saving,

$$
s_t = s y_t
$$

The parameter s represents saving rate.

Solow Model

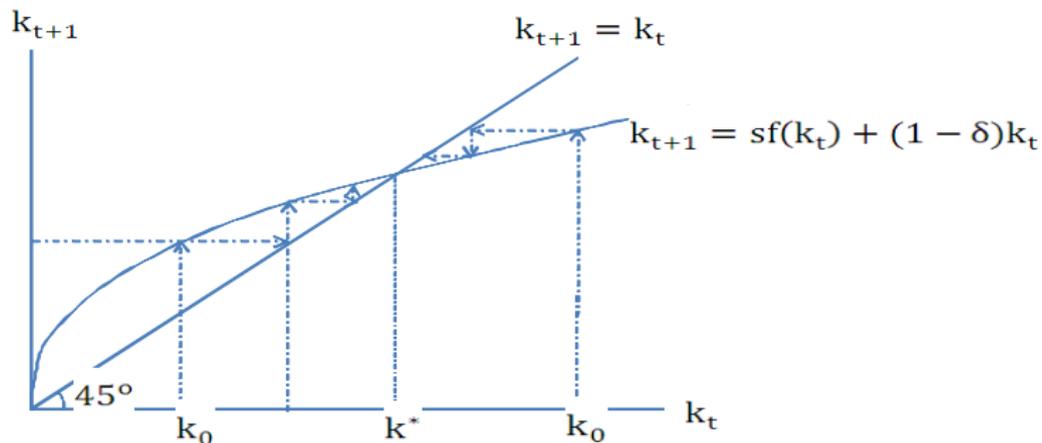- Substituting the consumption function into the capital accumulation equation, we have

$$
\begin{aligned}
k_{t+1} &= f(k_t) - (1-s) y_t + (1-\delta) k_t \\
&= f(k_t) - (1-s) f(k_t) + (1-\delta) k_t
\end{aligned}
$$

Hence,

$$
k_{t+1} = sf(k_t) + (1-\delta) k_t.
$$

This is Solow model.

Solow Model

Solow Model

Definition

The steady state is the points at which $\{(c_t^*, y_t^*, k_t^*)\}$ satisfies

$$c_{t+1}^* = c_t^*, \ y_{t+1}^* = y_t^*, \ k_{t+1}^* = k_t^*$$

- For any initial capital stock, economy eventually converges to the steady state. It means

  1. Economic growth rate eventually converges to 0.
  2. If the steady state is the same across countries, the poor countries eventually catch up.

- A key assumption to derive this result is $f''(k) < 0$.

Solow Model

- On the steady state,

$$
\begin{aligned}
k^* &= sf(k^*) + (1 - \delta) k^* \\
sf(k^*) &= \delta k^* \\
\frac{k^*}{f(k^*)} &= \frac{s}{\delta}
\end{aligned}
$$

- Example: $f(k) = k^\alpha$

The Review of High School Mathematics

- Remember

$$
\begin{aligned}
k^{\alpha+\beta} &= k^\alpha k^\beta \\
\left(k^\alpha\right)^\beta &= k^{\alpha\beta} \\
(kl)^\alpha &= k^\alpha l^\alpha \\
k^{-1} &= \frac{1}{k}
\end{aligned}
$$

- Therefore

$$
\begin{aligned}
k^{\alpha-\beta} &= k^\alpha k^{-\beta} = k^\alpha \left(k^\beta\right)^{-1} = \frac{k^\alpha}{k^\beta} \\
\left(\frac{k}{l}\right)^\alpha &= k^\alpha \left(\frac{1}{l}\right)^\alpha = k^\alpha l^{-\alpha} = k^\alpha \left(l^\alpha\right)^{-1} = \frac{k^\alpha}{l^\alpha}
\end{aligned}
$$

Solow Model

- Example: $f(k) = k^{\alpha}$

$$\frac{s}{\delta} = \frac{k^*}{f(k^*)} = \frac{k^*}{(k^*)^{\alpha}} = (k^*)^{1-\alpha}$$

$$k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = (k^*)^{\alpha} = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$c^* = (1-s)y^* = (1-s)\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$
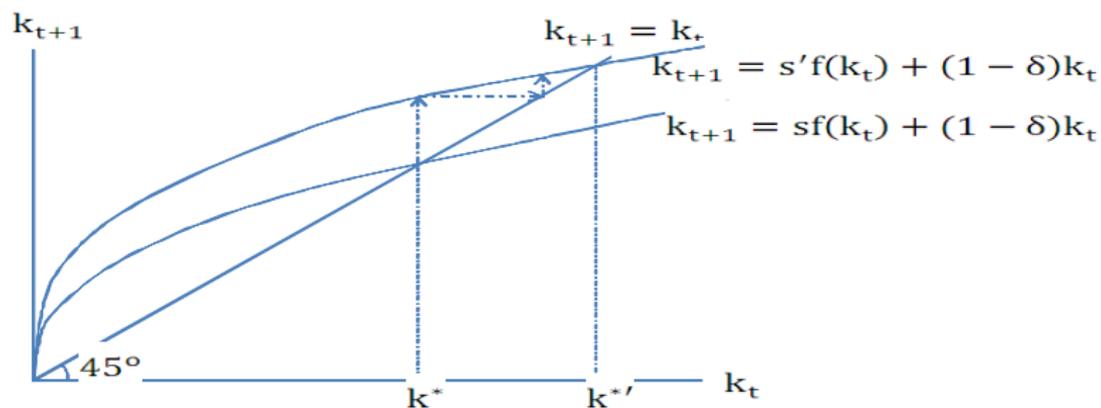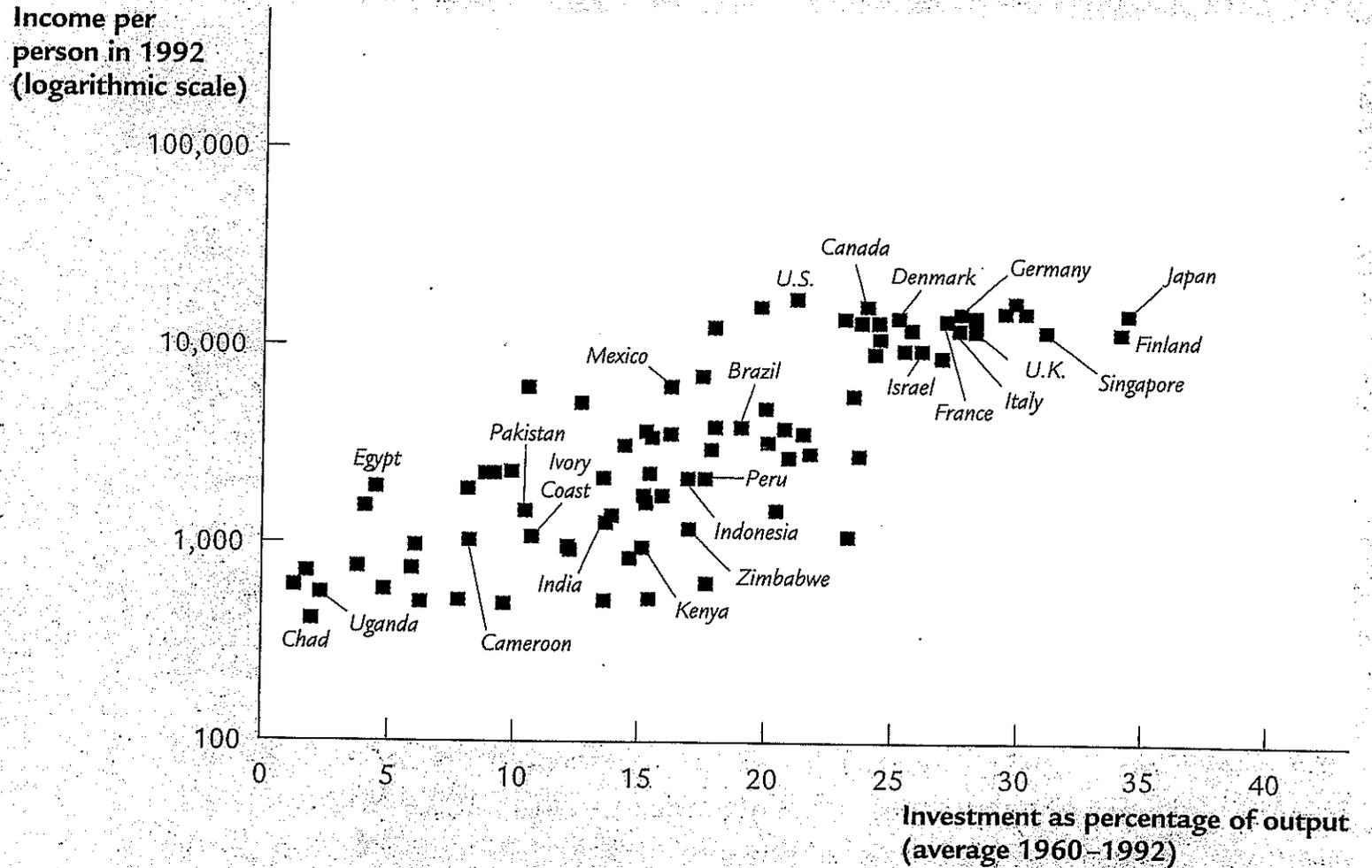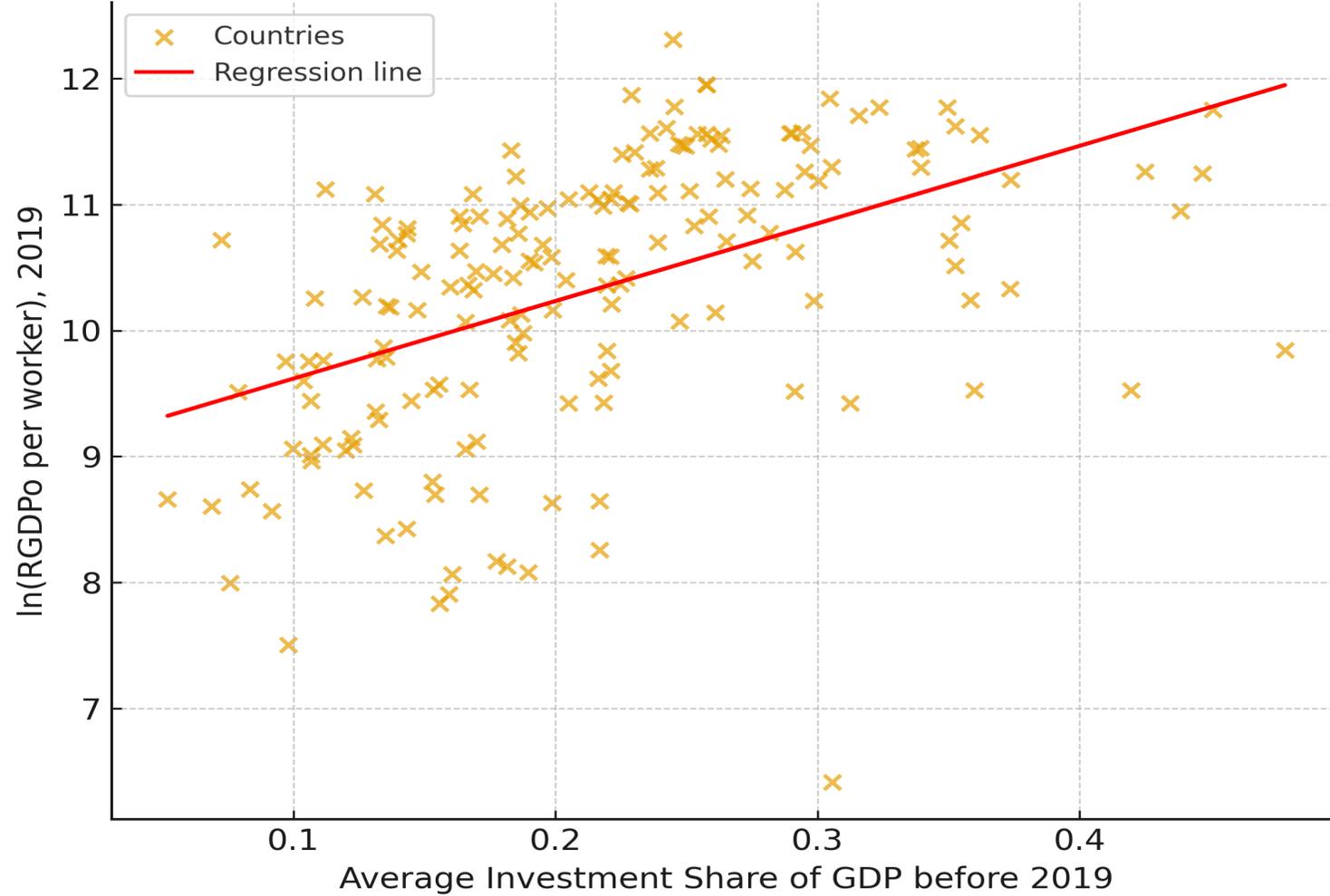
- The impact of s:

$$s'>s$$

figure 4-6

International Evidence on Investment Rates and Income per Person This scatterplot shows the experience of 84 countries, each represented by a single point. The horizontal axis shows the country's rate of investment, and the vertical axis shows the country's income per person. High investment is associated with high income per person, as the Solow model predicts.

Log of RGDPo per worker vs Historical Investment Share (2019)

Solow Model

- The impact of δ:

$$\delta' > \delta$$
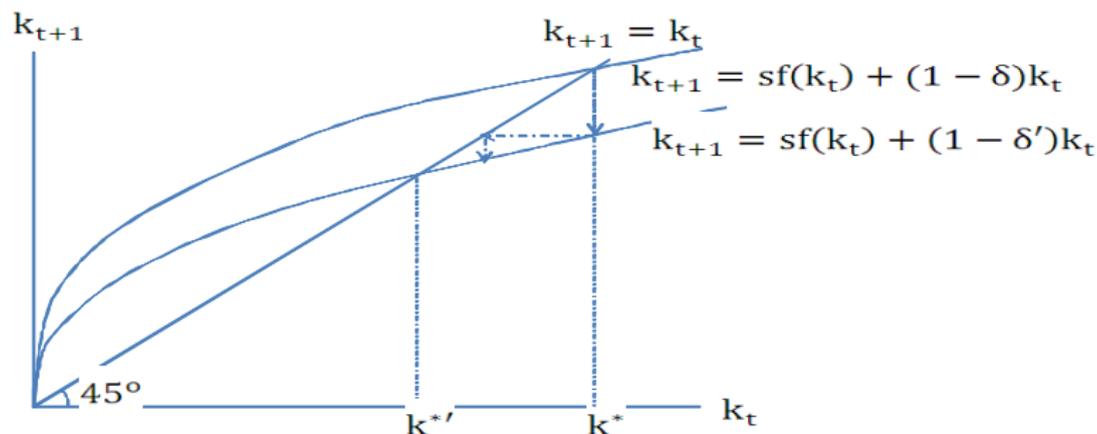
Solow Model

- An increase in s and a decrease in δ increase k^* and, therefore, y^*.
- Both changes cannot influence the long run growth rate.

The Extended Solow Model

- Solow model predicts that economic growth rate eventually converges to 0. This is not what we observe in data.
- In order to match the theory with data, we introduce technology growth and population growth.
  - Assume that

$$
\begin{aligned}
Y_t &= F\left(K_t, T_t N_t\right) \\
T_{t+1} &= \left(1 + g\right) T_t \\
N_{t+1} &= \left(1 + n\right) N_t
\end{aligned}
$$

  where T_t and N_t denote technology and population at date t, respectively. We assume labor augmenting technological progress.
  - Labor augmenting technological progress means that an increase in the technology has the same effect as an increase in population.
  - We assume the labor augmenting technological progress, because it leads the theory to data.

The Extended Solow Model

- GDP per unit of effective labor

$$Y_t = F\left(K_t, T_t N_t\right)$$
$$= F\left(\frac{K_t}{T_t N_t}, 1\right) T_t N_t$$
$$y_{et} = f\left(k_{et}\right) \equiv F\left(k_{et}, 1\right)$$

where $y_{et} = \frac{Y_t}{T_t N_t}$ and $k_{et} = \frac{K_t}{T_t N_t}$ are GDP per unit of effective labor and capital per unit of effective labor.
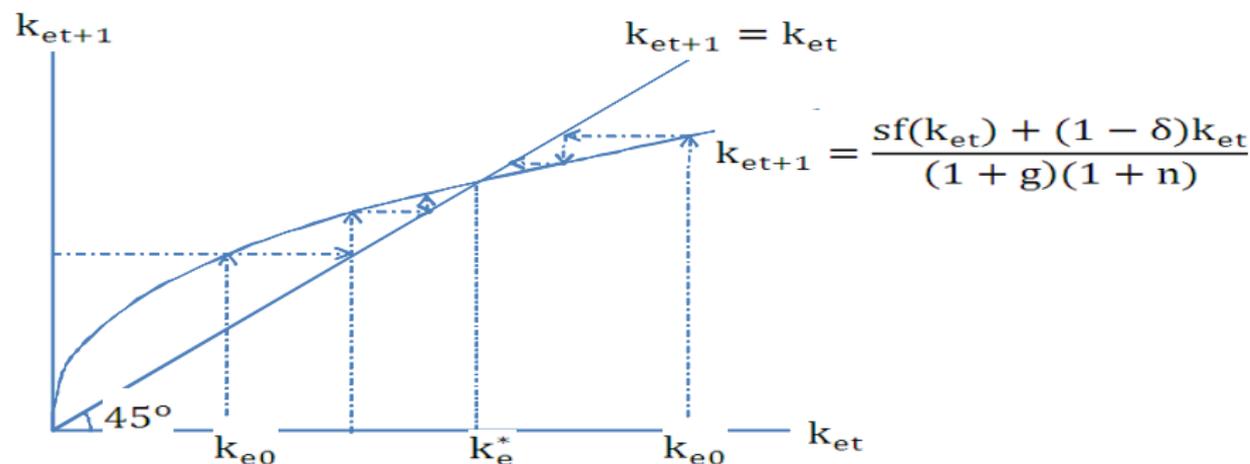
The Extended Solow Model

- Capital Accumulation

$$
\begin{aligned}
K_{t+1} &= I_t + (1 - \delta) K_t \\
&= Y_t - C_t + (1 - \delta) K_t \\
&= Y_t - (1 - s) Y_t + (1 - \delta) K_t \\
&= s Y_t + (1 - \delta) K_t \\
&= s y_{et} T_t N_t + (1 - \delta) k_{et} T_t N_t \\
\frac{K_{t+1}}{T_{t+1} N_{t+1}} \frac{T_{t+1} N_{t+1}}{T_t N_t} &= s y_{et} + (1 - \delta) k_{et} \\
k_{et+1} (1 + g) (1 + n) &= s f (k_{et}) + (1 - \delta) k_{et}
\end{aligned}
$$

- The extended Solow model

$$
k_{et+1} = \frac{s f (k_{et}) + (1 - \delta) k_{et}}{(1 + g) (1 + n)}
$$

The Extended Solow Model

The Extended Solow Model

> ## Definition
>
> The steady state is the points at which $\{(c_e^*, y_e^*, k_e^*)\}$ satisfies
>
> $$c_{et+1}^* = c_{et}^*, \ y_{et+1}^* = y_{et}^*, \ k_{et+1}^* = k_{et}^*$$
>
> where $c_{et} = \frac{C_t}{T_t N_t}$, $y_{et} = \frac{Y_t}{T_t N_t}$, and $k_{et} = \frac{K_t}{T_t N_t}$.

- For any initial capital stock, economy eventually converges to the steady state.
- A key assumption to derive this result is also $f''(k) < 0$.

- On the steady state k^* must satisfy

$$
\begin{aligned}
k_e^* &= \frac{sf\left(k_e^*\right) + (1-\delta)\,k_e^*}{(1+g)\,(1+n)} \\
(1+g)\,(1+n)\,k_e^* &= sf\left(k_e^*\right) + (1-\delta)\,k_e^* \\
(1+g+n+gn)\,k_e^* &= sf\left(k_e^*\right) + (1-\delta)\,k_e^*
\end{aligned}
$$

$$
sf\left(k_e^*\right) = (g+n+\delta+gn)\,k_e^*
$$

The Extended Solow Model

- Example $f\left(k^{e}\right)=\left(k^{e}\right)^{\alpha}$

$$
\begin{aligned}
s\left(k_{e}^{*}\right)^{\alpha} &= \left(g+n+\delta+gn\right) k_{e}^{*} \\
\frac{k_{e}^{*}}{\left(k_{e}^{*}\right)^{\alpha}} &= \frac{s}{g+n+\delta+gn} \\
\left(k_{e}^{*}\right)^{1-\alpha} &= \frac{s}{g+n+\delta+gn} \\
k_{e}^{*} &= \left[\frac{s}{g+n+\delta+gn}\right]^{\frac{1}{1-\alpha}}
\end{aligned}
$$

The Extended Solow Model

- The Impact of n :

$$n'>n$$
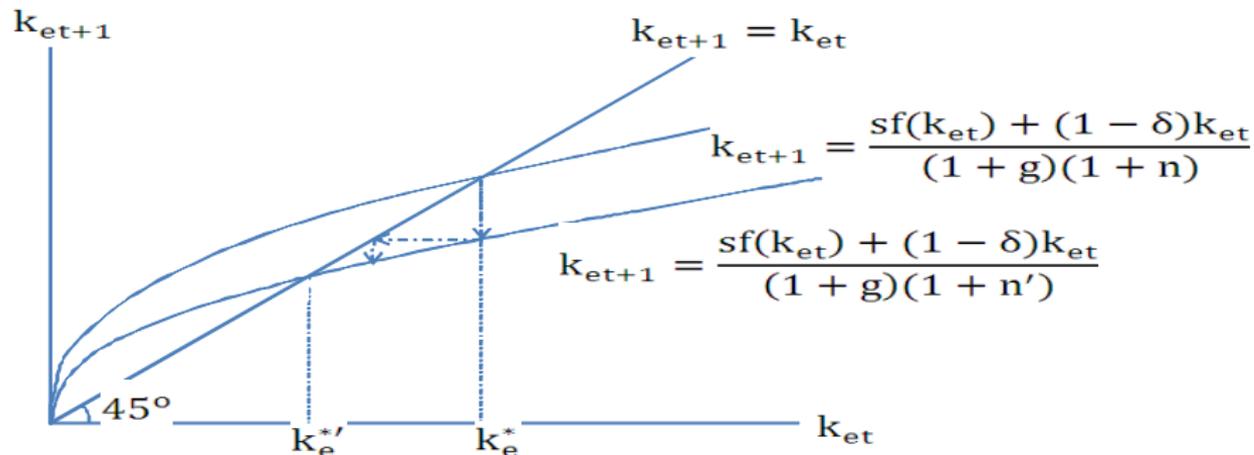
figure 4-13

Income per person in 1992 (logarithmic scale)

100,000

Germany

Denmark   U.S.

Canada

10,000

Israel

U.K.   Japan

Mexico

Italy   Finland   France   Singapore

Egypt   Brazil

Pakistan   Ivory Coast

Indonesia   Peru   Cameroon   Kenya

1,000

India   Zimbabwe

Chad   Uganda

100

0   1   2   3   4

Population growth (percent per year) (average 1960–1992)
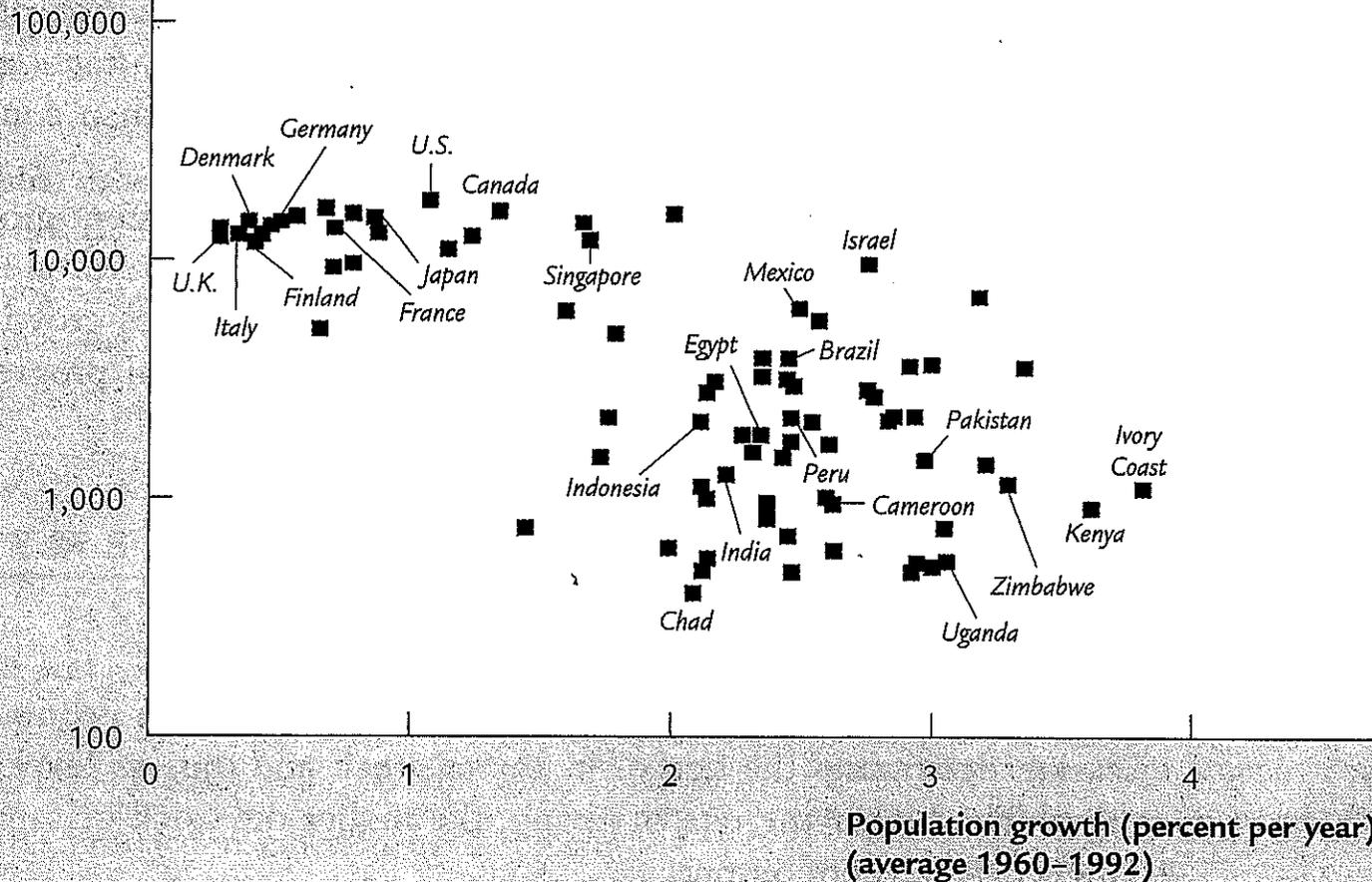
International Evidence on Population Growth and Income per Person This figure is a scatterplot of data from 84 countries. It shows that countries with high rates of population growth tend to have low levels of income per person, as the Solow model predicts.

Log of RGDPo per worker vs Historical Population Growth (2019)
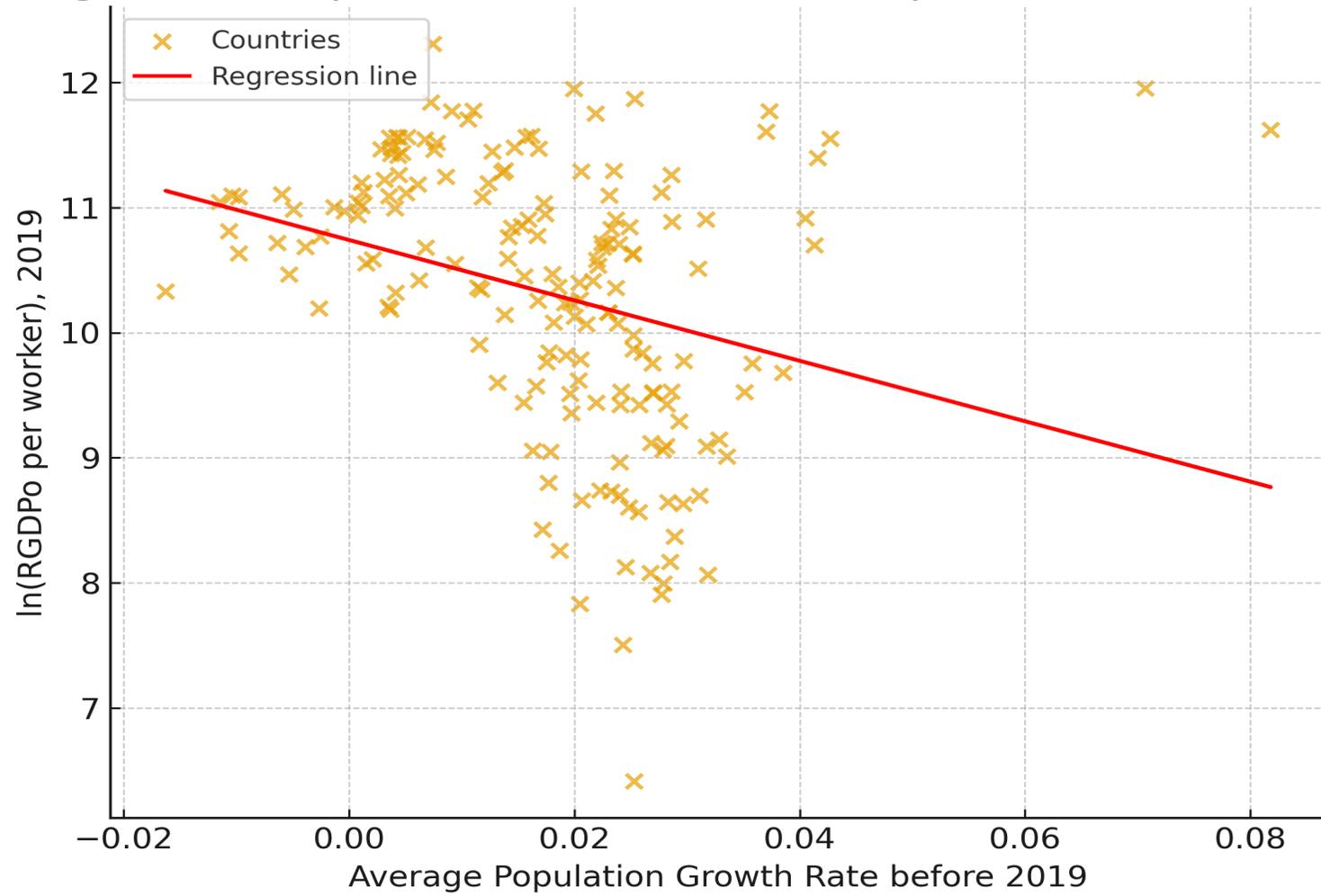
- An increase in n lowers, k_e^* and therefore, y_e^*.

The Extended Solow Model

- Golden Rule Level of Capital Stock

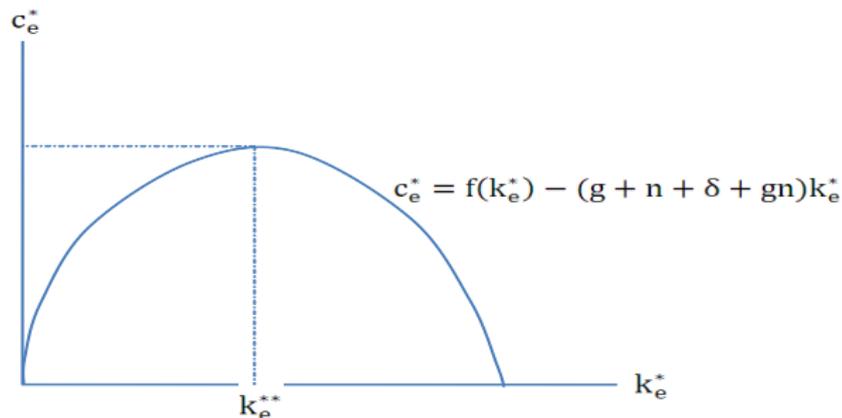$$sf\left(k_e^*\right) = \left(g + n + \delta + gn\right) k_e^*$$

$$
\begin{aligned}
c_e^* &= (1-s)\, f\left(k_e^*\right) \\
&= f\left(k_e^*\right) - \left(g + n + \delta + gn\right) k_e^*
\end{aligned}
$$

$$
\begin{aligned}
\frac{dc_e^*}{dk_e^*} &= f'\left(k_e^{**}\right) - \left(g + n + \delta + gn\right) = 0 \\
\frac{d^2 c_e^*}{d\left(k_e^*\right)^2} &= f''\left(k_e^*\right) < 0
\end{aligned}
$$

- k_e^{**} is the capital stock that maximizes consumption on the steady state.

The Golden Rule Level of Capital Stock

$$c_e^* = f(k_e^*) - (g + n + \delta + gn)k_e^*$$

The Extended Solow Model

- Example: $f(k_e) = (k_e)^\alpha$

$$\alpha (k_e^{**})^{\alpha-1} = g + n + \delta + gn$$
$$(k_e^{**})^{1-\alpha} = \frac{\alpha}{g + n + \delta + gn}$$
$$k_e^{**} = \left[\frac{\alpha}{g + n + \delta + gn}\right]^{\frac{1}{1-\alpha}}$$

Note that

$$k_e^* = \left[\frac{s}{g + n + \delta + gn}\right]^{\frac{1}{1-\alpha}}$$

Hence if $s = \alpha$, the golden rule level of capital stock is attained.

国内粗貯蓄率の国際比較
（2011年～2013年の平均、40％以上は赤字、出所:総務省）

	国内総生産に対する割合（%）			国内粗貯蓄率
	民間最終消費支出	政府最終消費支出	財貨・サービスの純輸出	
アジア				
日本	61%	20%	−2%	19%
イスラエル	56%	23%	0%	21%
イラン	40%	11%	7%	49%
インド	57%	12%	−6%	31%
インドネシア	55%	9%	−1%	36%
韓国	51%	15%	3%	34%
サウジアラビア	29%	20%	24%	51%
シンガポール	38%	10%	24%	52%
タイ	53%	16%	2%	31%
中国	36%	14%	3%	50%
トルコ	71%	15%	−7%	15%
フィリピン	73%	11%	−4%	16%
マレーシア	49%	14%	12%	37%
北アメリカ				
アメリカ合衆国	69%	20%	−4%	12%
カナダ	55%	22%	−2%	23%
メキシコ	67%	12%	−1%	21%
南アメリカ				
アルゼンチン	65%	15%	1%	20%
ブラジル	62%	21%	−1%	17%
ヨーロッパ				
イギリス	65%	21%	−2%	15%
イタリア	61%	19%	0%	20%
オーストリア	54%	20%	3%	26%
オランダ	45%	26%	9%	29%
ギリシャ	74%	17%	−5%	9%
スイス	54%	11%	10%	35%
スウェーデン	47%	26%	5%	28%
デンマーク	49%	27%	6%	24%
ドイツ	56%	19%	6%	25%
フィンランド	55%	24%	−1%	21%
フランス	55%	24%	−2%	21%
ベルギー	52%	24%	1%	24%
ポーランド	61%	18%	0%	21%
ポルトガル	66%	19%	−1%	15%
ロシア	50%	19%	7%	31%
アフリカ				
エジプト	79%	11%	−6%	9%
南アフリカ	60%	20%	−1%	20%
オセアニア				
オーストラリア	55%	18%	0%	27%

Kaldor's Stylized Facts (1963)

- Kaldor (1963) pointed out 6 stylized facts of economic growth. These facts are repeatedly observed by aggregate data of OECD countries.
- I would like to examine how the extended Solow growth model explains these stylized facts.
- As the extended Solow model predicts that economy eventually converges to the steady state, we expect that the behavior of real economy can be approximated by the behavior on the steady state. Hence, we compare the prediction of the theory on the steady state with Kaldor's facts.

- Definition

$$\ln k \equiv \log_e k$$

where $e = \lim_{m \to \infty} \left(1 + \frac{1}{m}\right)^m = 2.71828...$ It is useful to use e as a base. The useful properties of e and $\ln$ are

$$\frac{de^t}{dt} = e^t, \; \frac{d \ln k}{dk} = \frac{1}{k}$$

$$\text{If } g \approx 0, \; \ln(1+g) \approx g$$

- The review of high school mathematics

$$\ln kl = \ln k + \ln l$$
$$\ln k^\alpha = \alpha \ln k$$
$$\ln e = 1, \ln 1 = 0$$

Therefore

$$\ln \frac{k}{l} = \ln kl^{-1} = \ln k + \ln l^{-1} = \ln k - \ln l$$

Mathematical Preparation for the Analysis of Growth Rate

Lemma

Suppose that the growth rate of x, $g_x \approx 0$, then g_x is nearly expressed as follows

$$g_x \approx \ln x_{t+1} - \ln x_t$$

Proof.

Suppose that $x_{t+1} = (1 + g_x) x_t$,

$$
\begin{aligned}
\ln x_{t+1} - \ln x_t &= \ln (1 + g_x) x_t - \ln x_t \\
&= \ln (1 + g_x) + \ln x_t - \ln x_t \\
&= \ln (1 + g_x) \\
&\approx g_x \text{ if } g_x \approx 0
\end{aligned}
$$

$\square$

- **Remark:** If the time interval is not 1 but is almost 0, we can prove that the relationship is exact for any g_x:
  - Define g_x so that $x_{t+\Delta} = (1 + g_x \Delta) x_t$. Then, from the previous lemma, for any g_x there is a small Δ so that $g_x \Delta \approx \ln x_{t+\Delta} - \ln x_t$. It is shown that for any g_x,

$$\lim_{\Delta \to 0} \frac{\ln x_{t+\Delta} - \ln x_t}{\Delta} = \frac{d \ln x_t}{dt} = g_x, \ where \ g_x \equiv \frac{\frac{dx_t}{dt}}{x_t}$$

    Hence, we treat $g_x \approx \ln x_{t+1} - \ln x_t$ as if $g_x = \ln x_{t+1} - \ln x_t$ below.

Lemma

$$g_{xy} \approx g_x + g_y$$
$$g_{\frac{x}{y}} \approx g_x - g_y$$
$$g_{x^\alpha} \approx \alpha g_x$$

Proof.

$$
\begin{aligned}
g_{xy} &\approx \ln x_{t+1} y_{t+1} - \ln x_t y_t \\
&= \ln x_{t+1} + \ln y_{t+1} - \left[\ln x_t + \ln y_t\right] \\
&= \ln x_{t+1} - \ln x_t + \ln y_{t+1} - \ln y_t \\
&\approx g_x + g_y
\end{aligned}
$$

$\square$

Proof.

$$
\begin{aligned}
g_{x^\alpha} &\approx \ln x_{t+1}^\alpha - \ln x_t^\alpha \\
&= \alpha \ln x_{t+1} - \alpha \ln x_t \\
&= \alpha \left[\ln x_{t+1} - \ln x_t \right] \\
&\approx \alpha g_x
\end{aligned}
$$

$$
\begin{aligned}
g_{\frac{x}{y}} &= g_{xy^{-1}} \\
&\approx g_x + g_{y^{-1}} \\
&\approx g_x - g_y
\end{aligned}
$$

$\square$

Kaldor's Stylized Facts (1963)

- *The growth rate of GDP per capita is nearly constant:*

$$g_y = g_{y_e^* T} \approx g_{y_e^*} + g_T = g_T = g$$

- *The growth rate of capital per capita is nearly constant:.*

$$g_k = g_{k_e^* T} \approx g_{k_e^*} + g_T = g_T = g$$

- *The growth rate of output per worker differs substantially across countries.*

Hence in order to fit the theory to data, g must be constant and differs among countries.

Figure 1.2 Natural Logarithm of Per Capita Real GDP

Here, the slope of the graph is approximately equal to the growth rate of per capita real GDP. Excluding the Great Depression and World War II, the growth rate of per capita real GDP is remarkably close to being constant for the period 1900–2011. That is, a straight line would fit the graph fairly well.
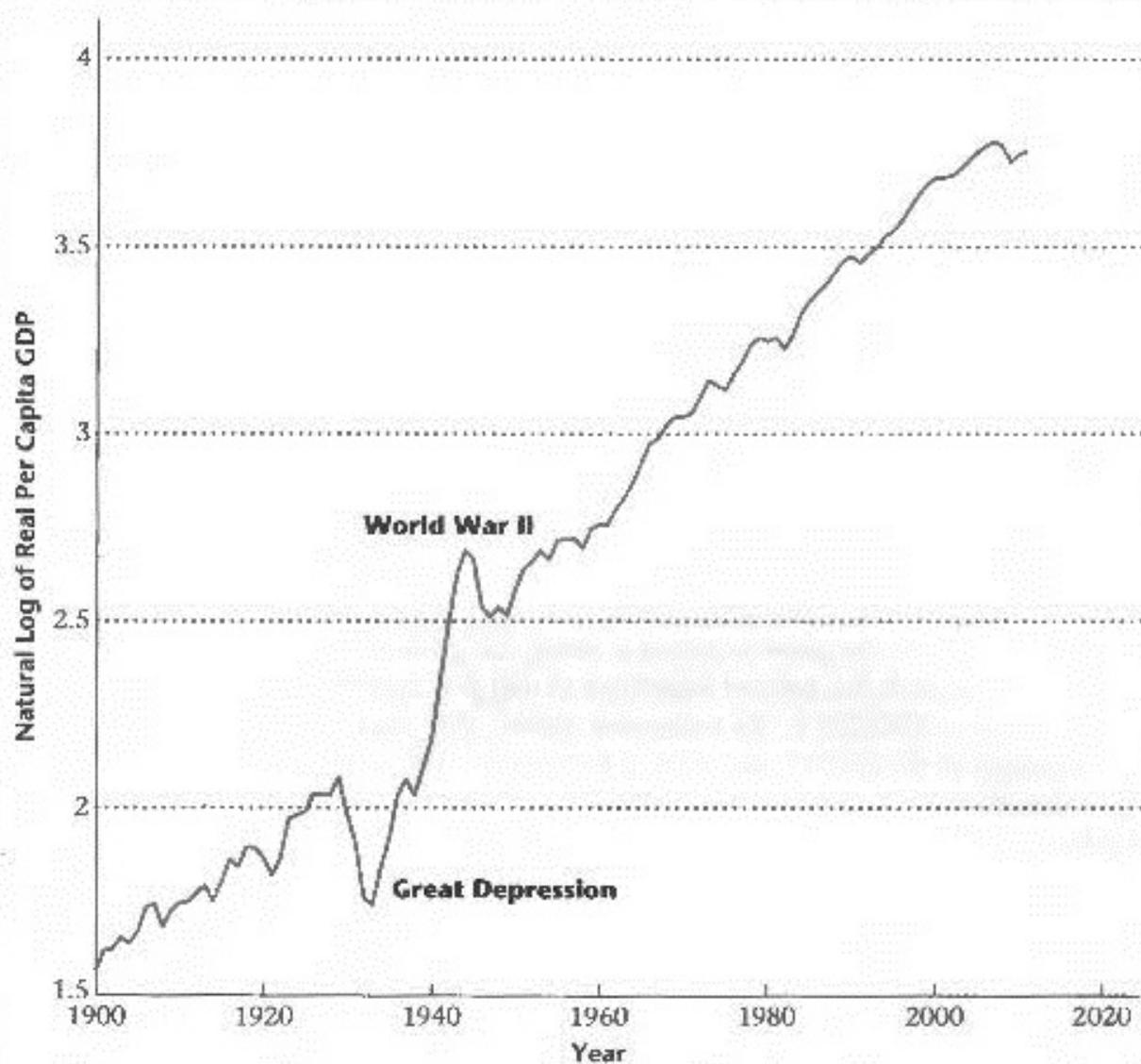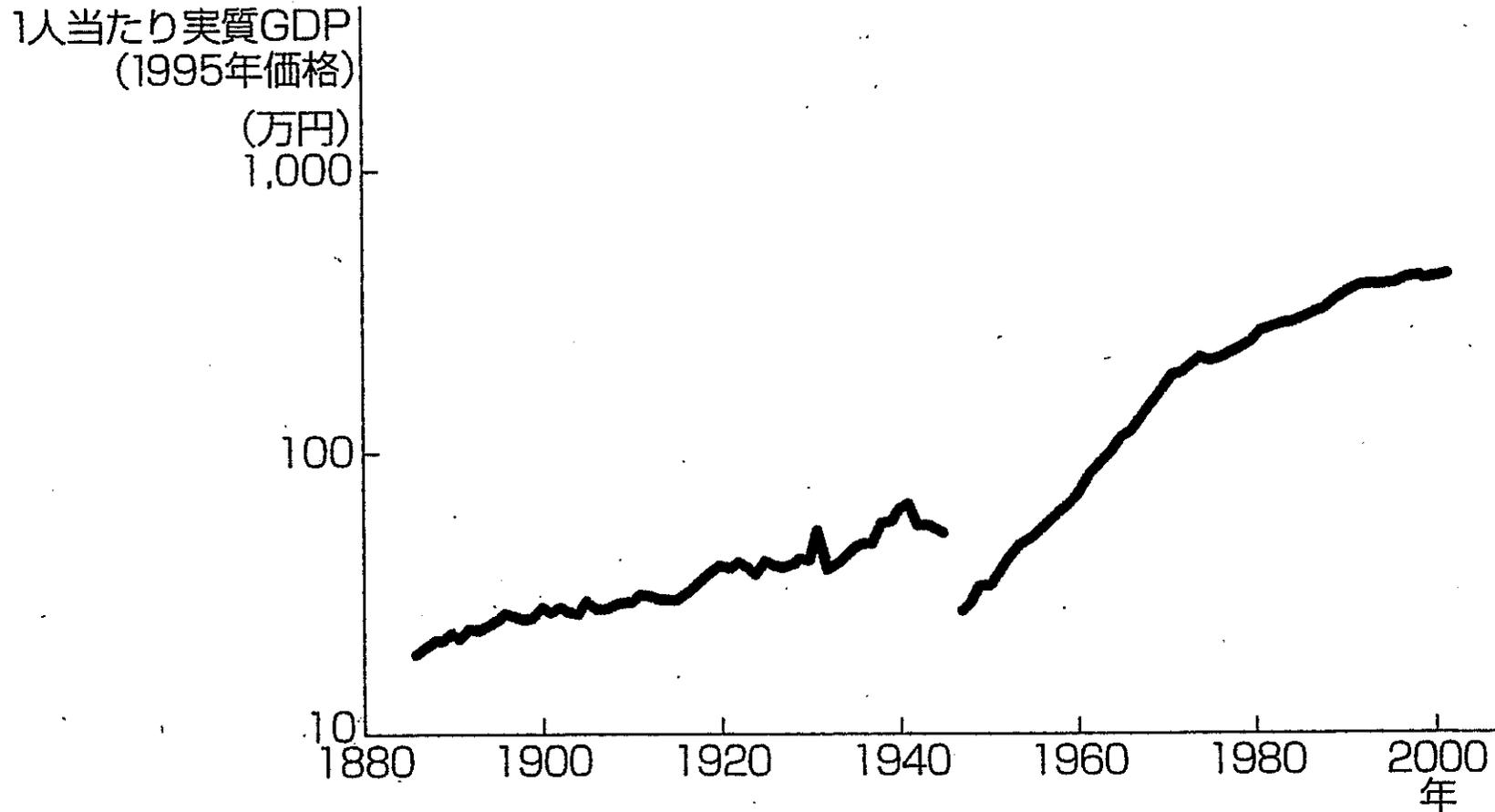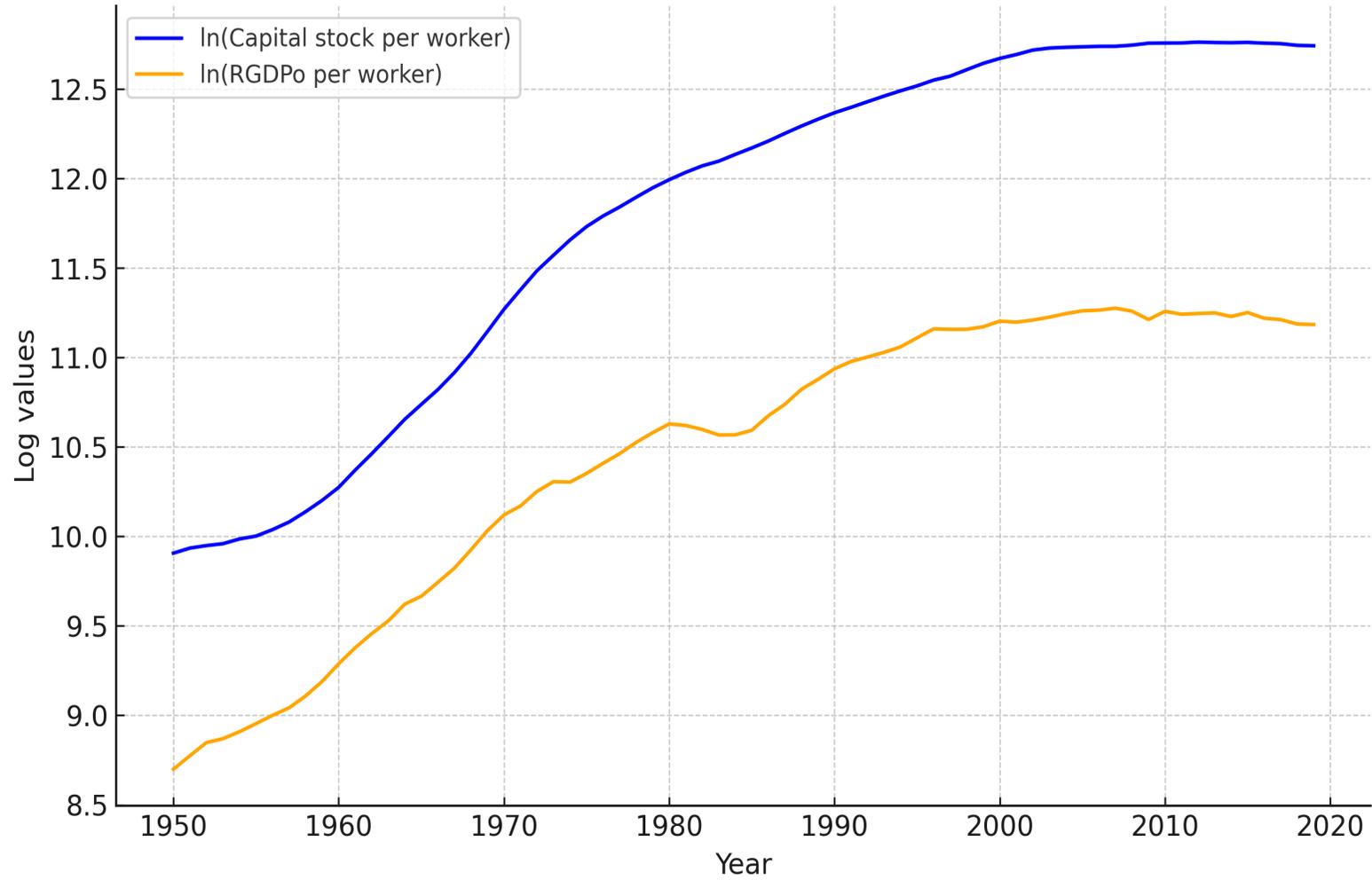
図 1-1′　日本経済の1人当たり実質GDP

1人当たり実質GDP
（1995年価格）
（万円）

　20世紀における日本経済の急速な成長を反映して，1人当たり実質GDPの現在の値は1900年当時の10倍を超えている．第2次世界大戦の破壊の影響とその後の高度成長および1970年代以降の成長率低下が顕著である．短期的な好不況ももちろん観察できるが，戦後においては実質GDPが減少した厳しい不況は希である．

（注）　暦年データ（1945～51年のみは会計年度）．

（出所）　日本統計協会『日本長期統計総覧』，大川一司ほか編『長期経済統計』，『経済統計年報』．

Japan: Log of Capital Stock per Worker and RGDPo per Worker (PWT10.0)

Kaldor's Stylized Facts (1963)

- *The rate of return to capital is nearly constant:*

$$r_t = f'(k_e^*) = const$$

- *The ratio of physical capital to output is nearly constant:*

$$\frac{K_t}{Y_t} = \frac{k_e^* T_t N_t}{y_e^* T_t N_t} = \frac{k_e^*}{y_e^*} = const$$

- *The shares of labor and physical capital are nearly constant:* Note that the marginal productivity of capital and labor are

$$\frac{r_t K_t}{Y_t} = r_t \frac{K_t}{Y_t} = const$$

Note that

$$1 = \frac{r_t K_t + w_t L_t}{Y_t}$$

Hence

$$\frac{w_t L_t}{Y_t} = 1 - \frac{r_t K_t}{Y_t} = const$$

Figure 1.12 Real Interest Rate

The figure shows a measure of the real interest rate, which here is the short-term nominal interest rate minus the actual rate of inflation. Monetary policy can have a short-run effect on the real interest rate; for example, the low real interest rates during the 1990–1991, 2001, and 2008–2009 recessions can be attributed to monetary policy actions.
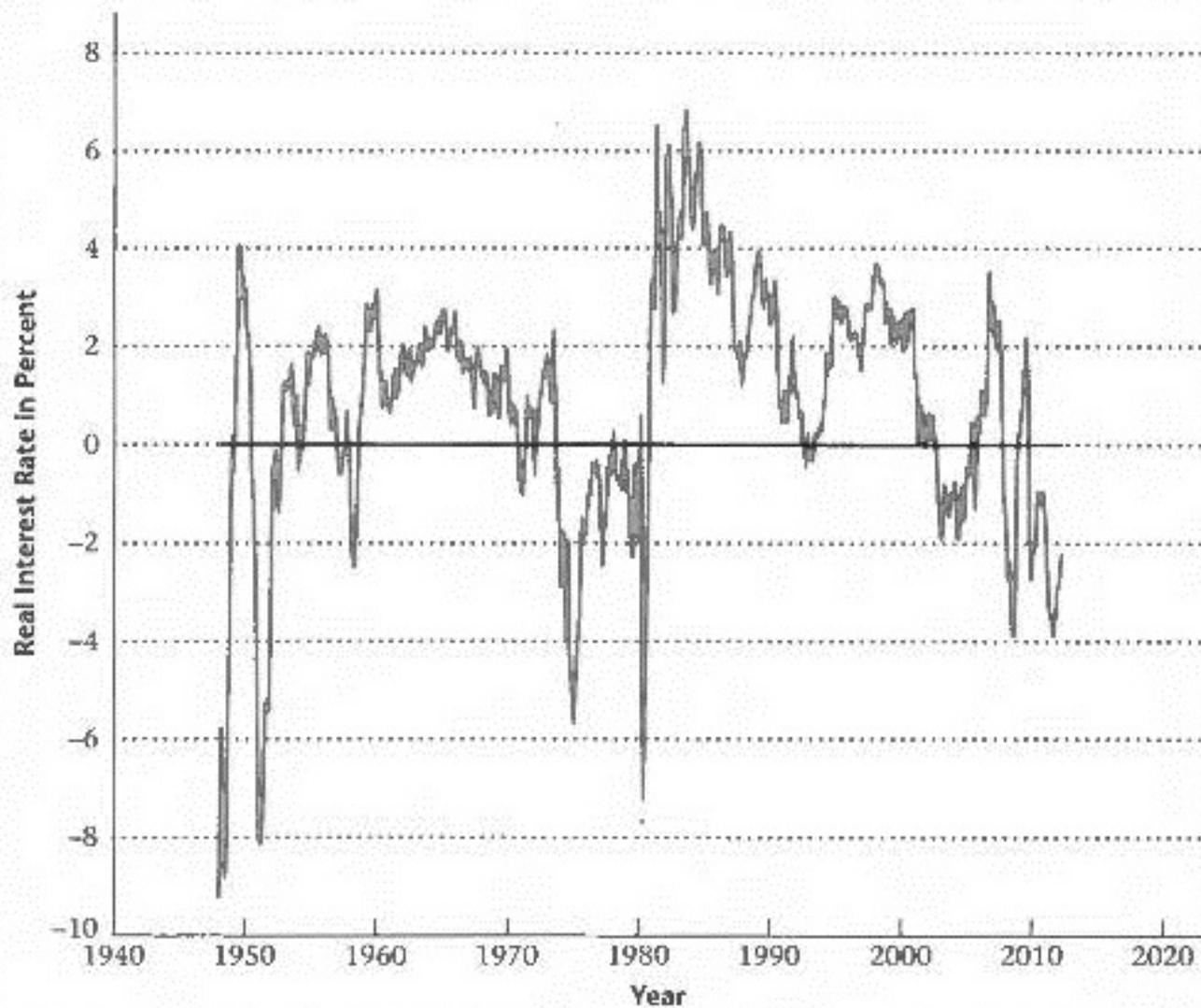
実質金利
（出所：内閣府）
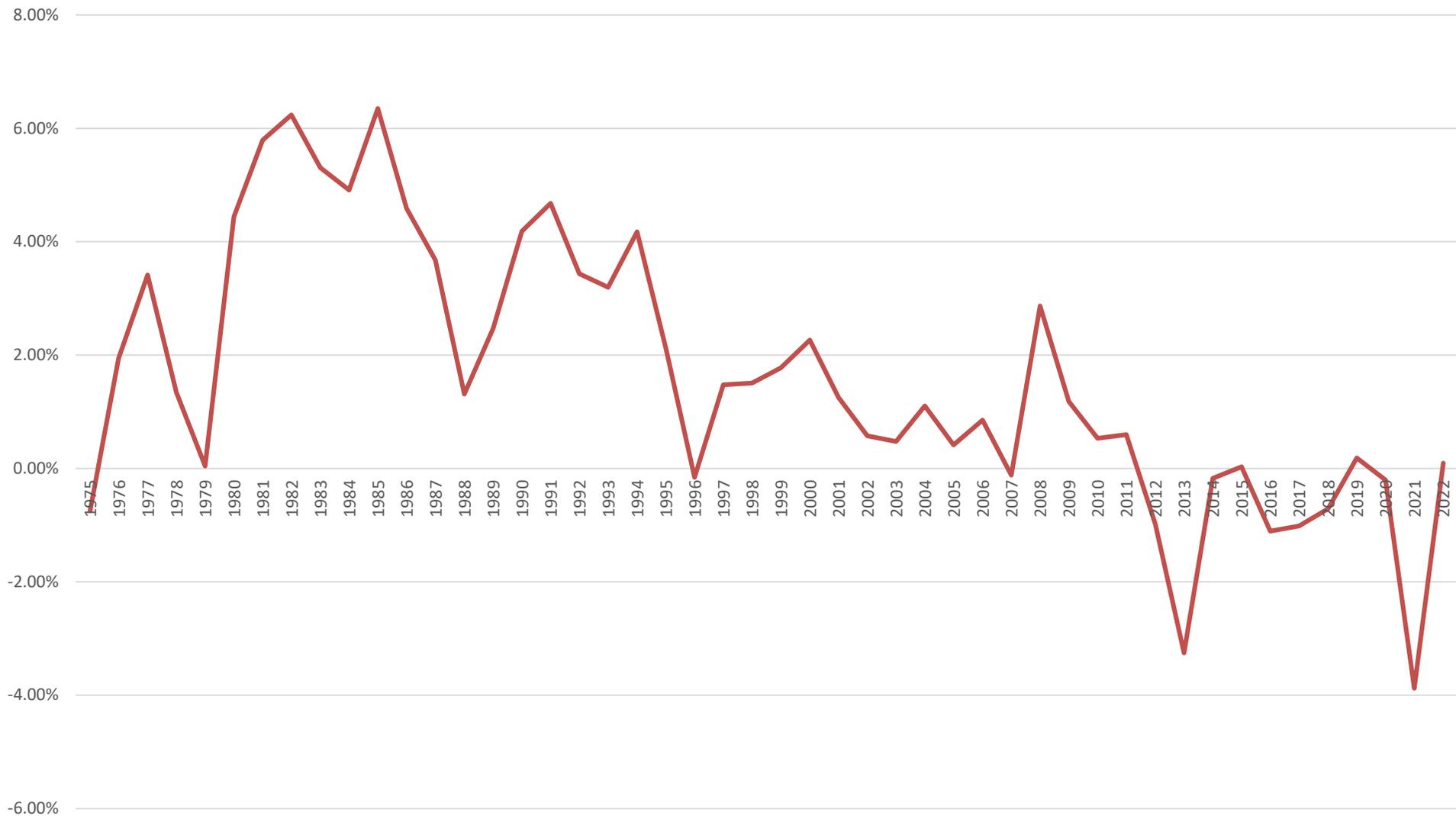
実質金利（5年物国債利回りと消費者物価上昇率から求めた事後的なもの、右目盛り）

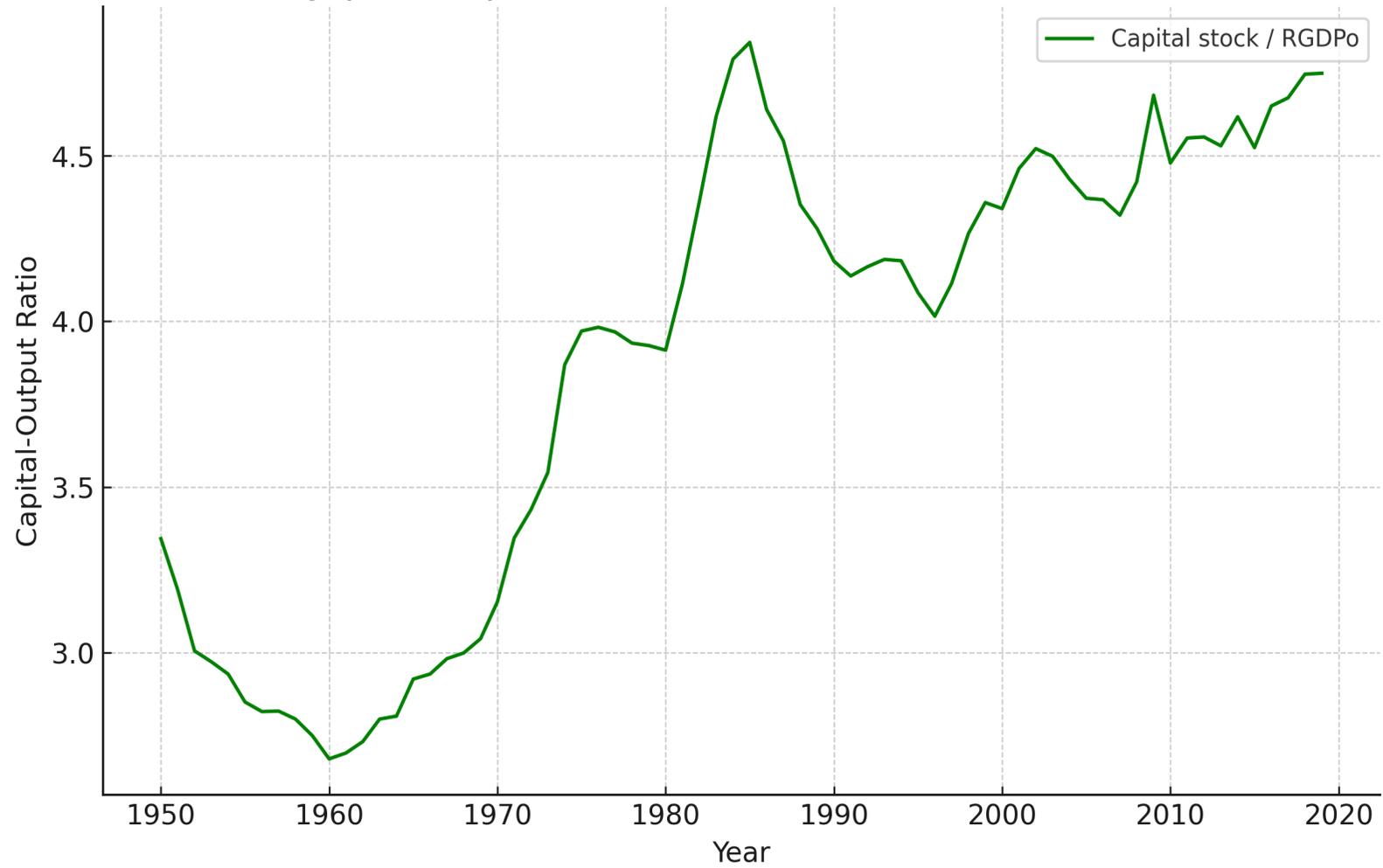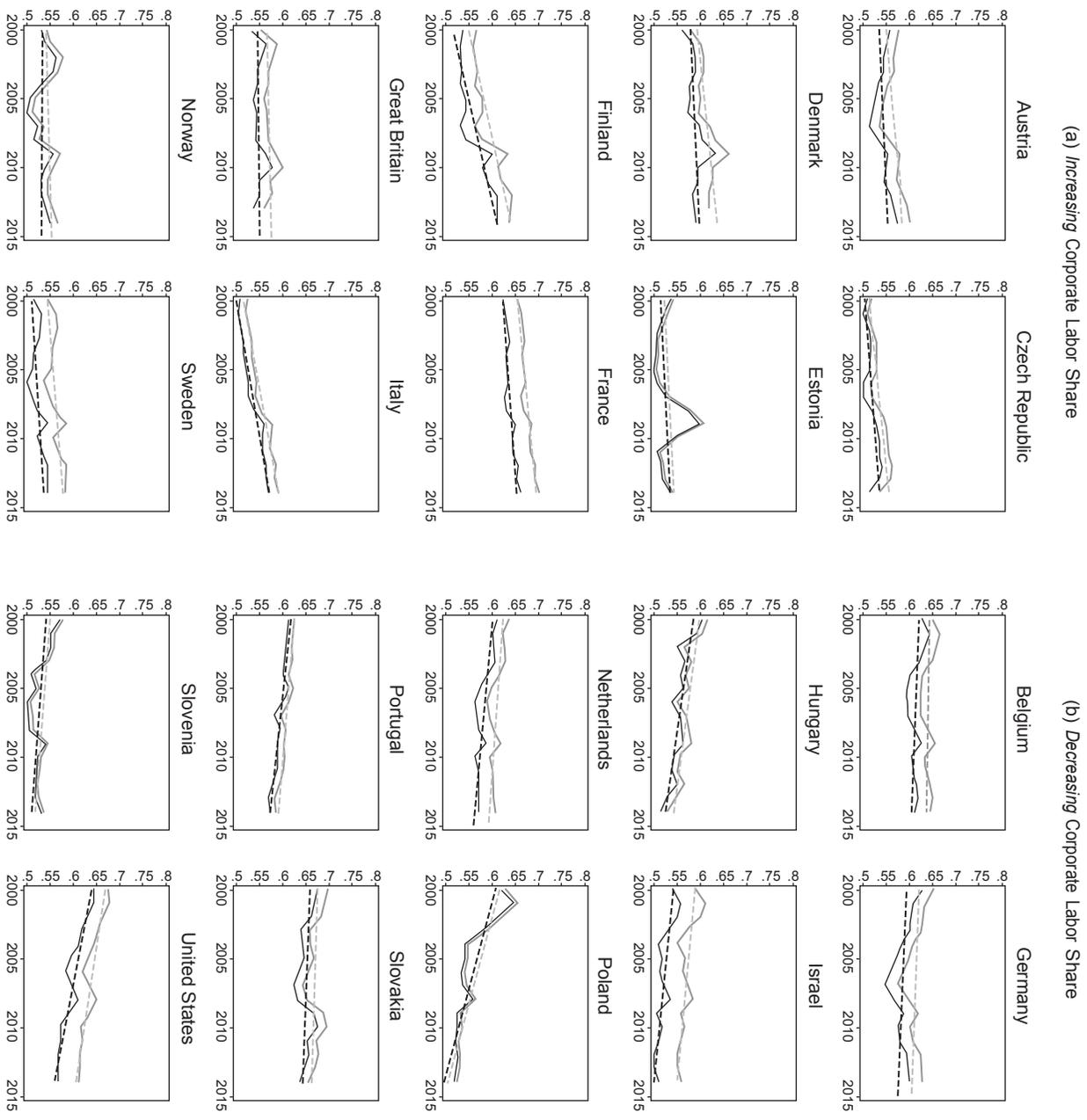Japan: Capital Stock to RGDPo Ratio (PWT10.0)

FIGURE 2

LABOR SHARE BY COUNTRY, CORPORATE SECTOR, 2000-2014

(a) *Increasing* Corporate Labor Share

(b) *Decreasing* Corporate Labor Share

NOTES: In each of these panels we show by country the time-series of the corporate labor shares LS (——) and LSx (——) for the sample period 2000-2014. The first and second (third and fourth) columns show the results for the countries with increasing (decreasing) corporate labor share LSx over the sample period. See the construction of these series in Section 2.
SOURCE: Aum et al. (2018).

Growth Accounting

- Assume that

$$y_{et} = (k_{et})^{\alpha}.$$

Then

$$\frac{y_t}{T_t} = \left(\frac{k_t}{T_t}\right)^{\alpha}$$
$$y_t = T_t^{1-\alpha}(k_t)^{\alpha}$$

Hence

$$g_{y_t} = g_{T_t^{1-\alpha}k_t^{\alpha}} \approx (1-\alpha)\,g_{T_t} + \alpha g_{k_t}$$

$$g_{y_t} = \alpha g_{k_t} + R(t)\,,\ \text{where}\ R(t) = (1-\alpha)\,g_{T_t}$$

Growth Accounting

- Note that α can be estimated as follows:

$$\alpha = \frac{\frac{r_t K_t}{Y_t} + \frac{r_{t+1} K_{t+1}}{Y_{t+1}}}{2} = 1 - \frac{\frac{w_t N_t}{Y_t} + \frac{w_{t+1} N_{t+1}}{Y_{t+1}}}{2}$$

So we can estimate

$$R\left(t\right) = g_{y_t} - \frac{\frac{r_t K_t}{Y_t} + \frac{r_{t+1} K_{t+1}}{Y_{t+1}}}{2} g_{k_t} = g_{y_t} - \left(1 - \frac{\frac{w_t N_t}{Y_t} + \frac{w_{t+1} N_{t+1}}{Y_{t+1}}}{2}\right) g_{k_t}$$

- $R\left(t\right)$ is called the Solow residual or the growth rate of the Total Factor Productivity (TFP). It reflects that all sources of growth other than the contribution of capital accumulation.

Growth Accounting

- Remarks
  1. The equation for Solow residual can be derived without assuming $f(k) = k^{\alpha}$. We don't need constant returns to scale as well.
  2. This is the simplest case. Usually, we can include other inputs as well.
  3. Growth accounting can be applied for several analysis.
     1. Young (1994) finds that the high growth rate of Hong Kong, Singapore, South Korea, and Taiwan over the past three decades is almost entirely due to rising investment, increase in labor force participation, and increase in the level of education, but not to rapid technological progress.
     2. Productivity slow down puzzle. $R(t)$ became small after 1970s.

Figure 4.18 The Solow Residual for the United States

The Solow residual is a measure of total factor productivity, and it is calculated here using a Cobb–Douglas production function. Measured total factor productivity has increased over time, and it also fluctuates about trend, as shown for the period 1948–2010.
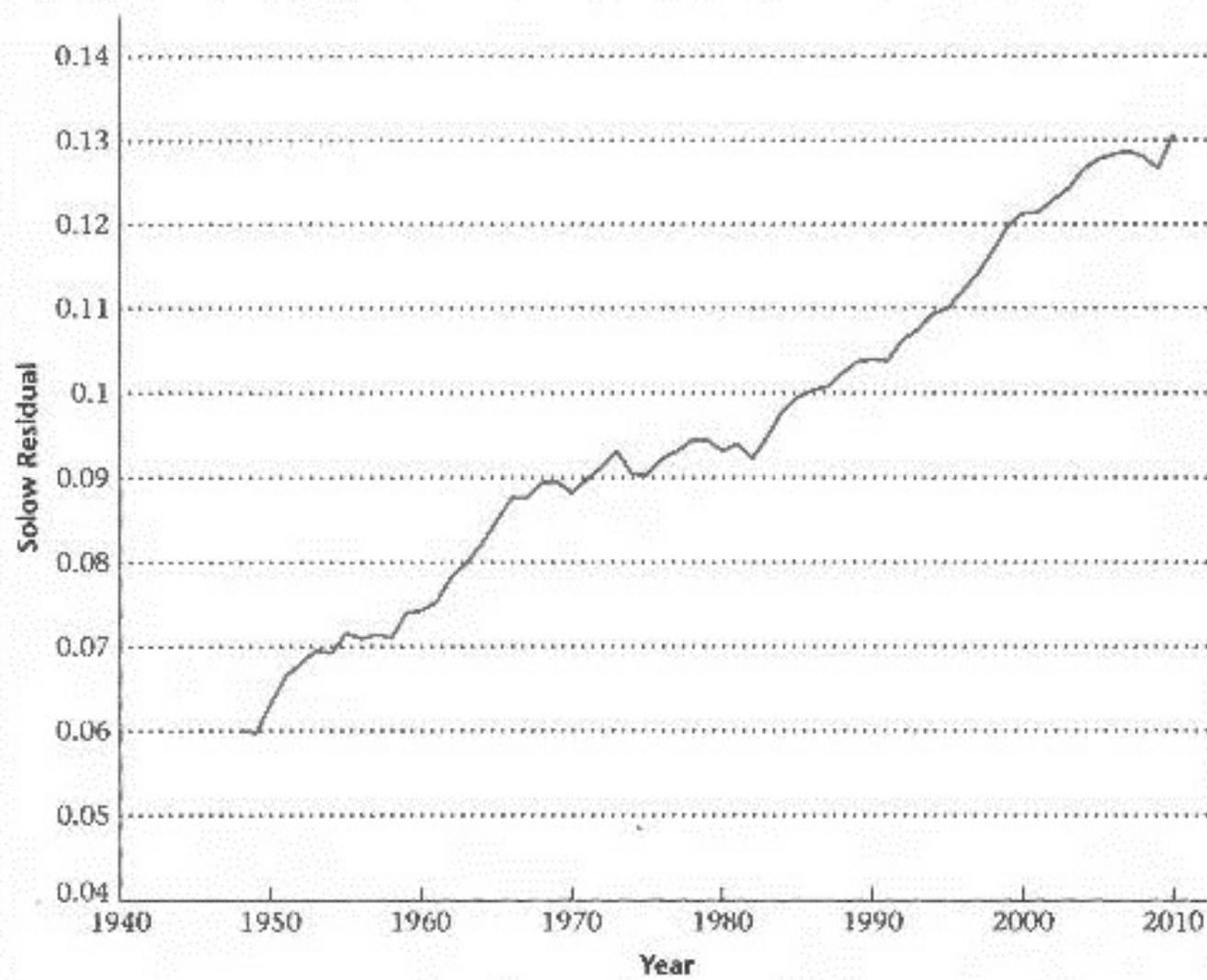
Solow Model and Income Differences

- In order for the Solow model on the steady state to explain the long run behavior of developed countries, we need to assume that g is constant and differs across countries.
- In this section, we ask a question whether the Solow model can explain the development facts.
    - First, I show what the new stylized facts are.
    - The next, I would like to ask if the Solow model can explain these facts.

Development Facts

- Parente and Prescott (1993) pointed out four main stylized facts.

    1. Income difference across countries is large.
    2. Wealth distribution has shifted up.
    3. Relative Income distribution does not show convergence.
    4. There have been development miracles and disasters.

- Durlauf and Quah (1998) also pointed out that

    1. Relative Income distribution across countries shows two peaks.

Development Facts

- Acemoglu (2009) points out that
  1. The relative ranking of countries has changed little between 1960 and 2000.
  2. The origins of the current cross-country differences in income per capita occur between the late eighteen century and early twentieth centuries.

- Barro and Sala–i–Martin (1995) shows that
  1. There is no evidence of absolute convergence.
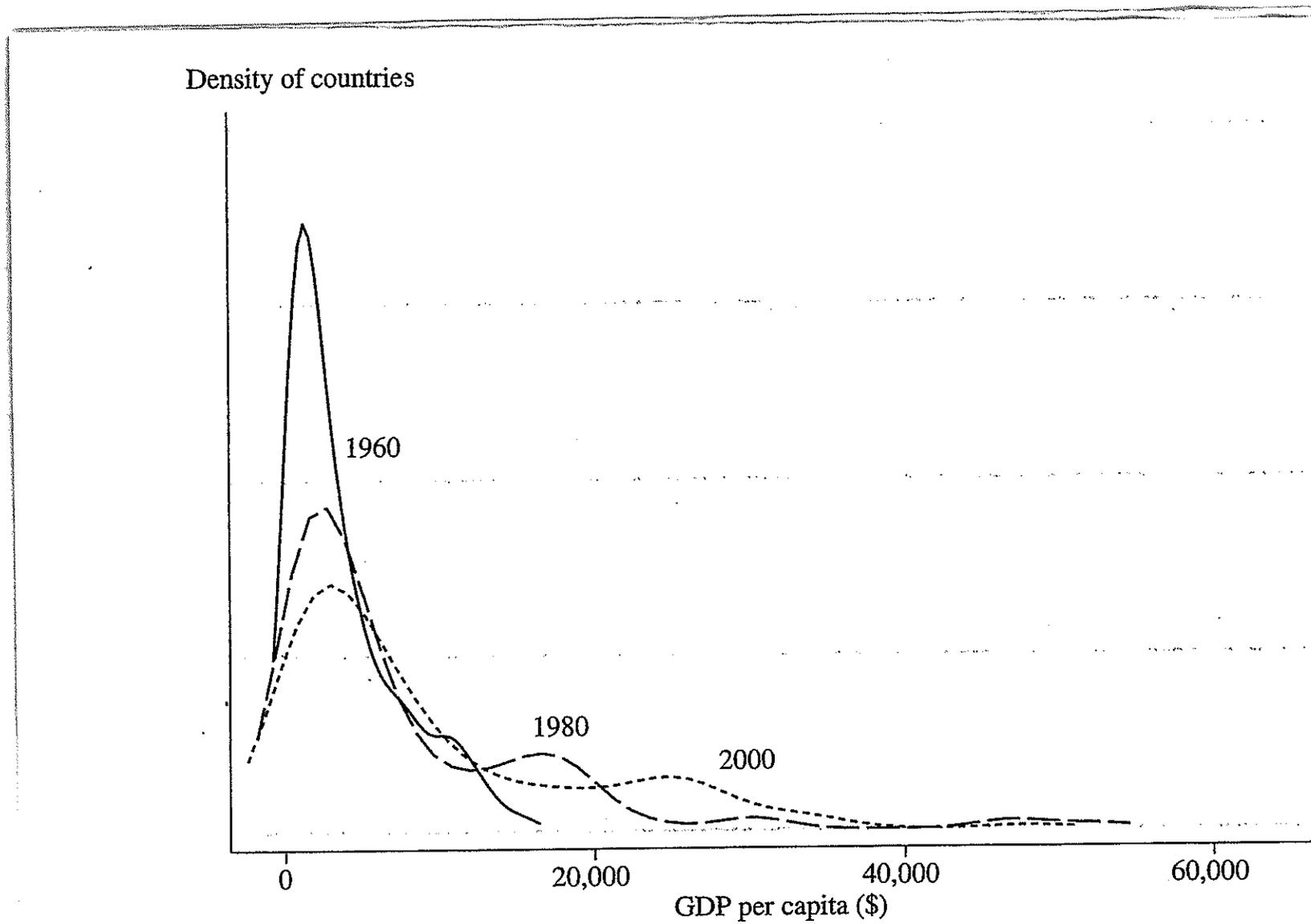 2. There is evidences of conditional convergence.

FIGURE 1.1 Estimates of the distribution of countries according to PPP-adjusted GDP per capita in 1960, 1980, and 2000.
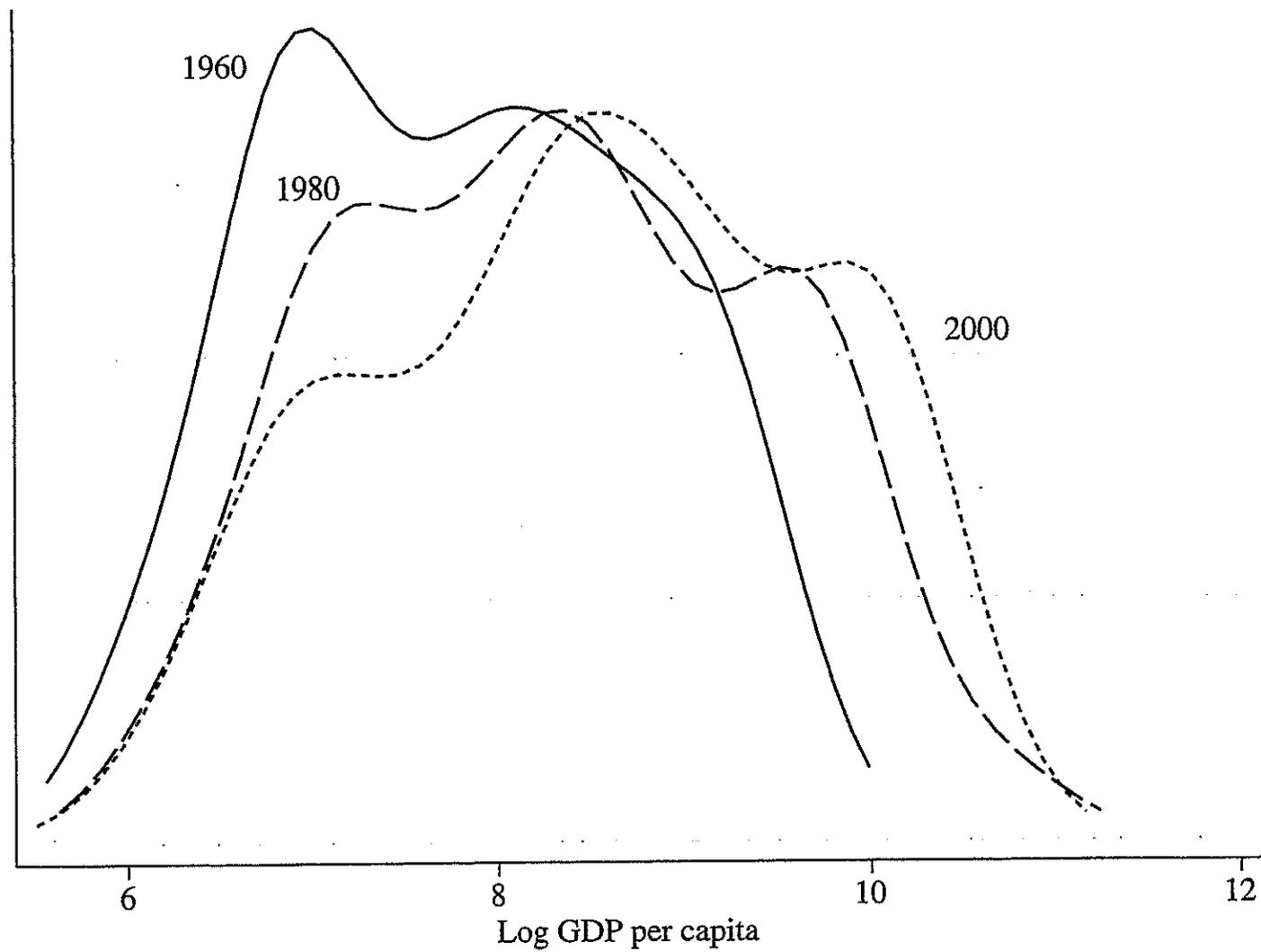
Density of countries

FIGURE 1.2 Estimates of the distribution of countries according to log GDP per capita (PPP adjusted) in 1960, 1980, and 2000.

Density of countries (weighted by population)

FIGURE 1.3 Estimates of the population-weighted distribution of countries according to log GDP per capita (PPP adjusted) in 1960, 1980, and 2000.
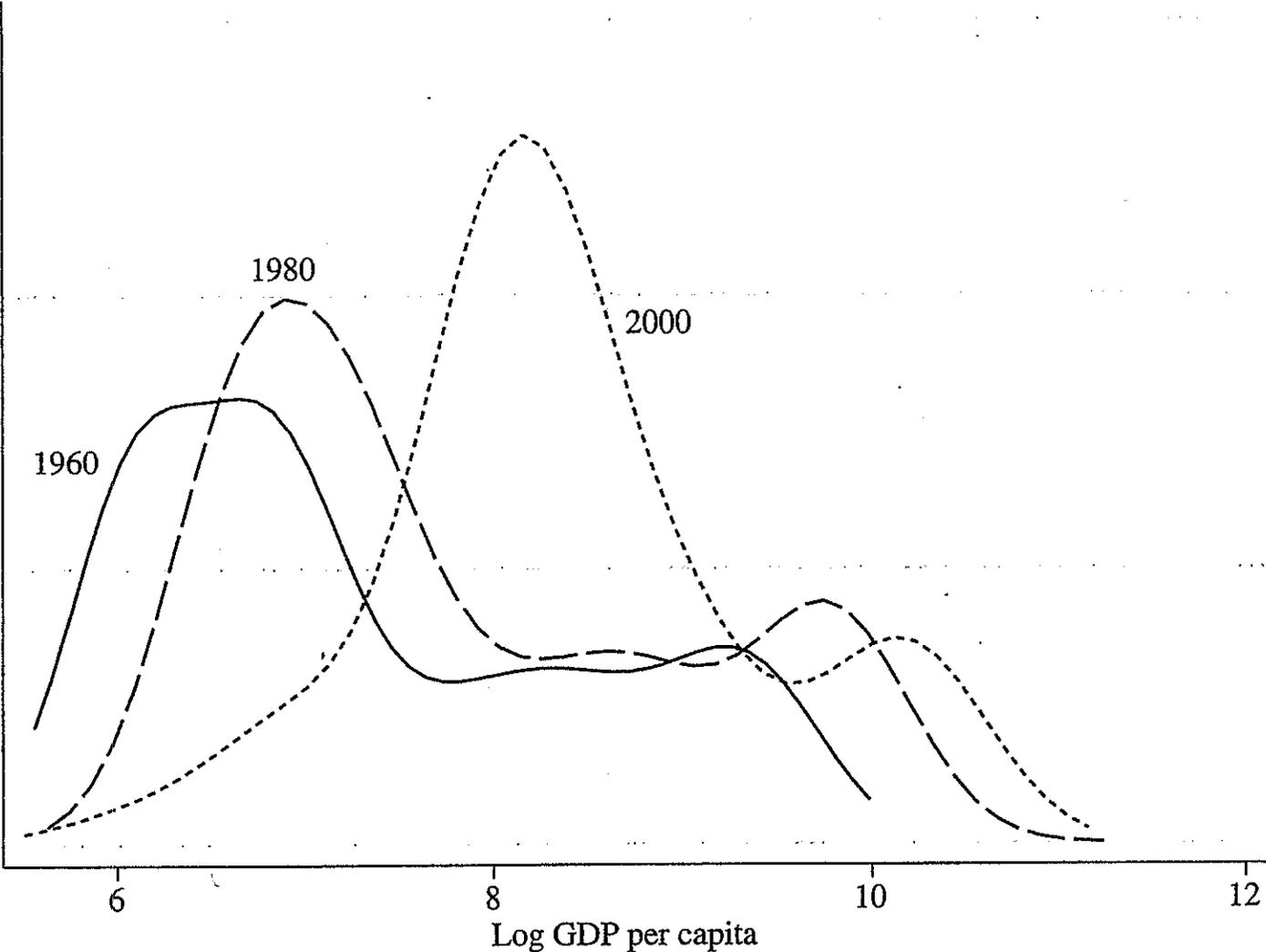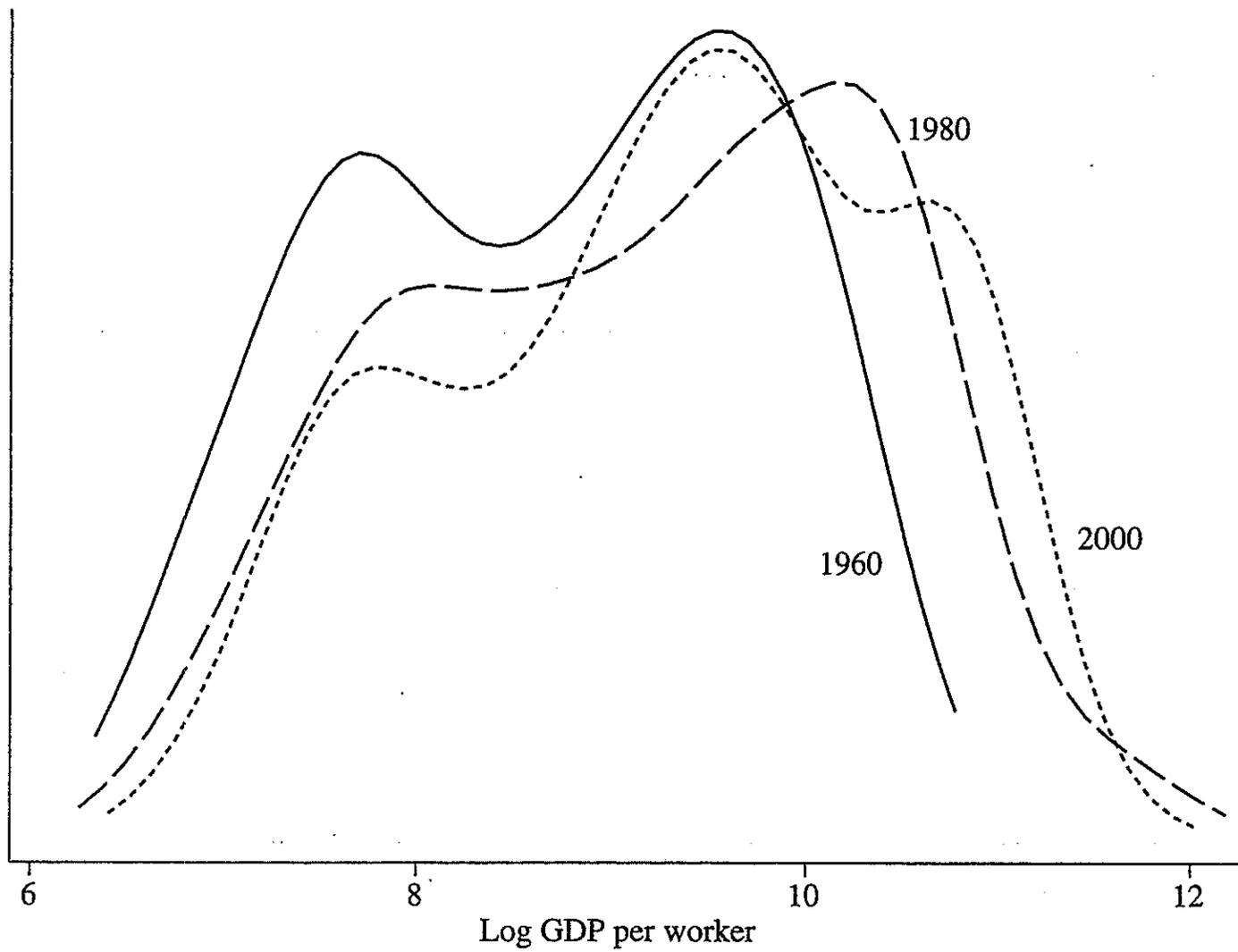
FIGURE 1.4 Estimates of the distribution of countries according to log GDP per worker (PPP adjusted) in 1960, 1980, and 2000.

Log GDP per worker relative to the United States, 2000
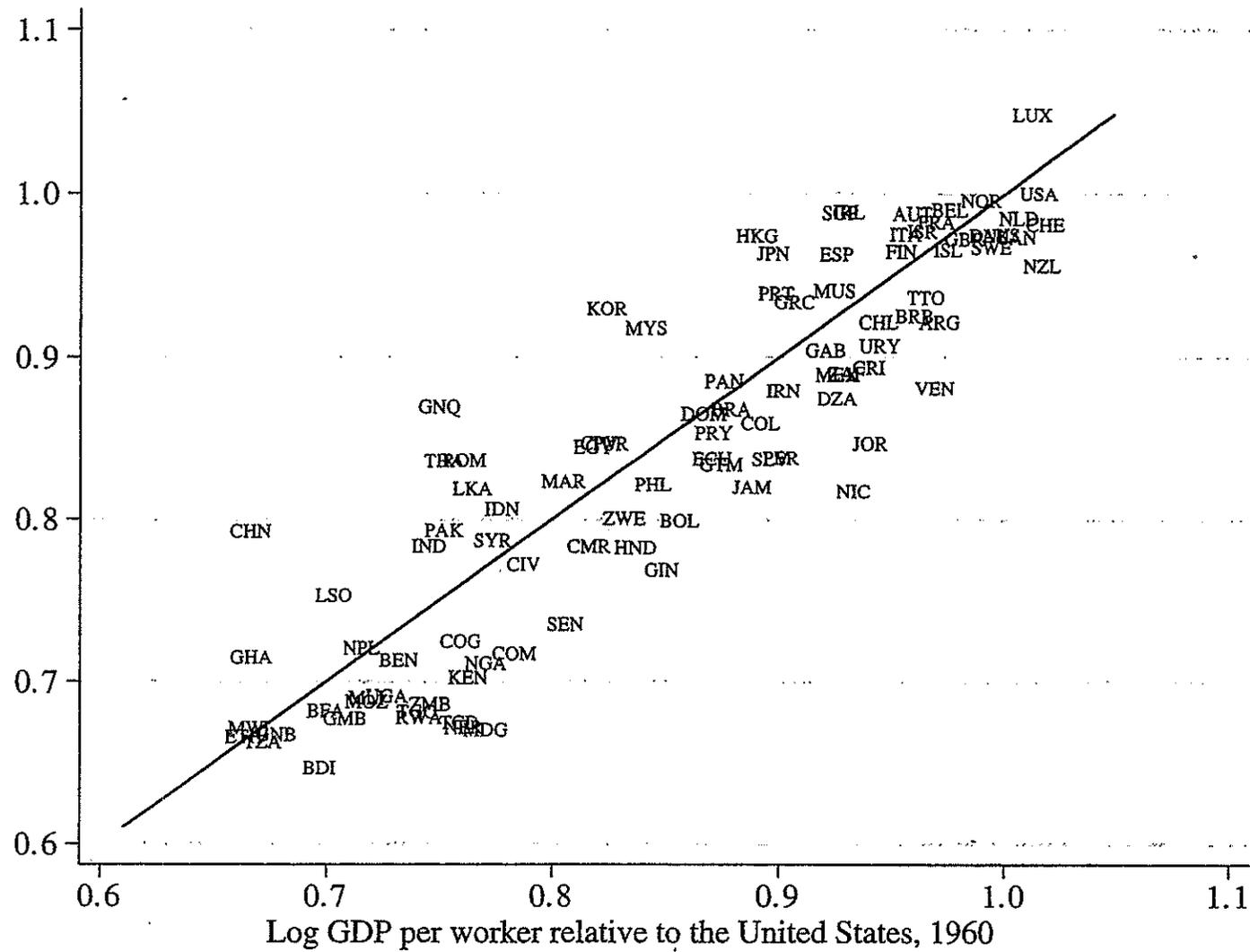
FIGURE 1.9 Log GDP per worker in 2000 versus log GDP per worker in 1960, together with the 45° line.

Log GDP per capita
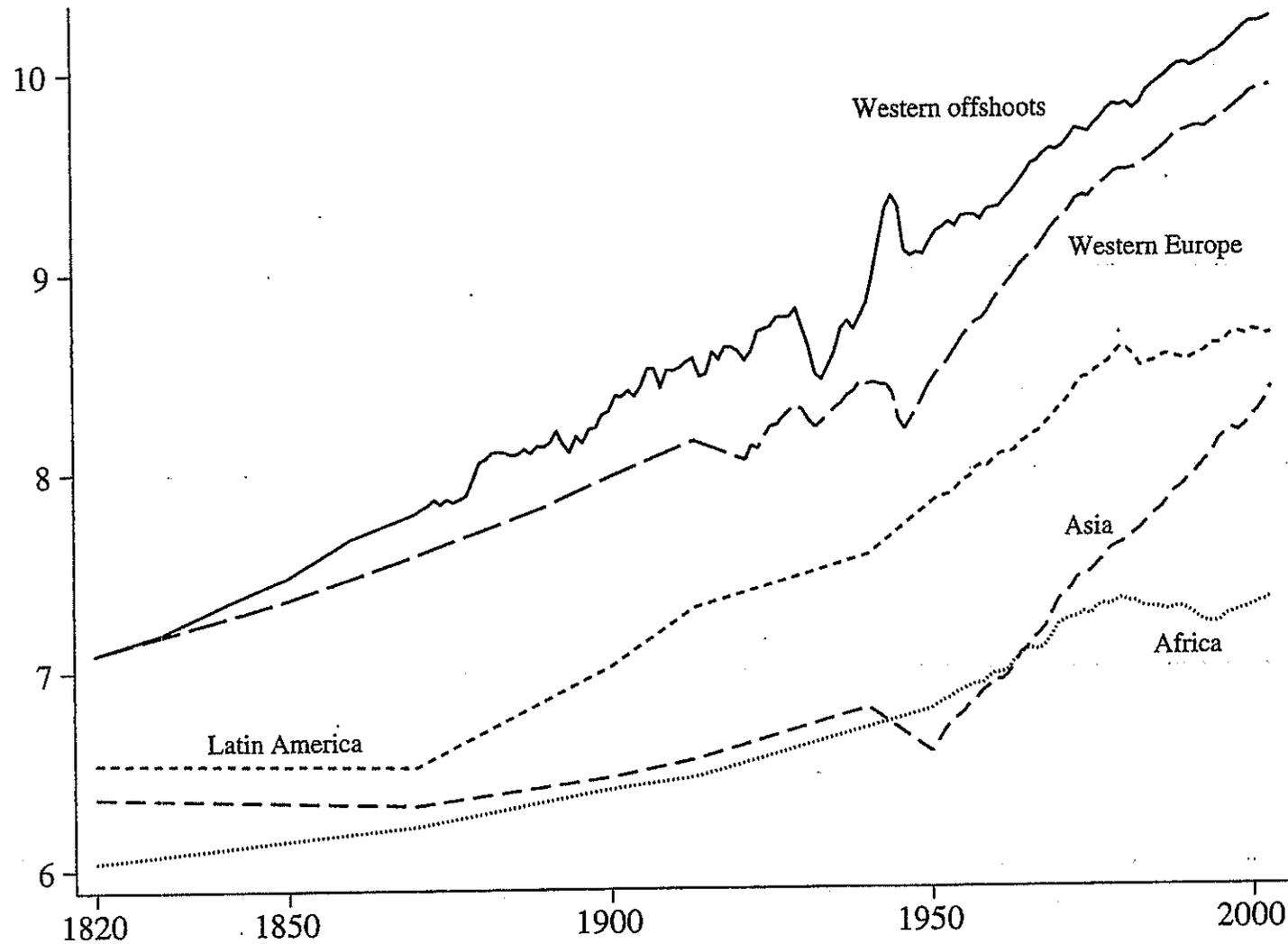
FIGURE 1.10 The evolution of average GDP per capita in Western offshoots, Western Europe, Latin America, Asia, and Africa, 1820–2000.

Log GDP per capita
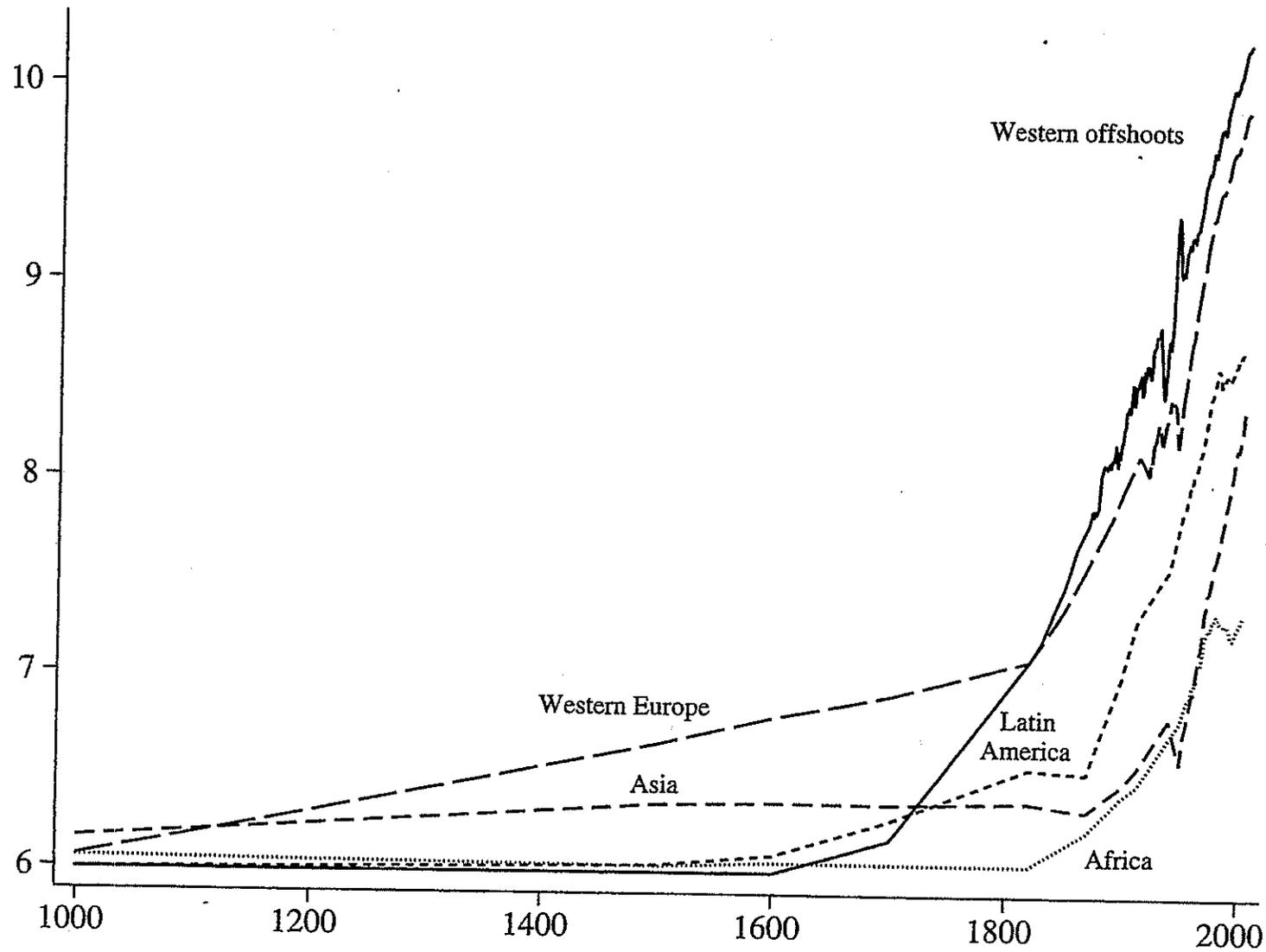
FIGURE 1.11   The evolution of average GDP per capita in Western offshoots, Western Europe, Latin America, Asia, and Africa, 1000–2000.

Average growth rate of GDP, 1960–2000
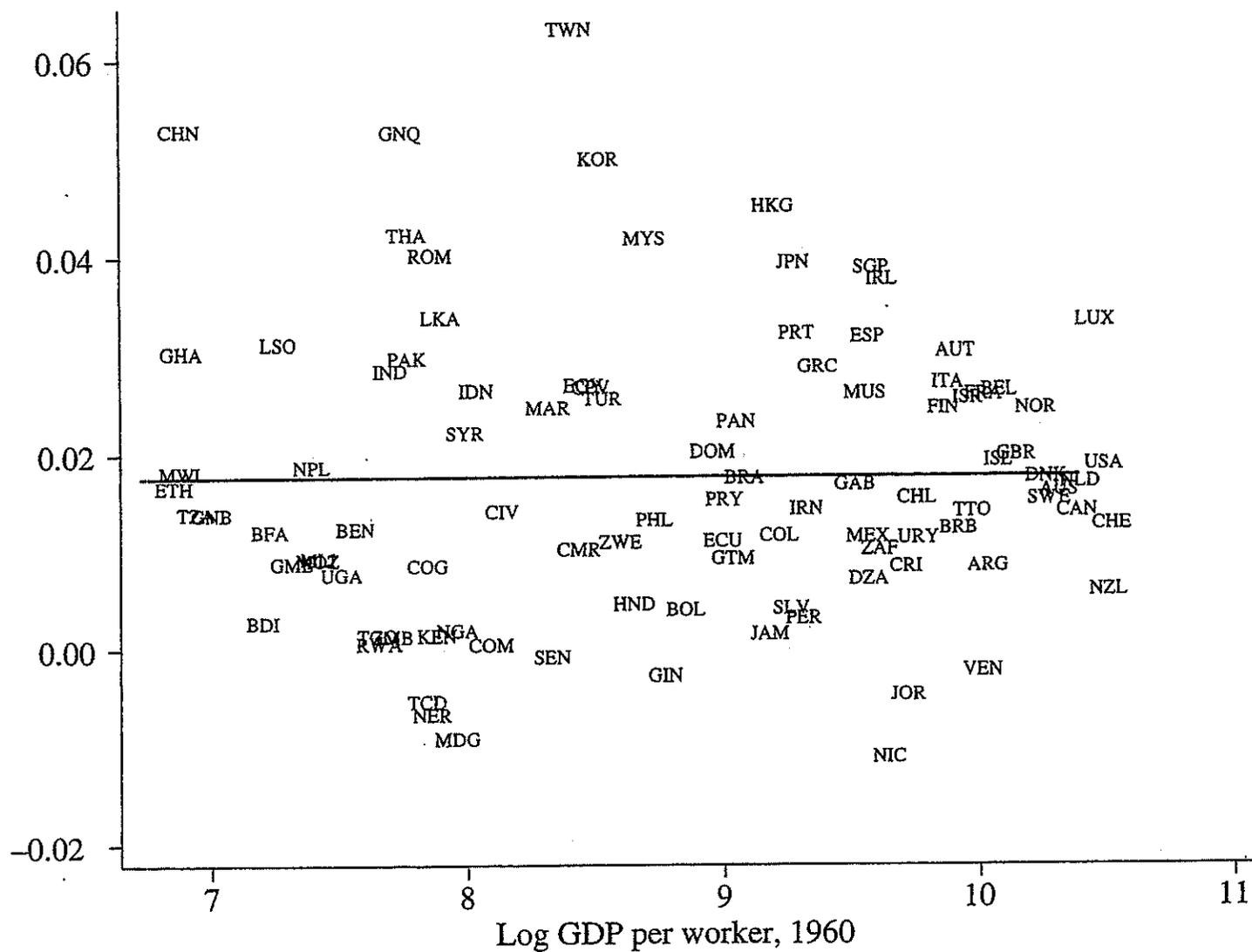
FIGURE 1.13 Annual growth rate of GDP per worker between 1960 and 2000 versus log GDP per worker in 1960 for the entire world.
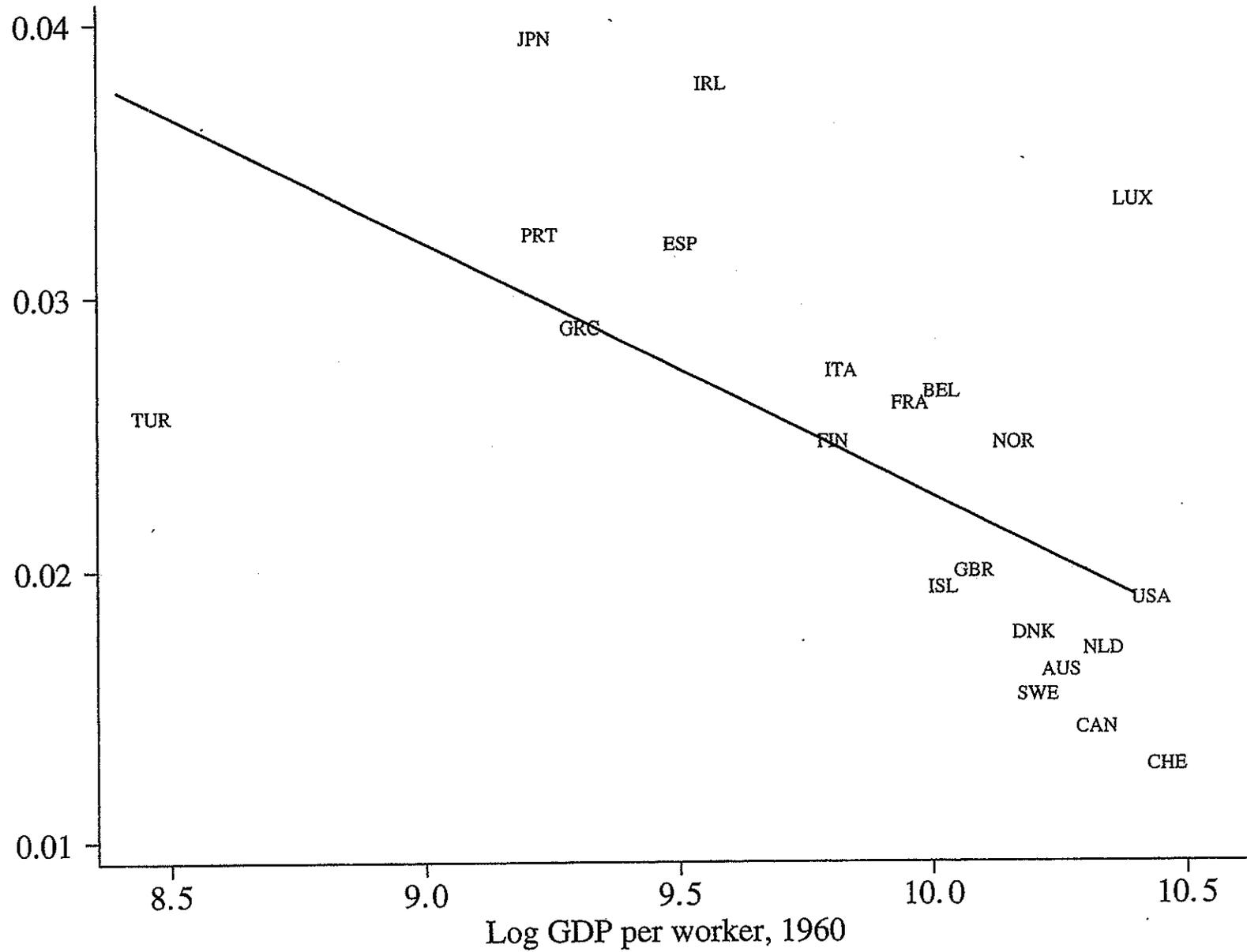
FIGURE 1.14 Annual growth rate of GDP per worker between 1960 and 2000 versus log GDP per worker in 1960 for core OECD countries.

Can the Solow model explain large income differences?

- Let me first examine whether the Solow model explains the first stylized fact: large income differences.

- Let me start with simple exercises.
    1. Suppose that T is the same across countries.
    2. Ask whether differences in capital stock per capita alone can explain differences in income per capita.

Can the Solow model explain large income differences?

- *Calibration Exercises 1 (Lucas (1990))*:

$$y_e = (k_e)^\alpha \Rightarrow k_e = (y_e)^{\frac{1}{\alpha}}$$

Choose two arbitrary countries, say US and India.

$$\frac{k_{us}\frac{1}{T}}{k_{India}\frac{1}{T}} = \frac{\left(y_{us}\frac{1}{T}\right)^{\frac{1}{\alpha}}}{\left(y_{india}\frac{1}{T}\right)^{\frac{1}{\alpha}}} \Rightarrow \frac{k_{us}}{k_{India}} = \left[\frac{y_{us}}{y_{india}}\right]^{\frac{1}{\alpha}}$$

According to Penn World Table 6.3, $\frac{y_{us}}{y_{india}} \approx 10$ in year 2007 and $\alpha = \frac{1}{3}$,

$$\frac{k_{us}}{k_{India}} = 10^3 = 1000$$

In order to explain large income differences, required differences in capital are too large.

Why are Required Capital Differences so Large?
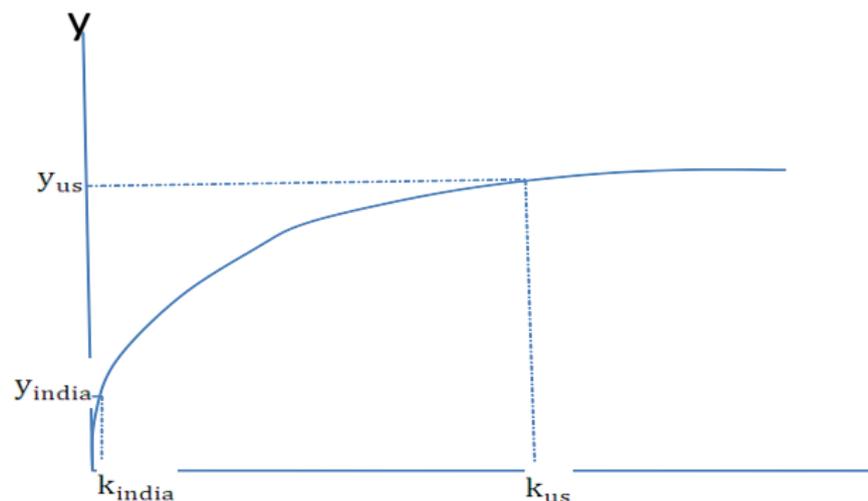
Can the Solow model explain large income differences?

- *Calibration Exercises 2 (Lucas (1990)):*

$$MPK = f'\left(k_e\right) = \alpha k_e^{a-1} = \alpha\left(y_e\right)^{\frac{\alpha-1}{\alpha}}$$

Choose again US and India

$$\frac{MPK_{us}}{MPK_{India}} = \frac{\alpha\left(y_{us}\frac{1}{T}\right)^{\frac{\alpha-1}{\alpha}}}{\alpha\left(y_{India}\frac{1}{T}\right)^{\frac{\alpha-1}{\alpha}}} = \left[\frac{y_{us}}{y_{india}}\right]^{\frac{\alpha-1}{\alpha}}$$

Since $y_{us}/y_{India} \approx 10$ and $\alpha = \frac{1}{3}$,

$$\frac{MPK_{us}}{MPK_{India}} = [10]^{\frac{\frac{1}{3}-1}{\frac{1}{3}}} = [10]^{\frac{-\frac{2}{3}}{\frac{1}{3}}} = [10]^{-2} = \frac{1}{100}$$

Attributing difference in output to difference in capital implies a huge variation in the rate of return on capital.

Can the Solow model explain large income differences?

Why are Required MPK Differences are so Large?
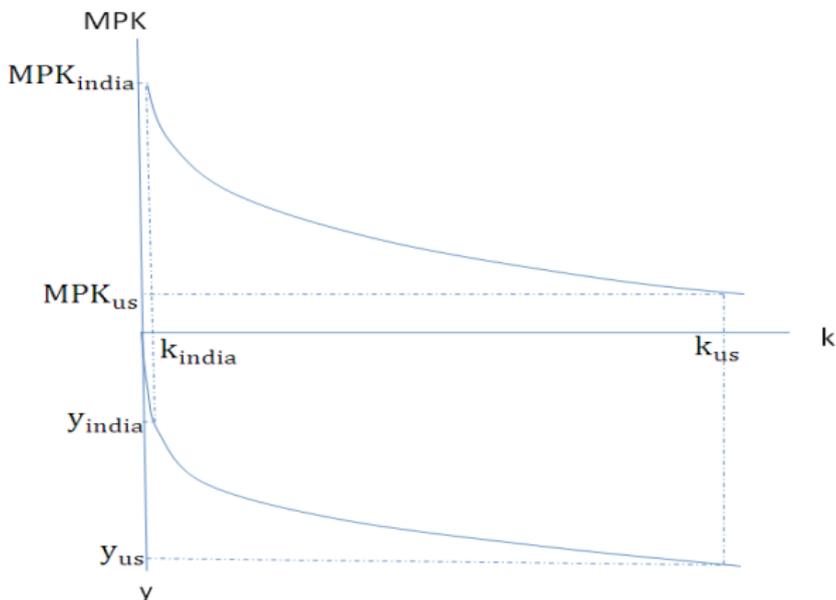
- Both examples indicate that
  - When $\alpha = \frac{1}{3}$, without productivity differences, the theory cannot explain large income differences.
  - If $\alpha \approx 1$, it may be possible to explain data. This may indicate the existence of unmeasured capital stock.

Can the Solow model explain large income differences?

- *Cross Country Regression (Mankiw, Romer and Weil (1992)):* Assume that every country has the same production function, $f(k_e) = k_e^\alpha$ and that all countries are on its steady state, then it is shown before that

$$
\begin{aligned}
k_e^* &= \left[\frac{s}{g + n + \delta + gn} \right]^{\frac{1}{1-\alpha}} \\
&\approx \left[\frac{s}{g + n + \delta} \right]^{\frac{1}{1-\alpha}}
\end{aligned}
$$

Hence

$$
\begin{aligned}
y_e &= (k_e^*)^\alpha \Rightarrow \frac{y_t}{T_t} = (k_e^*)^\alpha \Rightarrow y_t = T_t (k_e^*)^\alpha \\
y_t &= T_t \left[\frac{s}{g + n + \delta} \right]^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

Can the Solow model explain large income differences?

- Empirically Testable Equation

$$\ln y_t = \ln T_t + \frac{\alpha}{1-\alpha} \ln(s) - \frac{\alpha}{1-\alpha} \ln(g + n + \delta)$$

Suppose that $t = 0$ and

$$\ln T_0 = a + \varepsilon_i$$

where a is constant and ε_i is country specific shock.

$$\ln y_0 = a + \frac{\alpha}{1-\alpha} \ln(s_i) - \frac{\alpha}{1-\alpha} \ln(n_i + g + \delta) + \varepsilon_i$$

Assume that g and δ is constant, 0.05, across countries and s and n are independent of ε_i. This assumption implies that there are no productivity differences other than luck.

- *Data:* Penn World Table: Non-Oil countries, Non-Oil countries except for grade D countries and small population countries, OECD countries.
    1. *n*.. the average growth of working age population over 1960-1985, where working age is defined as 15 to 64.
    2. *s*...the average share of real investment in real GDP over 1960-1985.
    3. *y*...real GDP in 1985 divided by the working age population in that period.

TABLE I
Estimation of the Textbook Solow Model

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
$\ln(I/GDP)$	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\bar{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\bar{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. ($g + \delta$) is assumed to be 0.05.

Can the Solow model explain large income differences?

- The following 4 results are obtained by them.

  1. The coefficients on saving and population growth have predicted signs and 2 of 3 are significant.
  2. The restriction that the absolute values of the coefficients of $\ln(s)$ and $\ln(g + n + \delta)$ are the same cannot be rejected.
  3. High R^2.
  4. The estimated α is much higher than $1/3$.

- **Implication from results:** Although the Solow model has qualitatively correct predictions, but quantitatively, α is too small to explain huge income differences. This indicates the existence of unmeasured capital.

- **The problem of the estimation:** If productivity differences are not random, ε_i is correlated with s and n. The estimated parameters are biased upward. It indicates that the true α may be lower than the estimated α.

- **Conclusion:** Including the unmeasured capital as a part of T, evidence suggests that T must differ across countries in order to explain the large income differences.

- Students must hand assignment 3 in at the following lecture.

- In order to explain income differences across countries, we need to assume that T differs across countries.
- If the knowledge accumulation is the source of increase in T, why the poor countries do not imitate the knowledge in the developed countries?
- One possible explanation is that the use of knowledge demands human capital. Let me investigate this possibility.

Can Human Capital Explain Income Differences?

- *Hall and Jones (1999):* Assume that a country i has the production function:

$$Y_i = K_i^\alpha \left(T_i N_i \right)^{(1-\alpha)}$$

where $T_i = A_i h_i$. The variable A_i is the unobserved productivity and h_i is the level of human capital. Then

$$1 = \left(\frac{K_i}{Y_i} \right)^\alpha \left(\frac{T_i N_i}{Y_i} \right)^{(1-\alpha)}$$

$$y_i^{(1-\alpha)} = \left(\frac{K_i}{Y_i} \right)^\alpha \left(T_i \right)^{(1-\alpha)}$$

$$y_i = \left(\frac{K_i}{Y_i} \right)^{\frac{\alpha}{1-\alpha}} T_i$$

Hence

$$y_i = \left(\frac{K_i}{Y_i} \right)^{\frac{\alpha}{1-\alpha}} A_i h_i$$

Can Human Capital Explain Income Differences?

- y_i...National income and labor force data are taken from Summers and Heston (1991).
- They assume that

$$
\begin{aligned}
h_i &= \exp\left(0.134 \times E\right), \ \textit{if } E \leq 4, \\
&= \exp\left(0.134 \times 4 + 0.101 \times (E - 4)\right), \ \textit{if } 4 < E \leq 8, \\
&= \exp\left(0.134 \times 4 + 0.101 \times 4 + 0.068 \times (E - 8)\right), \ \textit{if } E > 8,
\end{aligned}
$$

where E is average educational attainment. The coefficients, 13.4, 10.1 and 6.8, are taken from previous research. Average educational attainment is measured in 1985 for the population aged 25 and over, as reported in Barro and Lee (1993).

Can Human Capital Explain Income Differences?

- Capital stock is estimated by the perpetual inventory method:

$$K_{t+1} = I_t + (1-\delta)\, K_t$$

where δ is assumed to be 0.06. We can recursively estimate capital stock when we know the initial value. Note that

$$
\begin{aligned}
K_{t+1} - K_t + \delta K_t &= I_t \\
g_K + \delta &= \frac{I_t}{K_t} \\
K_t &= \frac{I_t}{g_K + \delta}
\end{aligned}
$$

Using this relationship, the initial value at 1960 is estimated by

$$K_{1960} = \frac{I_{1960}}{g + \delta}$$

where g is the average geometric growth rate from 1960 to 1970.

- The parameter, α, is assumed to be $\frac{1}{3}$ and the variable, A_i, is estimated by the residual.

TABLE I
PRODUCTIVITY CALCULATIONS: RATIOS TO U. S. VALUES

Country	Y/L	Contribution from		
		$(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

The elements of this table are the empirical counterparts to the components of equation (3), all measured as ratios to the U. S. values. That is, the first column of data is the product of the other three columns.
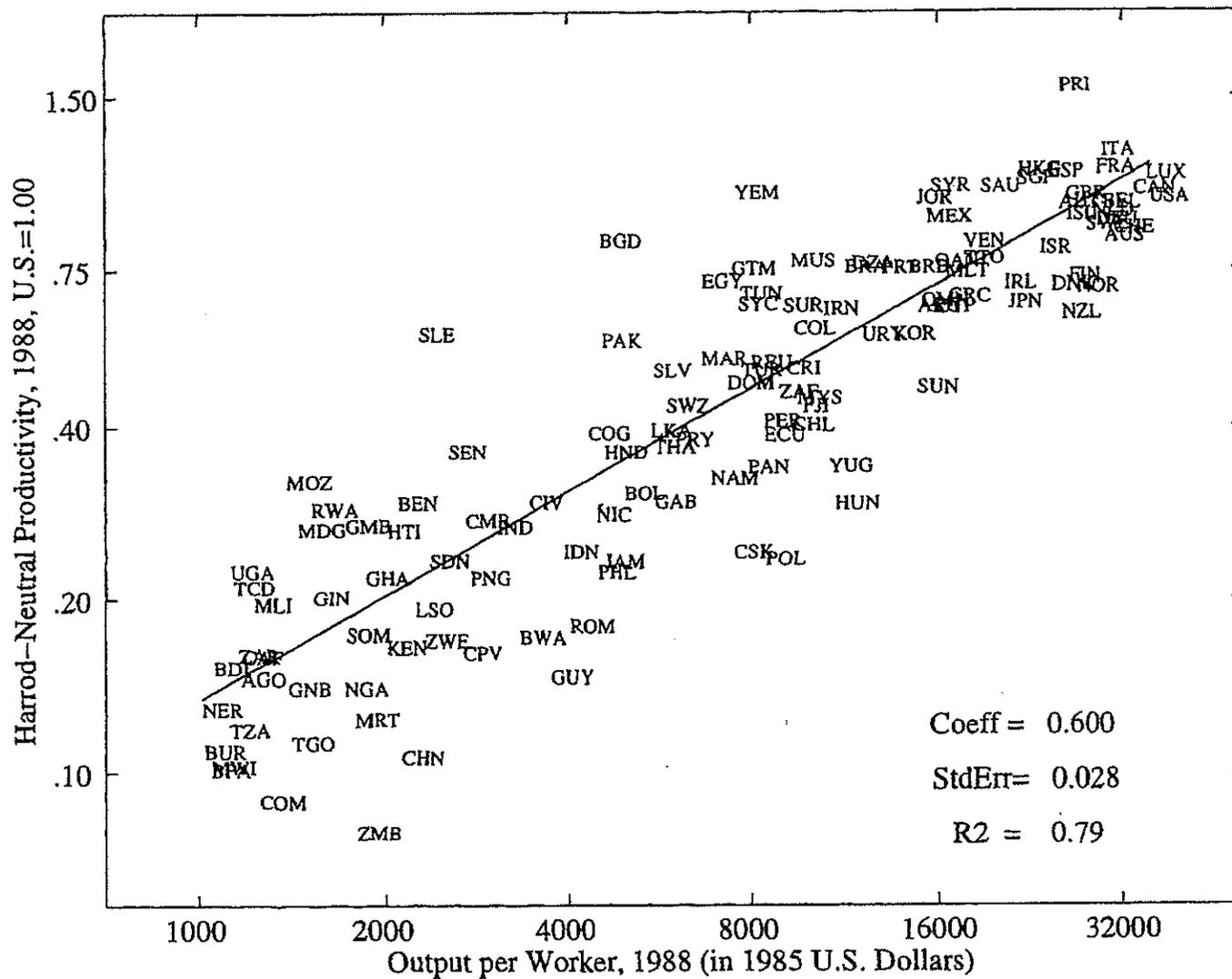
FIGURE I

Productivity and Output per Worker

Can Human Capital Explain Income Differences?

- Results
    1. Output per worker in the five richest countries is 31.7 times higher than output per worker in the five lowest countries.
    2. Capital intensity, human capital, productivity in the five richest countries are 1.8, 2.2 and 8.3 times larger than those in the five lowest countries, respectively.
    3. It shows that including human capital does not help much explaining a huge income differences.

- Questions on *Hall and Jones (1999)*:Clearly, school quality, on the job training, child-rearing, and prenatal care vary across countries. How can we estimate these effects?

Can Human Capital Explain Income Difference?

- *Hendricks (2002):* Hendricks use the wage data of immigrants in the US to estimate relative human capital. Assume that production function is

$$y_e = (k_e)^\alpha \Rightarrow \frac{Y}{AhN} = \left(\frac{K}{AhN}\right)^\alpha \Rightarrow Y = (K)^\alpha (Ah)^{1-\alpha} N^{1-\alpha}$$

Then

$$w = MPL = (1-\alpha)(K)^\alpha (Ah)^{1-\alpha} N^{-\alpha}$$

where $k_e = \frac{K_{us}}{A_{us} h_i N_{us}}$. Let me choose two countries, US and India.

$$
\begin{aligned}
\frac{w_{us}}{w_{india}} &= \frac{(1-\alpha)(K_{us})^\alpha (A_{us} h_{us})^{1-\alpha} N_{us}^{-\alpha}}{(1-\alpha)(K_{us})^\alpha (A_{us} h_{india})^{1-\alpha} N_{us}^{-\alpha}} = \left(\frac{h_{us}}{h_{india}}\right)^{(1-\alpha)} \\
\frac{h_{us}}{h_{india}} &= \left[\frac{w_{us}}{w_{india}}\right]^{\frac{1}{1-\alpha}}
\end{aligned}
$$

Can Human Capital Explain Income Difference?

- Using this estimate, Hendricks *(2002)* conduct the same calibration exercises as *Hall and Jones (1999) do.*
- He concludes that for countries below 40 percent of U.S. output per worker, less than half of the output gap relative to the United States is attributed to human and physical capital.
  - **Remark:** Discrimination and a country specific skill such as language strengthen his result because it means that the immigrants may be more productive in their home countries.
- **Issue:** the selection bias. If the average ability of immigrants are higher than the average ability of workers in their home countries, then the wage difference is lower than the average human capital difference between the US and their home country.
  - After considering the selection bias, he concluded that the selection bias cannot change his result.

Can Human Capital Explain Income Difference?

- *Can we dismiss human capital?* There are several reasons that human capital may be still important.
  - Externality: The exercises assume that there is no externality. But externality is important, the impact of human capital is larger.
  - Technology Adoption: Education may be important not only for production but also for stimulating the adoption of technology. In this case, education may have a growth effect.
  - The Influence on Population: Even in the high productive society, if population grows faster than productivity does, per capita income cannot increase. As investment in human capital increases the cost of raising children, it reduces the number of children and reduces population growth.

- Nonetheless, it is less likely that the lacks of physical and human capital provide a whole story for large income differences. Note that the lacks of physical capital and human capital cannot explain development miracles and disasters. We need to investigate a potential source of A.

Resource Allocation and Aggregate Productivity

- Many macroeconomists currently examine resource allocation as the potential source of A. If we think that the aggregate production function must be the result of several micro activities such as

$$F(K, AhL) = \sum_i^n F_i(K_i, A_i h_i L_i)$$

where i implies ith plant, the allocation of resources across plants can influence the aggregate productivity.

- Hsieh and Klenow (2009) investigate how much the misallocation of resources across plants influence the aggregate TFP by using microdata on manufacturing establishments in China, India and the United States. (US: 1977, 1982, 1087, 1992 and 1997, China: 1998-2005, India: 1987-1995.)
  - **Results:** When capital and labor are hypothetically reallocated to equate marginal products to the extent observed in the United States, they calculate manufacturing TFP gains of 30 %~50% in China and 40%~60% in India.

Resource Allocation and Productivity

- In reality, misallocation would be everywhere. But, it is difficult for a government to dictate better allocation. If it could, many communist countries could have survived. That is, it is impossible to point out every inefficiency in a world. For a policy purpose, we need to focus a particular misallocation that would influence productivity very much.

- **What kinds of misallocation are important?** As we discussed before, if we believe that the transfer of knowledge is the main source of productivity differences, the factors that influence technology adoption are likely to be important. We can provide two possible factors which can prevent the reallocation of resources to the technology diffusion.

Misallocation of Resources and Productivity Differences

- *Resistance to Technology Adoption:* Parente and Prescott (1994) show that the barrier to the adoption of new technology have substantial impacts on income differences. Parente and Prescott (1999) argue that monopoly right can be one such barrier. When government protects a particular company or industry, new comers cannot enter the market with new technologies. It hampers the adoption of new technology.

- Why do the resistance to technology adoption prevail?
  - Innovation and technology adoption are accompanied with creative destruction.
  - Creative destruction demands the replacement of resources.
    - In particular, If the incumbent has a large specific skill for old technology, the adoption of new technology makes the skill obsolete.
  - The reallocation of resources brings the conflicts of interests.
    - If preventing new entrance is cheaper than adopting new technology, they may resist new technology.

Misallocation of Resources and Productivity Differences

- *Misallocation of talent:* Baumol (1990) investigated historical evidence and provided the following three hypotheses:

  1. The social system, which determines the relative payoffs to different entrepreneurial activities, changes over time and across regions.
  2. Entrepreneurial behavior changes according to variations in the social system.
  3. The allocation of entrepreneurship between productive and unproductive activities has a large effect on the innovation of technology and dissemination of technological discoveries.

     - Productive activity...Activity to create new value.
     - Unproductive activity...Activity to transfer income from somebody to others.

- Murphy, Shleifer and Vishny (1991) provided a formal model that clarifies Baumol's hypotheses. It shows that 1) if talented people are misallocated to a declining industry or the sector that specializing transferring income, they may reduce the growth rate.

Fundamental Determinants of Differences in Economic Performance

- Both arguments suggest that in order for the misallocation to prevail, there must have an institutional arrangement to misallocate resources.
- This point is similar to the arguments by Acemoglu (2009). He said that investment in physical and human capital, and the reallocation of resources are proximate causes of income differences. But it does not answer to the deeper questions? Why do some countries invest more than others? Why do some countries attain better allocation.
- Acemoglu (2009) listed four candidates of fundamental causes. 1) Luck, 2) Geography, 3) Culture and 4) Institution.

Fundamental Determinants of Differences in Economic Performance

- Acemoglu, Johnson and Robinson (2001) provide evidence that
    1. More than 75 % of the income gap between relatively rich and relatively poor countries are explained by differences in economic institution (as proxied by the security of property rights.)
    2. Once the effect of institutions is controlled, there appears to be no effect of geographic variables.
    3. Once economic institution is taken into account, cultural variables do not appear to have a direct effect on economic growth and income per capita.
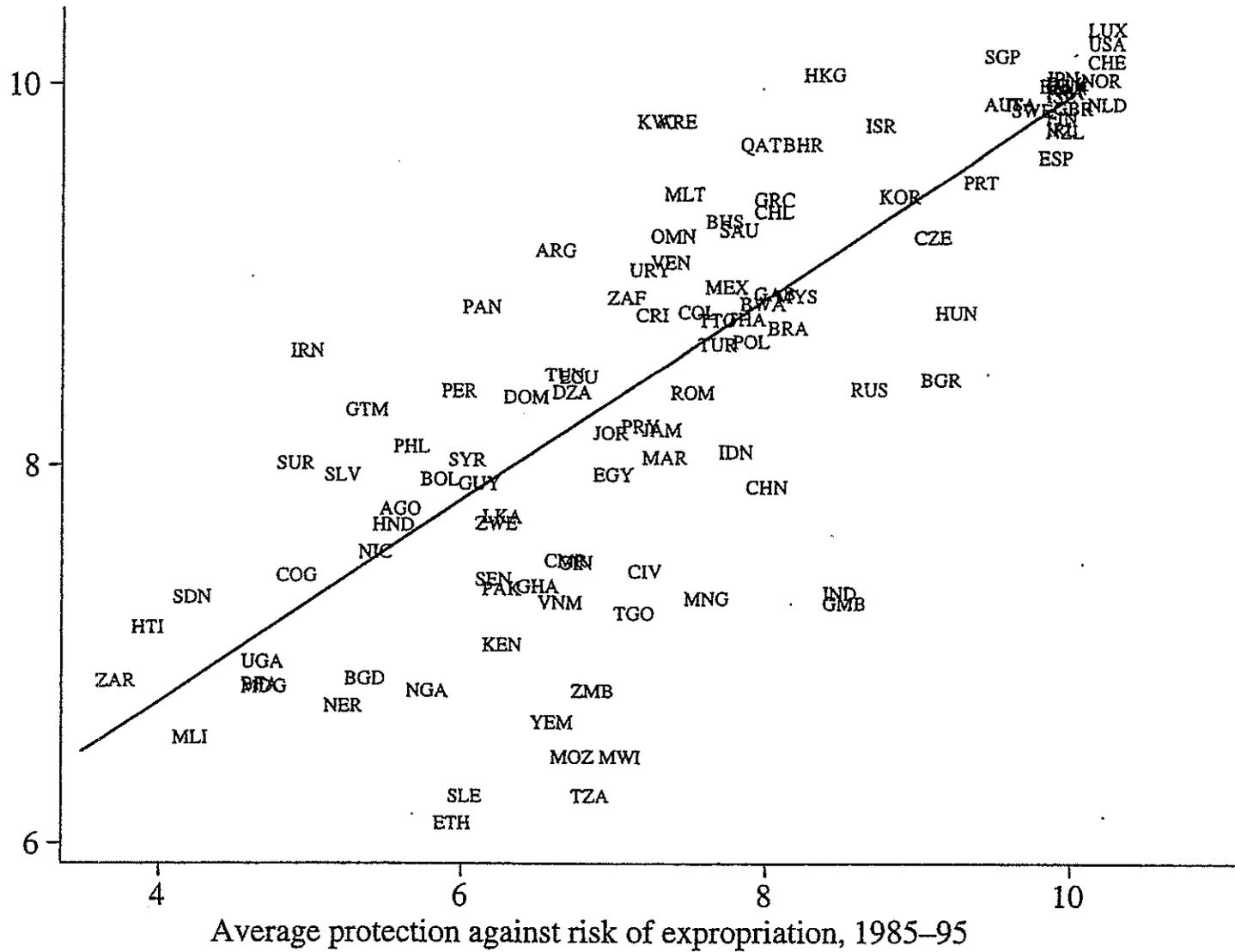
Log GDP per capita, 1995

LUX
USA
CHE
SGP
HKG
IRL NOR
AUT SWE GBR NLD
KWT RE ISR BEL DNK ISL FIN
QAT BHR NZL
ESP
MLT GRC KOR PRT
BHS CHL
OMN SAU CZE
ARG VEN
UR Y
ZAF MEX GAB SYS HUN
CRI COL BWA
PAN IT GHA BRA
TUR POL
IRN
TUN SU
PER DOM DZA ROM RUS BGR
GTM
JOR PRY AM
PHL IDN
SUR SLV SYR MAR
BOL GUY EGY CHN
AGO LKA
HND ZWE
NIC
COG CMR N CIV
SDN SEN GHA MNG IND
PAK VNM GMB
HTI NER TGO
UGA KEN
ZAR MDG BGD NGA ZMB
MLI YEM
MOZ MWI
SLE TZA
ETH

10

8

6

4   6   8   10

Average protection against risk of expropriation, 1985–95

FIGURE 4.1   Relationship between economic institutions, as measured by average expropriation risk between 1985 and 1995, and GDP per capita.

Fundamental Determinants of Differences in Economic Performance

- If better institution is important, why do not they choose better institution?
- Coase Theorem: If utility is transferable, and there is no liquidity constraint and no transaction costs, negotiation reaches a Pareto optimal allocation among parties who join the negotiation.
  - Example: A company introduces IT system to enhance productivity of the firm. It may substitute many clerks, and reduce labor costs. To do so, managers must fire workers. It is costly for workers. But managers can negotiate a retirement allowance with workers.
    1. Because utility is transferable, workers can agree if enough payment is offered.
    2. Because the introduction of IT system is assumed to be productive, a firm can offer enough payments to convince workers.
    3. Because there is no liquidity constraint, even though income is available in future, the firm can finance the payments.
    4. No transaction cost means that both parties cannot tell a lie, negotiation is costless and the contract is perfectly enforced.

Fundamental Determinants of Differences in Economic Performance

- The previous question can be rewritten as follows: Why cannot Coase Theorem be applied in a political world?

  1. Utility may not be transferable. Maybe, but in this case, enhancing economic growth is not a Pareto optimal policy. We need an additional value to justify economic growth.
  2. There might be a liquidity constraint. Possibly, but there is a developed international financial system. So they can relatively easily borrow money if the return is huge.
  3. The existence of transaction cost. This is possible. But what kinds of transaction costs are important?
  4. Coase theorem guarantees Pareto optimal allocation only among parties who join the negotiation. There might have an external effect of the negotiation.

- We discuss the points 3 and 4.

Fundamental Determinants of Differences in Economic Performance

- The external effect of negotiation: If the results of negotiation influence the third party, it is possible that the parties who can join the negotiation may steal some income from the third party.
  - For example, if all policies are determined by the negotiation among political elite from landlord, they would prevent enhancing manufacturing sectors, because their peasants or slave may move to manufacturing sectors. Enhancing manufacturing sectors can be Pareto improving, but landlords have no incentive to do so because they can keep their rent at the expense of peasants.

- But why does not the third party make an offer to negotiate with them?
  - There is a cost of organizing an interest group to make a reasonable negotiation with them. One of difficulty in organizing an interest group is free riders in the group: if some body incurs the cost of organizing group, others do not need to pay.

Fundamental Determinants of Differences in Economic Performance

- What kinds of transaction cost are important?
  - **Hold Up Problem:** There is no way for politicians to commit their statements. Without commitment, Coase theorem does not work.
    1. For example, if North Korea introduces the property rights and commit that they maintain investment friendly environment, many firms may invest in North Korea and it can potentially grow. However, after many firms making investment, they have always an incentive to steal the return from investment by raising tax or so. Moreover, because Kim Jong-un and other politicians are making a law, they can easily change a law. They don't have any method to commit their statements. Because no firms can trust politicians, they don't invest.
    2. Similarly, suppose that if some politicians in North Korea can peacefully replace Kim Jong-un from North Korea, they can potentially restructure the economy. For this purpose, they must commit that they provide his families enough income to guarantee their wonderful life. But, new politicians are in power, they may not keep their promise. As new politicians don't have any commitment device, Kim Jong-il cannot trust them and they never step down.

- Students must hand assignment 4 in at the next lecture.