Stabilization Policy

- Every country experiences boom and recession.
- The main question is whether government should or can actively stabilize aggregate demand for economy.
- I would like to investigate how demand stabilization policy affects real economy.
- This was a central issue of policy discussions. A school of economists advocates that a government should actively stabilize demand for economy; others disagree with this opinion. I would like to provide the framework to understand the reasons behind these policy discussions.

Stabilization Policy

- My lecture in this section is organized as follows.
  1. I introduce money and government to the basic model. For this purpose, I first discuss a money market.
  2. Incorporating money and government in the basic model, we discuss how a monetary policy and a fiscal policy influence GDP in the long run.
  3. I introduce a short run deviation of GDP, which creates the possibility that demand stabilization policy has a real impact.
  4. I discuss several explanations that enforce the economy to deviate from the long run equilibrium.
  5. I discuss stabilization policy and unemployment. We emphasize the importance of distinguishing the source of unemployment in the short run and long run.
  6. Finally, I discuss a difficulty of implementing stabilization policy. I show that how lack of commitment causes undesirable outcome.

What is Money?

- **Definition** Money is the stock of assets that can be readily used to make transactions.
- Suppose a person catches fish and wants to exchange them for rice.
  - Without money:
    - They must find someone who has rice and wants fish. In other words, trade is only possible when there is a double coincidence of wants.
  - With money:
    - The person only needs to find someone who has rice. This situation requires only a single coincidence of wants.
  - Thus, the existence of money increases the probability of finding a trading partner and makes transactions much easier.
- **Liquidity:** This property of making transactions easier is called liquidity.

What is Money?

- In a standard microeconomics textbook, money does not appear.
  - In traditional microeconomic theory, the process of finding a trading partner is not considered.
  - Instead, individuals are assumed to make decisions based solely on relative prices—for example, the price of apples in terms of oranges.
  - Therefore, economists tend to emphasize purchasing power—what one can buy with a monthly income—rather than the nominal amount of income itself.
- However, in reality, the quantity of money that exists within an economy has a significant impact on economic activity. This is one of the distinctive puzzles of macroeconomics.

What is Money?

- Money and Expectation
  1. When this person brings money (fiat currency) and finds someone who owns rice, why would the rice owner be willing to exchange their rice for money?
  2. Naturally, the rice owner expects that they will later be able to use that money (fiat currency) to obtain something they desire in the future—say, vegetables.
  3. But then, why would the person who owns vegetables be willing to exchange their vegetables for money? Of course, that person also expects that others will accept the same money in future transactions.
  4. Following this logic, we can see that fiat money holds value because members of society share a collective belief or expectation that it has value.
- Therefore, whether inflation occurs or not largely depends on people's expectations regarding the future value of money.

Money Market

- In this course, we will discuss money from a more practical perspective in order to understand monetary policy.
  - However, please keep in mind that there are complex and fundamental issues related to money, such as those we have just considered.
- Money has three functions
  1. a store of value
  2. a unit of account
  3. a medium of exchange.

Money Market

- **Money Supply:** Money supply includes both currency in the hands and deposits at banks that households can use on demand for transaction:

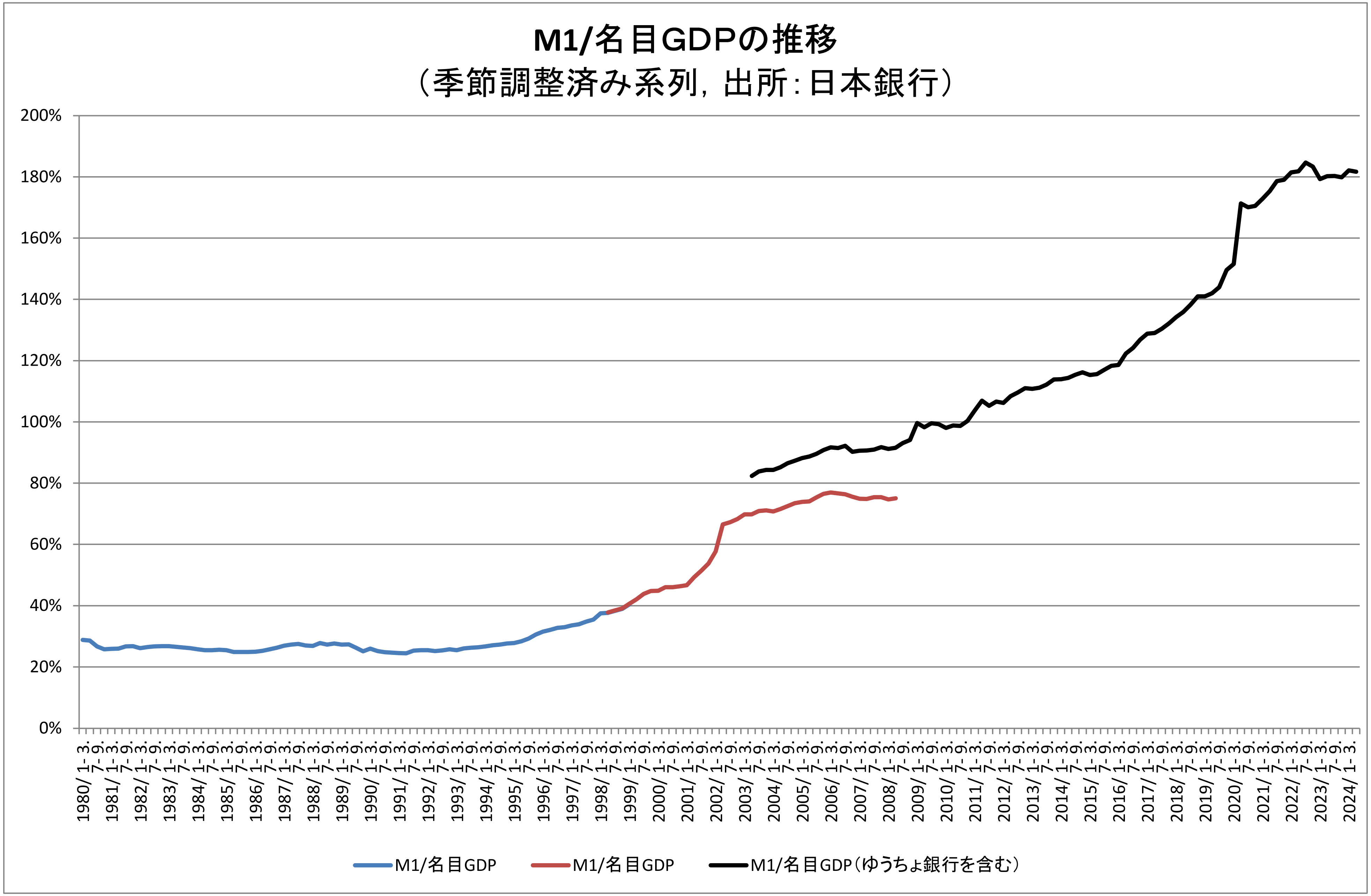
$$M^s = C + D \tag{2}$$

where M^s is money supply, C is currency and D is Deposit.

  - Currency consists of Bank of Japan notes and coins. Bank of Japan notes represent liabilities of the central bank.

Money Market

- Depending on the types of deposits included, several different definitions of the money supply are used:
  - M1 = Currency in circulation + deposit currency held at deposit-taking institutions (yokin toriatsukai kikan)
    - Deposit currency consists of checking accounts and savings accounts
  - M2 = Currency in circulation + deposits held at domestic banks
    - Deposits consists of deposit currency + time deposits (teiki yokin) + foreign currency deposits (gaika yokin)+ certificates of deposit (Jyotosei Yokin).
    - "Domestic banks" (kokunai ginkō) do not include Japan Post Bank (Yūcho Bank).
  - M3 = Currency in circulation + deposits held at all deposit-taking institutions (yokin toriatsukai kikan)
    - "Deposit-taking institutions" (yokin toriatsukai kikan) include domestic banks, Japan Post Bank (Yūcho Bank), credit cooperatives (Shinyo Kumiai), labor banks (Rodo Kinko), and agricultural cooperatives (Nogyo Kyodo Kumiai), among others.


M1/名目GDPの推移
（季節調整済み系列，出所：日本銀行）
200%
180%
160%
140%
120%
100%
80%
60%
40%
20%
0%
M1/名目GDP
M1/名目GDP
M1/名目GDP（ゆうちょ銀行を含む）

Money Market

- **A Model of Money Supply:** The total amount of YEN that is supplied by Bank of Japan is called the monetary base, denoted by B and the monetary base equals currency plus reserves

$$B = C + R \tag{3}$$

- Using equation (2) and equation (3),

$$\frac{M^s}{B} = \frac{C + D}{C + R}$$

Hence

$$M^s = \frac{cd + 1}{cd + rd} B$$

where $cd = C/D$ and $rd = R/D$. The parameter $\frac{cd+1}{cd+rd}$ is called money multiplier, which is a decreasing function of rd.

- This equation implies that Bank of Japan can control money supply by changing monetary base and reserve-deposit ratio.

Money Market

- How does Bank of Japan control B? There are three instruments:
  1. **Open-market operations:** Bank of Japan can sell or buy government bonds. When it buys bonds in the market, it pays yen for the bonds. Therefore, monetary base goes up. On the other hand, when it sells, it receives yen. Hence monetary base goes down.
  2. **Operations at exchange market:** Bank of Japan can sell or buy dollars. When it buys dollars in the market, it pays yen for the dollars. Therefore, monetary base goes up. On the other hand, when it sells, it receives yen. Hence monetary base goes down.
  3. **Change in the Discount Rate:** When commercial banks find that they do not have enough reserves, they can ask Bank of Japan to discount their bills. When Bank of Japan reduces the discount rate, commercial banks can easily borrow money from Bank of Japan and it increases monetary base.

Money Market

- **Interbank Lending Market:** If a bank cannot hold the required amount of liquidity asset as reserves, it will need to borrow money in the interbank market to cover the shortfall.
- **Financial Crisis:** During Financial Crisis, there are some difficulties in this interbank system. A bank defaults. Other banks that made loans to this bank may default too. All banks are worried about the risk of default and the transaction in the interbank lending market becomes low. It is said that low transaction volume in this market was a major factor to the financial crisis of 2007.

- **Banking System and Money Supply:** Banking System influences the amount of money supply.
  - Example: Banks are required to keep a proportion of deposit by law in that depositors can always withdraw money. The deposits that banks have received but have not lent out are called reserves, denoted by R. Suppose that you deposit 1000 yen, $R = rd \times 1000$ *yen* where rd is the reserve deposit ratio. Hence, the bank can use $(1 - rd) \times 1000yen$ to make loans and $(1 - rd) \times 1000yen$ goes to public as currency. Since money supply is sum of currency and deposits, money supply is $(1 - rd) \times 1000yen + 1000yen$. In this way, the banking system influences money supply.

信用創造のメカニズム

- 例

銀行

- 1000円 → 1000円 → 700円

預金通貨　現金通貨

↓

300円

預金準備

マネタリーベース＝７００円＋３００円＝１０００円

貨幣量＝７００円＋１０００円＝１７００円

- **Money Demand:** Why do people demand money? There is the benefits and costs of holding money.
  1. The cost of holding money is that you miss interest. Instead of holding money, you can buy stocks or bonds. If you buy bonds, you can earn interest. But if you hold money in your pockets, you miss this opportunity. Therefore, the higher the interest rate, the smaller the demand for money.
  2. Then what is the benefit of holding money? It makes our transactions smoother. When you made a deal with your business partners, you must pay money. Hence we expect that when we have more transactions, we must demand more money. Since transactions increase when real GDP is larger, we expect that the demand for money is an increasing function of real GDP.

Money Market

- The discussions suggest the following money demand function.

$$\frac{M^d}{P} = L(\rho, Y)$$

where M^d is the demand for money and P is the aggregate price. The above equation implies that a decrease in the interest rate and an increase in real GDP raise the demand for money.

- For a simple analysis, we assume that the money demand is proportional to real GDP:

$$\begin{aligned}
\frac{M^d}{P} &= k(\rho) Y, \ \ k'(\rho) = \frac{dk}{d\rho} < 0 \\
\frac{m^d}{P} &= k(\rho) y
\end{aligned}$$

where $m^d = \frac{M^d}{N}$ and $y = \frac{Y}{N}$, and $k(\rho)$ is called Marshall's k. Marshall's k measures how much money people want to hold for every income.

Money Market

- Money market is

$$\frac{m^s}{P} = \frac{m^d}{P} = k(\rho)\, y$$

where $m^s = \frac{M^s}{N}$.

- **Government Sector:** Assume that government must finance government expenditure G_t, and government bond, B_t and transfer payments to households, TR_t, such as welfare for the poor and Social Security payments for the elderly, by a lump sum tax, with which consumers must pay fixed cost, ς_t, to government.

$$
\begin{aligned}
B_{t+1} &= (1+\rho) B_t + G_t + TR_t - \varsigma_t \\
\frac{B_{t+1}}{N_{t+1}} \frac{N_{t+1}}{N_t} &= (1+\rho) \frac{B_t}{N_t} + \frac{G_t}{N_t} + \frac{TR_t}{N_t} - \frac{\varsigma_t}{N_t} \\
\frac{B_{t+1}}{N_{t+1}} (1+n) &= (1+\rho) \frac{B_t}{N_t} + \frac{G_t}{N_t} + \frac{TR_t}{N_t} - \frac{\varsigma_t}{N_t} \\
b_{t+1} &= \frac{1+\rho}{1+n} b_t + \frac{\hat{g}_t + tr_t - \tau_t}{1+n}
\end{aligned}
$$

where $b_t = \frac{B_t}{N_t}$, $\hat{g}_t = \frac{G_t}{N_t}$, $tr_t = \frac{TR_t}{N_t}$ and $\tau_t = \frac{\varsigma_t}{N_t}$.

Government Sector

- For simplicity, we assume that the government maintains a primary balance of zero and that $\hat{g}_t$, tr_t and τ_t are constant.

$$\hat{g} = \tau^n, \ \tau^n \equiv \tau - tr$$

where τ^n is the net tax. Then

$$b_{t+1} = \frac{1+\rho}{1+n} b_t$$

- For the moment, we also assume that $n = 0$. Then

$$b_{t+1} = (1+\rho) \, b_t$$

Government Sector

- Following Ono's (2006) critique, we assume that θ proportion of $\hat{g}$ increases value added. That is, $(1-\theta)$ proportion of $\hat{g}$ can be seen as a hidden income transfer, tr^h, to somebody:

$$tr^h = (1-\theta)\,\hat{g}.$$

  - Note that current national account presumes $\theta = 1$.
  - For simplicity, we assume that θ is constant.

Firm

- Assumption: Constant returns to scale and the steady state.
  - Moreover, because we ignore capital accumulation from our following analysis, we simply assume $k^* = \frac{K}{TN} = 1 \Rightarrow k = \frac{K}{N} = T$.
  - Note that ignoring capital accumulation simplifies our explanation, but makes us impossible to analyze the long run impact of demand stabilization policy through capital accumulation.

$$Y \leq F(K, TlN) = F\left(1, \frac{TNl}{K}\right) K = F(1, l) K$$

Define ϕ

$$\phi(l) \equiv F(1, l)$$

Then

$$Y \leq \phi(l) K$$

Moreover,

$$y = \frac{Y}{N} \leq \phi(l) \frac{K}{N} = \phi(l) T$$

Firm

- Profit maximization Problem

$$
\begin{aligned}
\Pi &= \max_{K,L} \{PY - WlN - RK\} \\
&= \max_{l,K} \left\{ P\phi(l)K - \frac{Wl}{PT}\frac{TN}{K}PK - \frac{R}{P}PK \right\} \\
&= \max_{l,K} \left\{ P\phi(l)K - \frac{W}{PT}lPK - \frac{R}{P}PK \right\} \\
&= \max_{K} \pi_k PK \\
\textit{where } \pi_k &= \max_{l} \left\{ \phi(l) - \frac{w}{T}l - r \right\}
\end{aligned}
$$

where $w = \frac{W}{P}$ and $r = \frac{R}{P}$.

Firm

- Assumptions on the aggregate production function
  1. $\phi(0) = 0$
  2. $\phi'(l) = \frac{d\phi(l)}{dl} > 0.$
     - When the firm employs more capital per workers, it increases output per workers.
  3. $\phi''(l) = \frac{d^2\phi(l)}{dl^2} < 0$
     - This means that the marginal productivity of capital per workers is diminishing.
  4. Inada Conditions: technical conditions.

$$\lim_{l \to 0} \phi'(l) = \infty, \lim_{l \to \infty} \phi'(l) = 0.$$

Firm

- Profit maximization with respect to l

$$\pi_k = \max_l \left\{ \phi(l) - \frac{w}{T} l - r \right\}$$

- First Order Conditions

$$\frac{w}{T} = \phi'(l) \Rightarrow w = \phi'(l)\, T \Rightarrow l \textit{ is determined}.$$

- π_k

$$\pi_k = \phi(l) - \frac{w}{T} l - r = \phi(l) - \phi'(l)\, l - r \Rightarrow \pi_k \textit{ is determined}.$$

Firm

- Profit maximization with respect to K

$$\Pi = \max_K \pi_k PK$$

- Capital Demand Function

$$\begin{aligned} K &= 0 \ \mathit{if} \ \ \pi_k < 0, \ \ r > \phi(I) - \phi'(I)\, I \\ K &\in [0, \infty] \ \ \mathit{if} \ \ \pi_k = 0, \ \ r = \phi(I) - \phi'(I)\, I \\ K &= \infty \ \mathit{if} \ \ \pi_k > 0, \ \ r < \phi(I) - \phi'(I)\, I \end{aligned}$$

Firm

- 0 Economic Profit

$$\Pi = \pi_k PK = 0$$

When the market is competitive, more entrepreneurs will enter as long as economic profits are positive. Hence, in the long run, economic profit is 0.Hence

$$\Pi = 0, K > 0 \Rightarrow \pi_k = 0 \Rightarrow r = \phi(I) - \phi'(I) I$$

Household

- How long do they work?: Assume that everybody works $h(w)$ of time. Hence, the supply of labor is equal to $h(w)N$. We assume that $h'(\cdot) = \frac{dh(w)}{dw} > 0$.
- How much do they consume today?:
  - I take the consumption per capita, c as given.
  - Budget Constraint: Assume that $\left(\frac{m^s}{P}\right)_{+1} = \frac{m^s}{P}$. Then

$$
\begin{aligned}
a_{+1} + c + \left(\frac{m^s}{P}\right)_{+1} &= (1+\rho^a)\,a + \frac{m^s}{P} + wh(w) - \tau + tr + tr^h \\
a_{+1} + c &= (1+\rho^a)\,a + wh(w) - \tau^n + tr^h \\
\rho^a &= \max\{r-\delta, \rho\}
\end{aligned}
$$

where τ is tax rate and tr is income transfer.

Arbitrage Condition and Market Clearing Condition

- Arbitrage Condition implies,

$$r - \delta = \rho = \rho^a$$

- Labor Market Clearing Condition

$$lN = h(w)N \Rightarrow l = h(w)$$

- Capital Market Clearing Condition

$$\begin{aligned} kN + bN &= aN \\ b &= a - k \end{aligned}$$

Stabilization Policy in the Long Run

- Given $(a, c, k\,(= T)\,, M^s, \hat{g}, \theta, \delta)$, a market equilibrium with money and government consists of $\left(y, l, a_{+1}, \rho, r, w, \tau^n, P, tr^h, b, b_{+1}\right)$ which satisfies
- Individual Behavior
  - A Firm's Profit Maximization and the Production Function determine:

$$y = \phi\,(l)\;T$$

$$\begin{aligned} w &= \phi'\,(l)\;T \\ r &= \phi\,(l) - \phi'\,(l)\;l \end{aligned}$$

  - A Consumer's Budget Constraint

$$a_{+1} + c = (1+\rho)\,a + wh\,(w) - \tau^n + tr^h$$

Stabilization Policy in the Long Run

- Markets
  - An Arbitrage Condition
$$r - \delta = \rho$$
  - Labor market clearing conditions
$$l = h(w)$$
  - Capital market clearing conditions
$$b = a - k$$

Stabilization Policy in the Long Run

- Government and Money
  - Government's budget constraint determines τ^n and tr^h:

$$\begin{aligned} \tau^n &= \hat{g} \\ tr^h &= (1-\theta)\,\hat{g} \end{aligned}$$

  - The dynamics of debt.

$$b_{+1} = (1+\rho)\,b$$

  - The money market cleaning condition determines P:

$$\frac{m^s}{P} = k(\rho)\,y$$

- Supply Side of Equilibrium: Because $l = h(w)$,

$$
\begin{aligned}
y &= \phi(h(w))\,T \\
w &= \phi'(h(w))\,T \\
\rho &= \phi(h(w)) - \phi'(h(w))\,h(w) - \delta
\end{aligned}
$$

Stabilization Policy in the Long Run

- Goods market

$$
\begin{aligned}
a_{+1} + c &= (1+\rho) a + wh(w) - \tau^n + tr^h \\
k_{+1} + b_{+1} + c &= (1+\rho) a + wh(w) - \hat{g} + (1-\theta) \hat{g} \\
k_{+1} + (1+\rho) b + c &= (1+\rho) a + wh(w) - \hat{g} + (1-\theta) \hat{g} \\
k_{+1} + c &= (1+\rho)(k+b) + wh(w) - \theta\hat{g} \\
k_{+1} + c &= (1+r-\delta) k + wh(w) - \theta\hat{g} \\
k_{+1} + c &= \left(\phi(I) - \phi'(I) I + (1-\delta)\right) k + \phi'(I) TI - \theta\hat{g} \\
k_{+1} + c &= \left(\phi(I) - \phi'(I) I\right) T + (1-\delta) k \\
&\quad + \phi'(I) TI - \theta\hat{g} \\
k_{+1} + c &= \phi(I) T + (1-\delta) k - \theta\hat{g}
\end{aligned}
$$

$$
\begin{aligned}
y &= c + k_{+1} - (1-\delta) k + \theta\hat{g} \\
&= c + i + \theta\hat{g}
\end{aligned}
$$

Stabilization Policy in the Long Run

- Supply Side determines (y, w, ρ)

$$
\begin{aligned}
y &= \phi(h(w))\, T \\
w &= \phi'(h(w))\, T \\
\rho &= \phi(h(w)) - \phi'(h(w))\, h(w) - \delta
\end{aligned}
$$

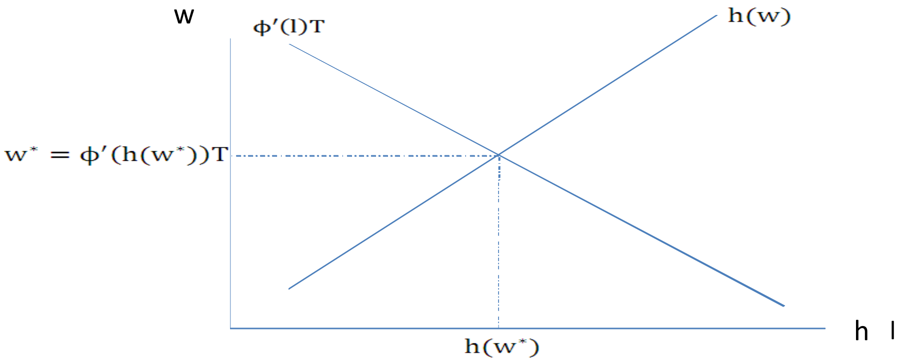
- Demand Side determines (i, P)

$$
\begin{aligned}
y &= c + i + \theta \hat{g} \\
\frac{m^s}{P} &= k(\rho)\, y
\end{aligned}
$$

- Remark: Note that current national income accounting presume that $\theta = 1$. In facts,

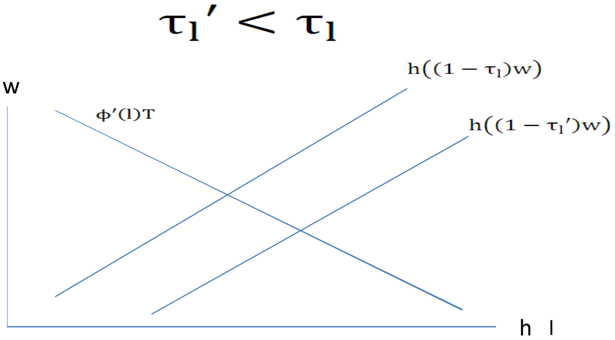
$$
\textit{the measured GDP per capita} = y + (1 - \theta)\, \hat{g} = c + i + \hat{g}
$$

Labor Market Equilibrium



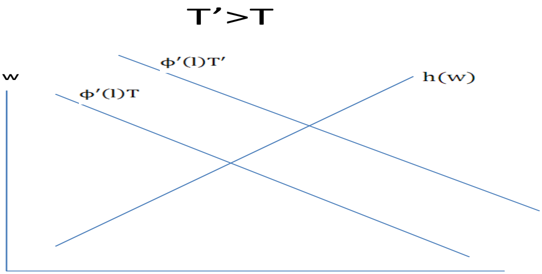
- **Stabilization Policy vs. Supply Side Policy:** Both monetary supply, (a change in m^s), and a fiscal policy, (a change in $\hat{g}$), cannot influence GDP per capita, y, the real interest rate, ρ, and the real wage rate w in the long run.
    1. Given a labor supply function $h(w)$, w is determined from $w = \phi'(h(w))\,T$.
    2. Given w, $y = \phi(h(w))\,T$ and $\rho = \phi(h(w)) - \phi'(h(w))\,h(w) - \delta$
- If a government wishes to influence GDP in the long run, the policy must have an impact on supply side. I will give two examples.

- Suppose that government levies a tax on labor income. In this case, households react to $(1-\tau_l)\,w$, where τ_l is a flat labor income tax rate. Hence, as a reduction in labor income tax increases $(1-\tau_l)\,w$, $h\,((1-\tau_l)\,w)$ would be larger.



Stabilization Policy in the Long Run

- Suppose that government expenditure increases productivity T. Because $\phi'(h(w))\,T$ implies that the marginal product of labor is larger, firms have more incentive to employ labor. Hence, $y = \phi(h(w))\,T$ would be larger.



Stabilization Policy in the Long Run

- **Fiscal Policy and Crowding Out:** How does fiscal policy influence an economy in the long run? Consider

$$y = c + i + \theta \hat{g}.$$

As y is already given supply side, an increase in $\hat{g}$ must result in reductions of $c + i$. It leads to the following theorem.

Theorem

A permanent increase in public expenditure (which only affect demand side) will be offset by a permanent reduction of private expenditure in the long run (crowding out).

Stabilization Policy in the Long Run

- **Monetary Policy and Inflation:** How does fiscal policy influence an economy in the long run? Consider

$$\frac{m^s}{P} = k(\rho)\, y,$$

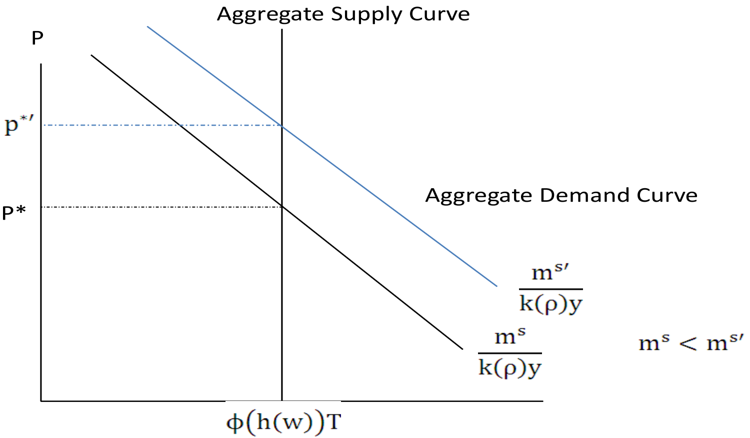
  Because ρ and y are already given by supply side, equation implies that an increase in money supply, m^s, causes inflation (= an increase in the price index, P).
- All variables measured in physical units, such as output and relative prices, are called real variables. Variables expressed in terms of money are called nominal variables. This result implies that money supply affects nominal variables, but it does not affect real variables. This is called *the neutrality of money*.

Theorem

An increase in money supply causes inflation, but does not affect real variables in the long run.

Stabilization Policy in the Long Run

Aggregate Demand and Aggregate Supply Curve



- **The quantity theory of money:** *The quantity theory of money is derived from money market equilibrium.*

$$
\begin{aligned}
m^s V(\rho) &= Py \\
M^s V(\rho) &= PY
\end{aligned}
$$

where $V(\rho) = \frac{1}{k(\rho)}$.

- The $V(\rho)$ is called the income velocity of money, which tells us the number of times money enters someone's' income during a given period of time. It measures the speed of transaction. Note that the income velocity of money is inverse of Marshall's k. If people wish to hold more money given income ($=$ large k), money cannot move much ($=$ small $V(\rho)$). If people hold little money in hand, money can frequently move around.

Money Market in the Long Run

- Note that

$$
\begin{aligned}
g_{M^sV} &= g_{PY} \\
g_{M^s} + g_V &= g_P + g_Y
\end{aligned}
$$

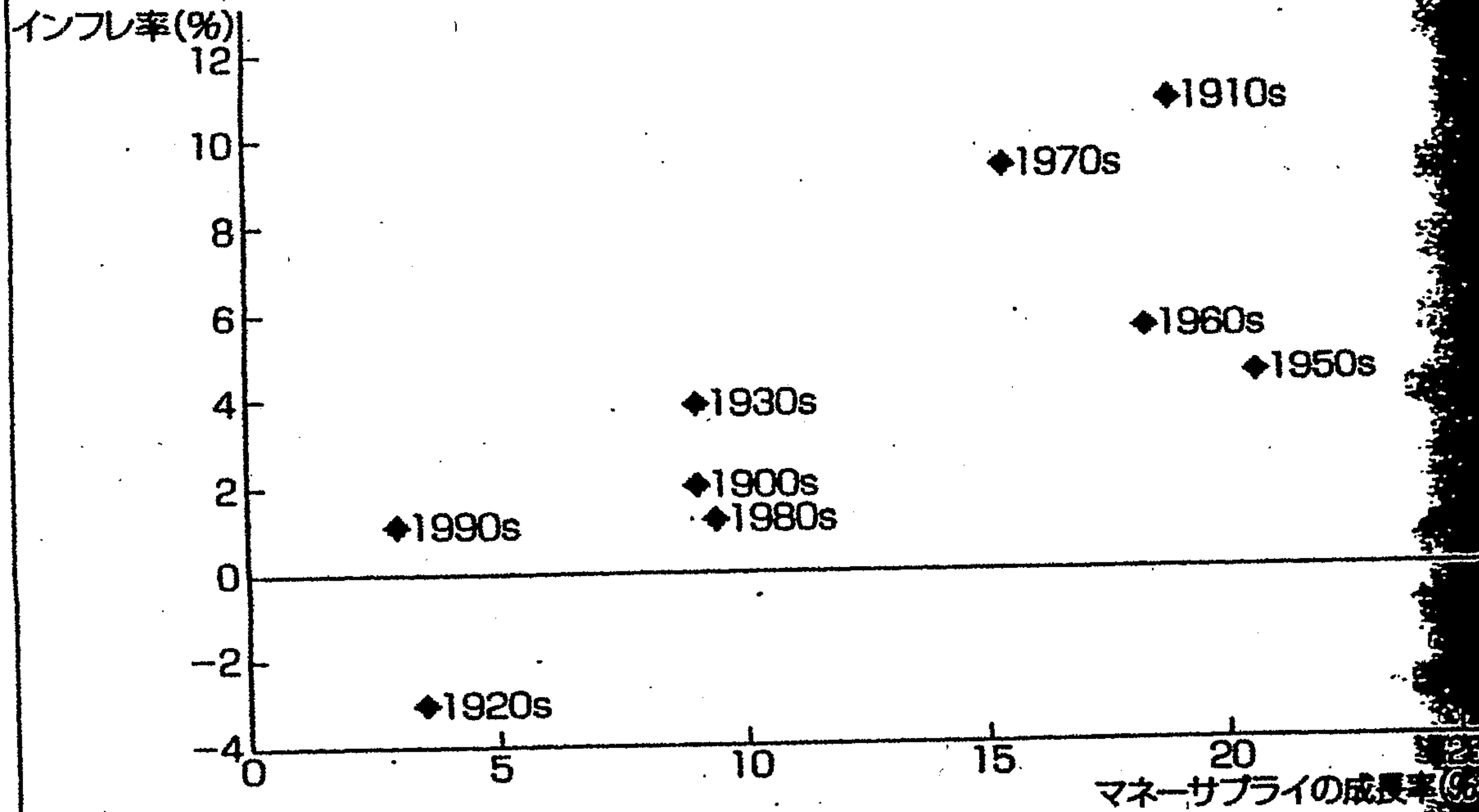
  Since the supply side determines ρ, ρ is given. If ρ is constant, V is constant. Hence, $g_V = 0$.

$$g_{M^s} = g_P + g_Y$$

  If an economy is in a steady state, we know that $g_Y = g_{yN} = g_y + g_N = g_T + g_N$. we may be able to assume that it is near constant.

- Hence, the quantity theory of money suggests that the central bank can control inflation rate g_p by changing g_{M^s}.

図 5-1′ 日本のインフレーションと貨幣成長：長期データ



日本の場合も，マネーサプライの成長率（M2）とインフレーションの間には正の相関関係がある。

（出所） マネーサプライについては藤野正三郎『日本のマネーサプライ』，日本銀行『経済統計年報』，物価指数は大川一司ほか編『長期経済統計』，『消費者物価指数年報』。

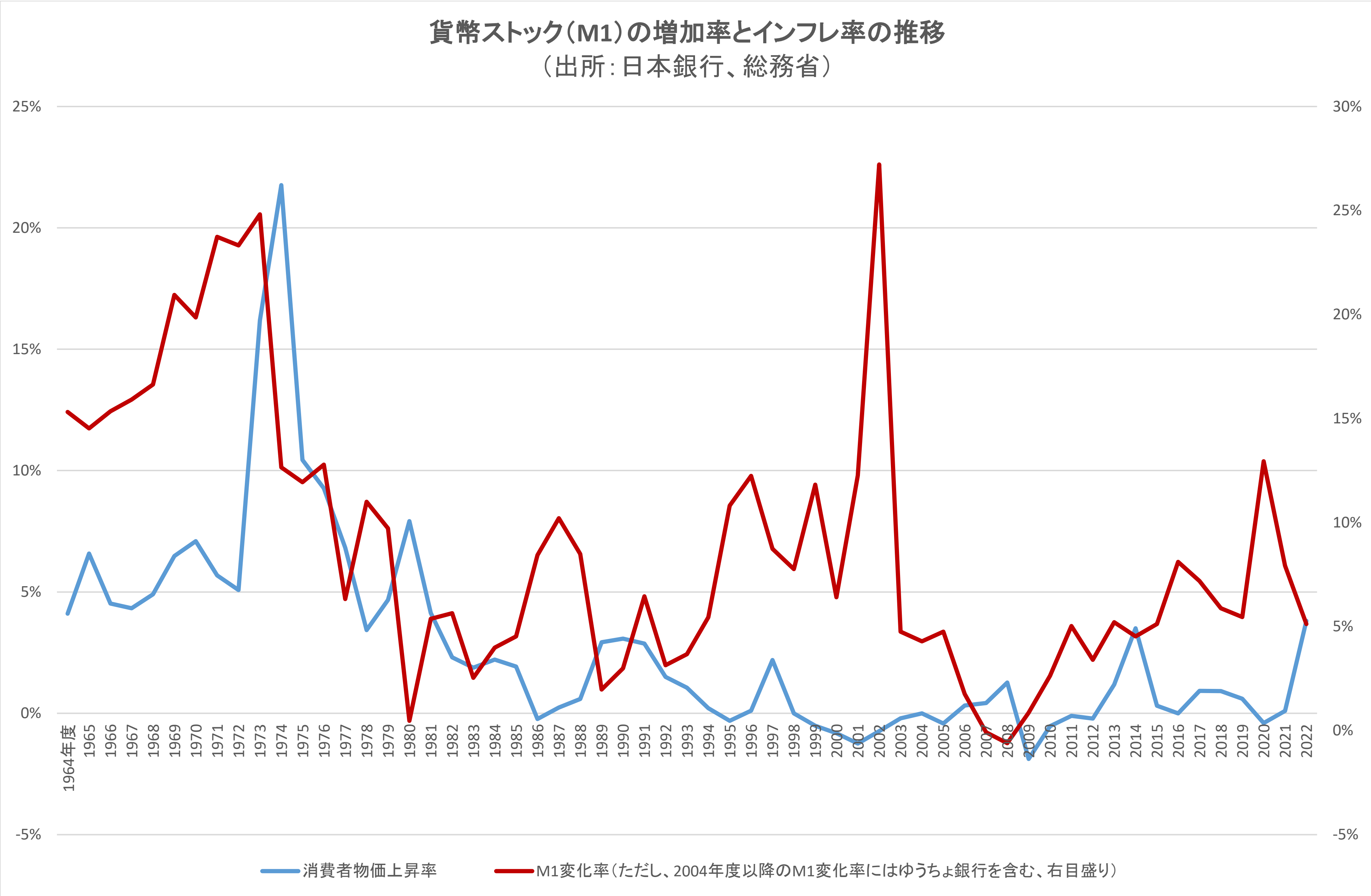
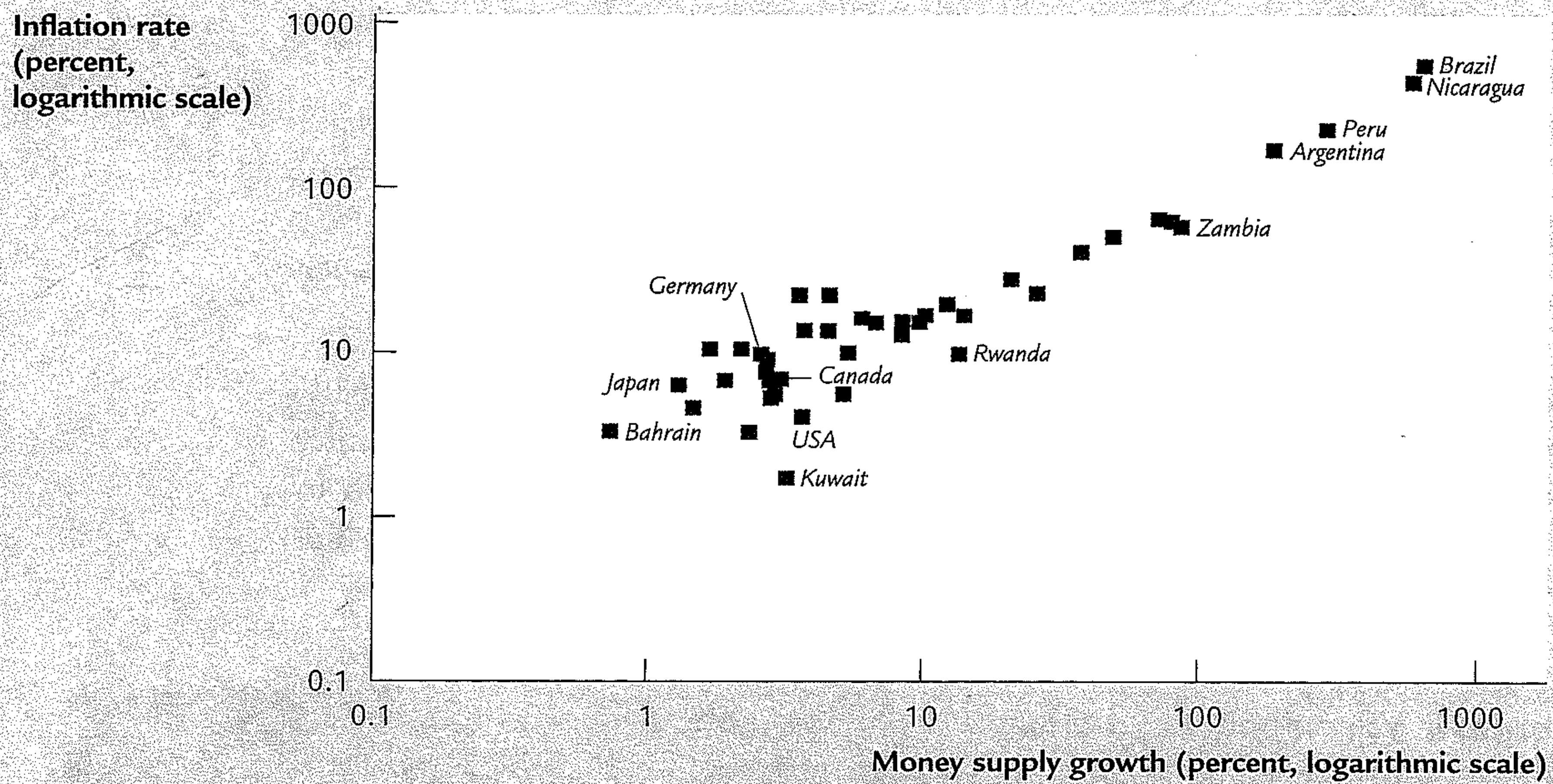

貨幣ストック（M1）の増加率とインフレ率の推移
（出所：日本銀行、総務省）
25%
20%
15%
10%
5%
0%
-5%
30%
25%
20%
15%
10%
5%
0%
-5%
1964年度
消費者物価上昇率
M1変化率（ただし、2004年度以降のM1変化率にはゆうちょ銀行を含む、右目盛り）

figure 7-2



International Data on Inflation and Money Growth In this scatterplot, each point represents a country. The horizontal axis shows the average growth in the money supply (as measured by currency plus demand deposits) during the 1980s, and the vertical axis shows the average rate of inflation (as measured by the GDP deflator). Once again, the positive correlation is evidence for the quantity theory's prediction that high money growth leads to high inflation.

Source: International Financial Statistics.

Money Market in the Long Run

- **Nominal Interest Rate and Real Interest Rate:** Alternative difficulty in controlling inflation arises when we realize difference between nominal interest rate and real interest rate. The real interest rate is defined as nominal interest rate minus the expected inflation rate:

$$\rho^r = \rho^n - g_P^e \tag{4}$$

  where ρ^r is the real interest rate, ρ^n is the nominal interest rate and g_P^e is the expected inflation rate.

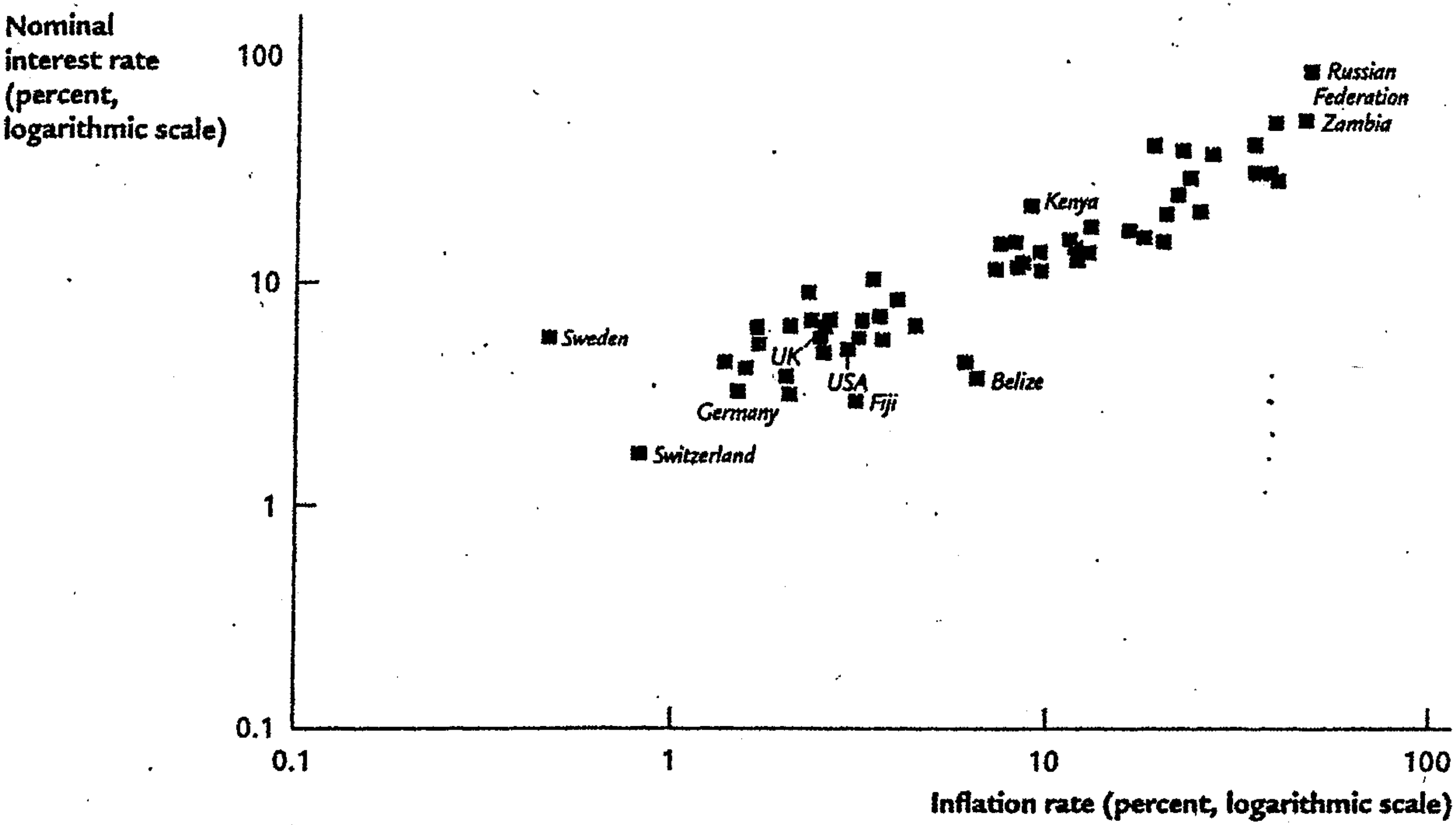
  - ρ^r influences saving decisions and investment decisions.
  - ρ^n is considered to be opportunity costs of holding money. Hence, money demand depends on ρ^n.

- **Fisher equation:** Rearranging equation

$$\rho^n = \rho^r + g_P^e$$

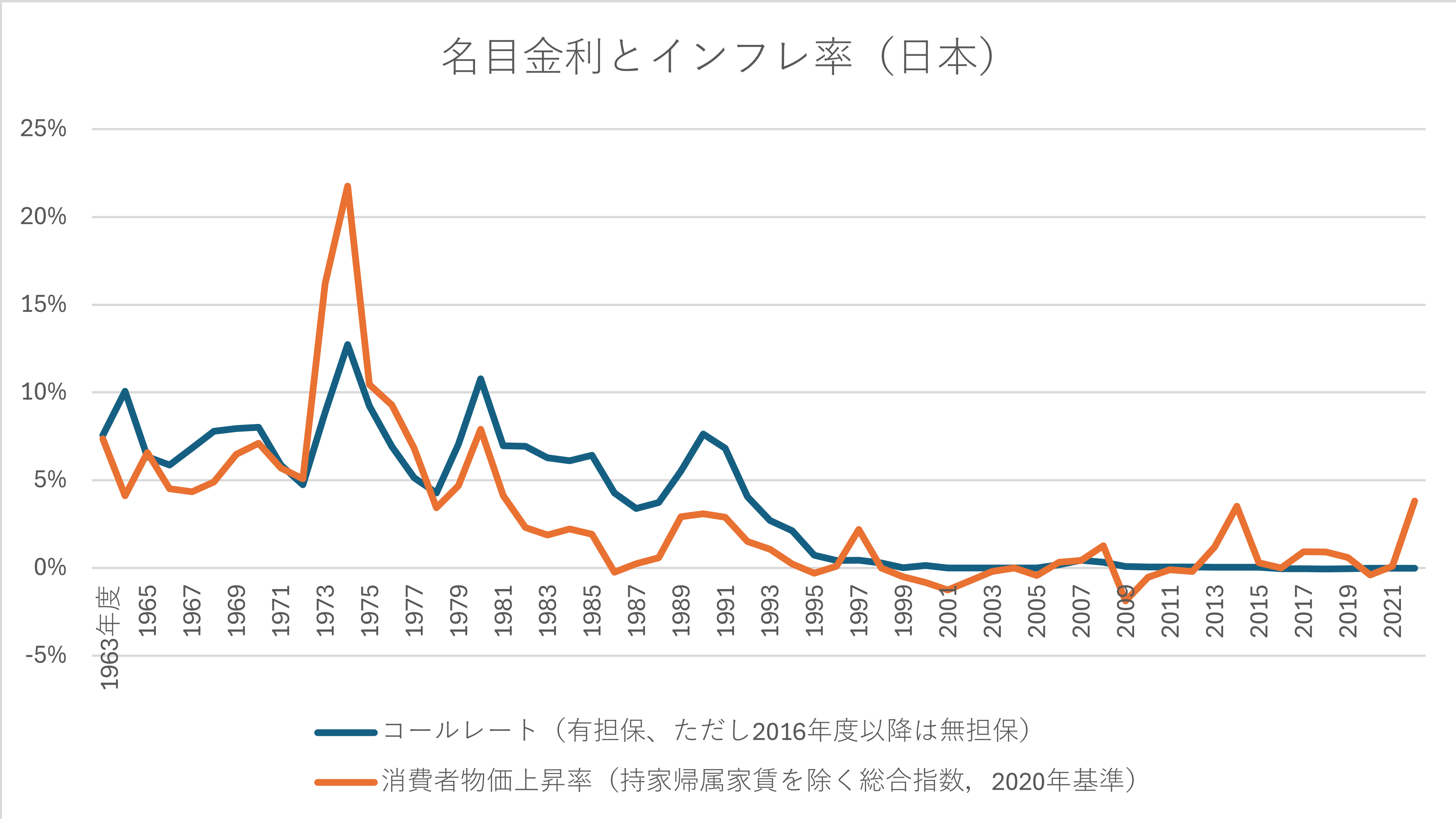
  This is called Fisher equation.

figure 7-4



Inflation and Nominal Interest Rates Across Countries This scatterplot exhibits the three-month nominal interest rate and the inflation rate (during the previous year) in 58 countries in 1996. The positive correlation between the inflation rate and the nominal interest rate is evidence for the Fisher effect.

Source: International Financial Statistics.


名目金利とインフレ率（日本）
25%
20%
15%
10%
5%
0%
-5%
1963年度
1965
1967
1969
1971
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
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2001
2003
2005
2007
2009
2011
2013
2015
2017
2019
2021
コールレート（有担保、ただし2016年度以降は無担保）
消費者物価上昇率（持家帰属家賃を除く総合指数，2020年基準）

- As Marshall's k is a decreasing function of ρ^n, and therefore the V is an increasing function of ρ^n.

$$M^s V\left(\rho^r + g_P^e\right) = PY, \ V'\left(\cdot\right) > 0.$$

- This equation implies that expected future inflation rate can influence actual inflation rate.

$$g_P^e \uparrow \Rightarrow \rho^n \uparrow \Rightarrow k\left(\rho^n\right) \downarrow \Rightarrow V\left(\rho^n\right) \uparrow \Rightarrow P \uparrow$$

  Hence, a careless expansionary monetary policy may induce a hyper inflation. So monetary policy must take into account how it influences people's expectation. But how?

Money Market in the Long Run

- Note that

$$
\begin{aligned}
\ln P_t + \ln Y &= \ln M_t + \ln V\left(\rho^r + g^e_{P,t}\right) \\
\ln P_t &= \ln M_t + \ln V\left(\rho^r + g^e_{P,t}\right) - \ln Y
\end{aligned}
$$

- Assume that $\ln V\left(\rho^r + g^e_P\right) = \gamma\left(\rho^r + g^e_P\right)$, and ρ^r and $\ln Y$ are constant over time. Then

$$
\begin{aligned}
g^e_{P,t} &= E\left[\ln P_{t+1}\right] - \ln P_t \\
&= E\left[\ln M_{t+1} + \ln V\left(\rho^r + g^e_{P,t+1}\right) - \ln Y\right] \\
&\quad - \left[\ln M_t + \ln V\left(\rho^r + g^e_{P,t}\right) - \ln Y\right] \\
&= E\left[\ln M_{t+1} + \gamma\left(\rho^r + g^e_{P,t+1}\right)\right] \\
&\quad - \left[\ln M_t + \gamma\left(\rho^r + g^e_{P,t}\right)\right] \\
(1+\gamma)\, g^e_{P,t} &= E\left[\ln M_{t+1}\right] - \ln M_t + \gamma g^e_{P,t+1}
\end{aligned}
$$

- The expected Inflation rate

$$g^e_{P,t} = \frac{E\left[\ln M_{t+1}\right] - \ln M_t + \gamma g^e_{P,t+1}}{1+\gamma}$$

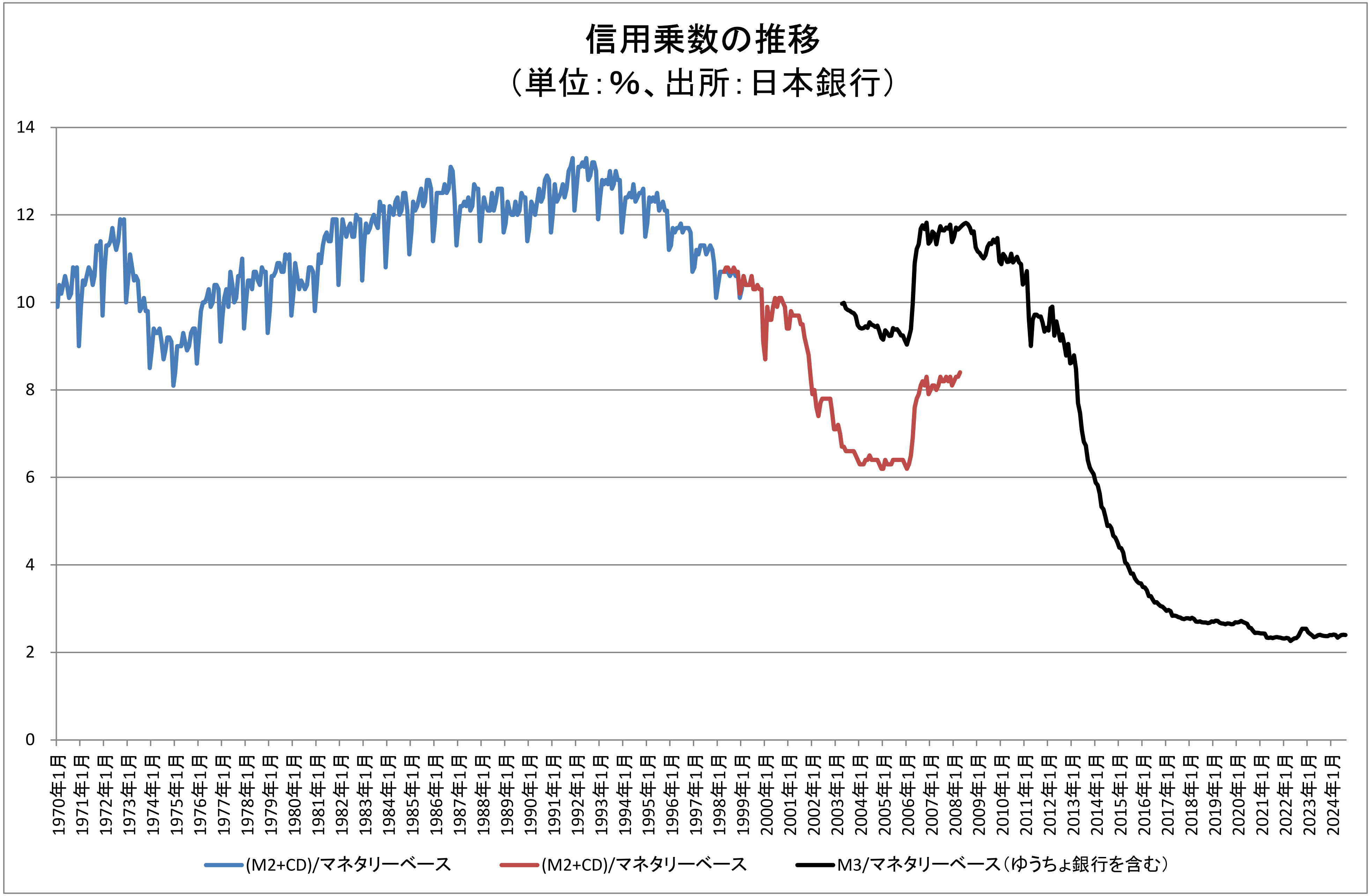
  - It means that the current expected inflation rate depends on the expectation on the money supply in the next period and the expected inflation rate in the next period. We can also imagine that the expected inflation rate in the next period would depend on the expectation on the money supply in two period later and so on. It indicates that in order to control people's expectation, the commitment on the future monetary policy is necessary.

Discussion

- **Can Bank of Japan really control money supply?:** So far we assume that the Bank of Japan can perfectly control money supply. Is it true? During Xmas season, we can observe an increase in money supply and nominal GDP. However, it is difficult to believe that an increase in money supply during Xmas causes an increase in nominal GDP. Natural interpretation is opposite: since people transact more during Xmas seasons, people demand more money.
- Note that there is a relationship between B and M^s.

$$M^s = \frac{cd + 1}{cd + rd} B$$

where $cd = C/D$ and $rd = R/D$. We so far assume that cd and rd are constant. But in fact these numbers are endogenous variables. In Japan, money multiplier has decreased since 1992. It means that although Bank of Japan has actively increased Monetary Base, its impacts on money supply has decline during the period.



信用乗数の推移
（単位：%、出所：日本銀行）
14
12
10
8
6
4
2
0
1970年1月
1971年1月
1972年1月
1973年1月
1974年1月
1975年1月
1976年1月
1977年1月
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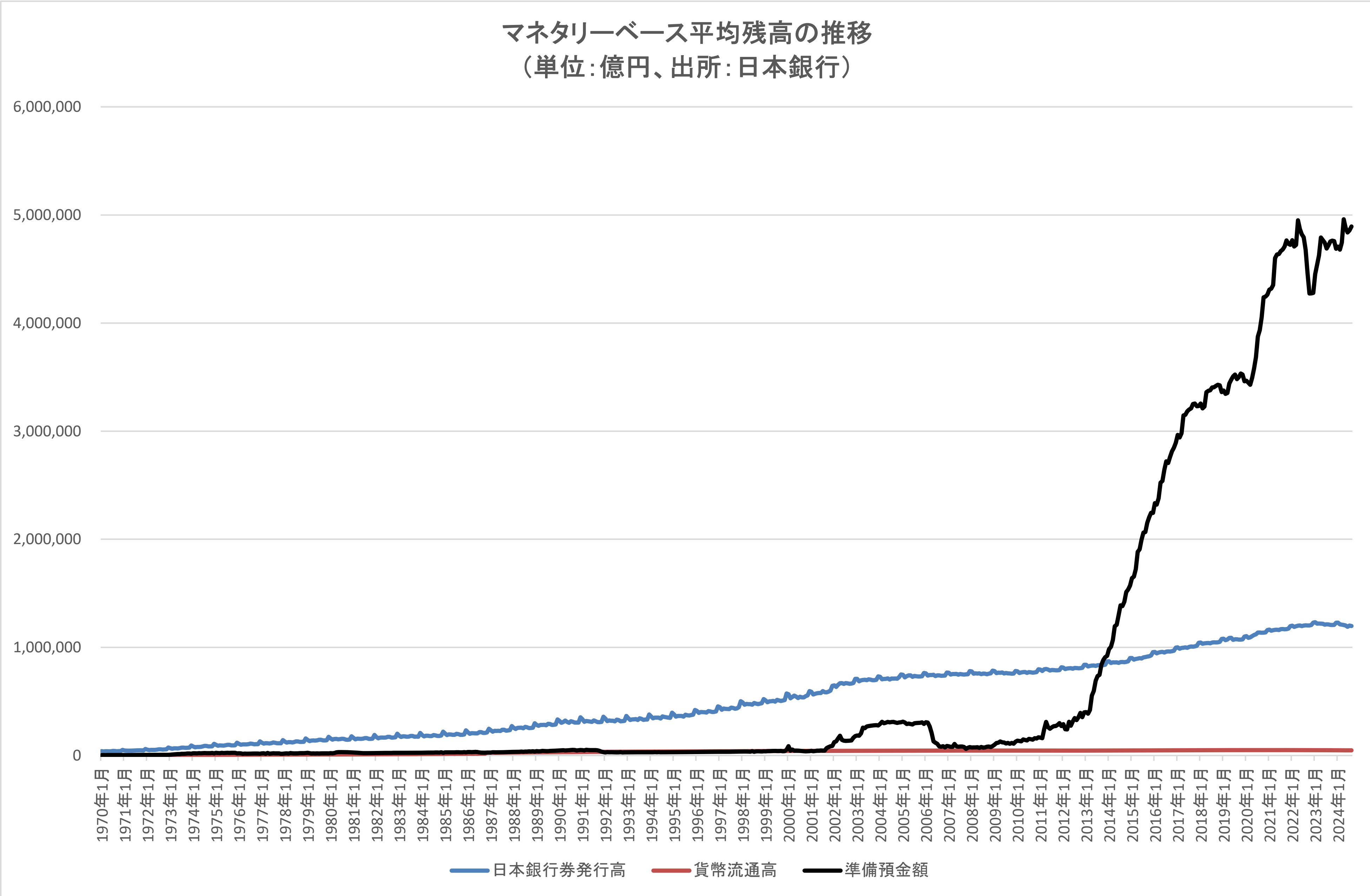
2014年1月
2015年1月
2016年1月
2017年1月
2018年1月
2019年1月
2020年1月
2021年1月
2022年1月
2023年1月
2024年1月
(M2+CD)/マネタリーベース
(M2+CD)/マネタリーベース
M3/マネタリーベース（ゆうちょ銀行を含む）



マネタリーベース平均残高の推移
（単位：億円、出所：日本銀行）
6,000,000
5,000,000
4,000,000
3,000,000
2,000,000
1,000,000
0
1970年1月
1971年1月
1972年1月
1973年1月
1974年1月
1975年1月
1976年1月
1977年1月
1978年1月
1979年1月
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2017年1月
2018年1月
2019年1月
2020年1月
2021年1月
2022年1月
2023年1月
2024年1月
日本銀行券発行高
貨幣流通高
準備預金額

Discussion

- **Interest rate as a policy target:** While it is questionable that Bank of Japan can control Money supply, people in Bank of Japan typically believe that they rather accurately control a short term nominal interest rate. In fact, the bank of Japan actually pledge to keep nominal interest rates at around 0 (or minus) until recently. Let us discuss what would happen if the nominal interest rate is 0 in the long run.
- **Zero interest rate policy in the long run:** Zero interest rate policy is likely to induce deflation in the long run. Because of Fisher equation,

$$
\begin{aligned}
0 &= \rho^r + g_P^e \\
g_P^e &= -\rho^r < 0
\end{aligned}
$$

  - Key observation is that the real interest rate is not influenced by monetary policy in the long run. As we discuss later, in the short run, lowering nominal interest rate lowers real interest rate, and, therefore, encourages investment. But, as real interest rate is constant in the

Discussion

- **Friedman Rule:** Friedman argues that an optimal nominal interest rate is 0. In this case, real interest rate is equivalent to deflation and the money supply must be adjusted to satisfy $\frac{M^s}{P} = k(0)\,Y$. Because the opportunity cost of holding money is 0, as far as the price system works well, everybody can enjoy the benefits of money without cost. What is wrong with this?
- **Cost of Deflation:** There are two potential problems on Friedman rule.
  1. Friedman rule assumes that everybody accepts a reduction in nominal wage payment. If not, we should observe more unemployed workers or bankrupt companies.
  2. Because debt is based on nominal value, lower nominal wage means that debtors will face a difficulty in paying their interest.
     - In Japan, the largest debtor is government. Hence, government must suffer from the cost of deflation, which causes higher tax rate in future.

Assignment

- Students must hand assignment 5 in at the next lecture.

Adjustment Cost of Investment and Stabilization Policy

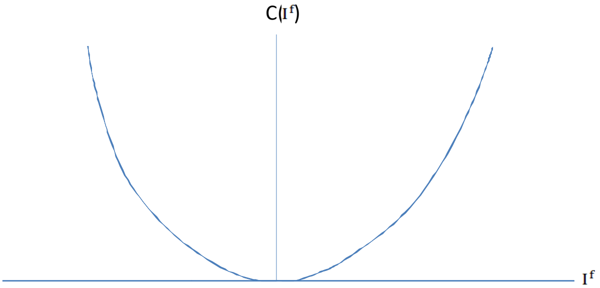
- So far I assumed that firms can instantaneously make any investment. This may be an innocuous assumption in the long run. However, the assumption may be questionable in a short run problem. This section introduces the adjustment cost of investment.
- When there is no adjustment cost of investment, one unit of output is transferred to produce one unit of investment. Hence the value of capital is also equal to 1.
- When the cost of investment exists, the value of capital deviates from one. Assume that q is the marginal value of capital and $C\left(I^f\right)$ is the adjustment cost of investment. The firm's investment problem can be written as follows.

$$\max_{I^f} qK' - I, \; I = I^f + C\left(I^f\right)$$

$$K' = I^f + (1-\delta) K$$

where $C(0) = C'(0) = 0$, $C'\left(I^f\right) > 0$ and $C''\left(I^f\right) > 0$.

Adjustment Cost of Investment



Adjustment Cost of Investment and Stabilization Policy

- The first order condition is

$$q = C'\left(I^f\right) + 1$$

Note that this equation implies that if there is no adjustment cost, the value of capital, q, is equal to 1. The marginal value capital, q, is called the marginal q.

- Because $C'\left(I^f\right)$ is large when I^f is large,

$$(q-1)\uparrow \Rightarrow C'\left(I^f\right)\uparrow \Rightarrow I^f\uparrow \Rightarrow I\uparrow$$

- Hence investment can be written as an increasing function of $q-1$.

$$I = \psi(q-1),\ \psi'(\cdot) > 0$$

Detour to the Q theory of Investment

- Q **Theory of Investment:** With an additional technical condition on the adjustment cost ($C\left(I^f\right) = C^*\left(I^f, K\right)$ and C^* is CRSs in I^f and K), it is known that the marginal q can be estimated by the Tobin's Q where

$$q = Tobin's\ Q \equiv \frac{Market\ Value\ of\ a\ Firm}{Replancement\ Cost\ of\ Capital\ Stock}$$

and there is $\psi^*(q-1)$ such that

$$I = \psi(q-1) = \psi^*(q-1)K.$$

  - It means that if the market value of firm is greater than replacement cost of capital, then the firm should invest, if not, it should sell some assets. In this way, Tobin's Q can be considered as the proxies of the importance of the expected investment opportunities. This is called the Q theory of investment.

- **What influences q?** Let us maintain the current assumption $\left(C\left(I^f\right)\right.$ does not depend on $\left.K\right)$ and discuss what influences the expected investment opportunities. If you have one unit of capital stock in your hand, you can sell it and earn q_t. If you save q_t in your bank, your account at the next year would be $(1+\rho)\,q_t$. On the other hand, if you keep the capital, you expect to receive rental price r^e_{t+1} at date $t+1$ and you expect to sell the capital in the next year by the price q^e_{t+1}. Because δ portion of capital is assumed to be depreciated, you will expect to earn $q_{t+1}(1-\delta)$ by selling the capital. Hence, an arbitrage condition implies

$$\begin{aligned}(1+\rho)\,q_t &= r^e_{t+1} + q^e_{t+1}(1-\delta)\\ r^e_{t+1} &= E\left[\phi\left(h\left(w_{t+1}\right)\right) - \phi'\left(h\left(w_{t+1}\right)\right)h\left(w_{t+1}\right)\right]\end{aligned}$$

  - Hence, the expectation on the future marginal productivity of capital must influence the expected investment opportunities.

- Assume that there is no capital gain and loss for simple analysis and economy is stationary:

$$q_{t+1} = q_t = q,\ \ r^e_{t+1} = r^e = E\left[\phi\left(h\left(w\right)\right) - \phi'\left(h\left(w\right)\right)h\left(w\right)\right]$$

Then

$$\begin{aligned}
(1+\rho)\,q &= r^e - \delta q + q \\
q &= \frac{r^e}{\rho+\delta} \\
r^e &= E\left[\phi\left(h\left(w\right)\right) - \phi'\left(h\left(w\right)\right)h\left(w\right)\right]
\end{aligned}$$

Adjustment Cost of Investment and Stabilization Policy

- Hence

$$I = \psi\left(\frac{r^e}{\rho+\delta} - 1\right), \; r^e = E\left[\phi\left(h\left(w\right)\right) - \phi'\left(h\left(w\right)\right)h\left(w\right)\right]$$

- Note that

$$\begin{aligned} r^e - \delta &= \rho \Rightarrow \frac{r^e}{\rho+\delta} = 1 \Rightarrow I = 0 \\ r^e - \delta &> \rho \Rightarrow \frac{r^e}{\rho+\delta} > 1 \Rightarrow I > 0 \\ r^e - \delta &< \rho \Rightarrow \frac{r^e}{\rho+\delta} < 1 \Rightarrow I < 0 \end{aligned}$$

Hence, the long run condition can be seen as the situation that any adjustment is finished.

Adjustment Cost of Investment and Stabilization Policy

- Define $\iota(\cdot)$ such that

$$i = \frac{\psi\left(\frac{r^e}{\rho+\delta} - 1\right)}{N} \equiv \iota(\rho), \ \iota(r^e - \delta) = 0, \ \iota'(\rho) < 0.$$

  - **Remark:** This function ignores that the expected investment opportunities influence investment decision. Hence, the following analysis presume that the expected future returns on the investment $r^e = E[\phi(h(w)) - \phi'(h(w))h(w)]$, does not influence investment.

- Because $r^e - \delta \neq \rho$ in general, the marginal product of capital, $\phi(h(w)) - \phi'(h(w))h(w)$, does not determine the interest rate.

Adjustment Cost of Investment and Stabilization Policy

- Supply Side determines (y, w)

$$\begin{aligned} y &= \phi(h(w))\,T \\ w &= \phi'(h(w))\,T \end{aligned}$$

- Demand Side determines (ρ, P)

$$\begin{aligned} y &= c + \iota(\rho) + \theta\hat{g} \\ \frac{m^s}{P} &= k(\rho)\,y \end{aligned}$$

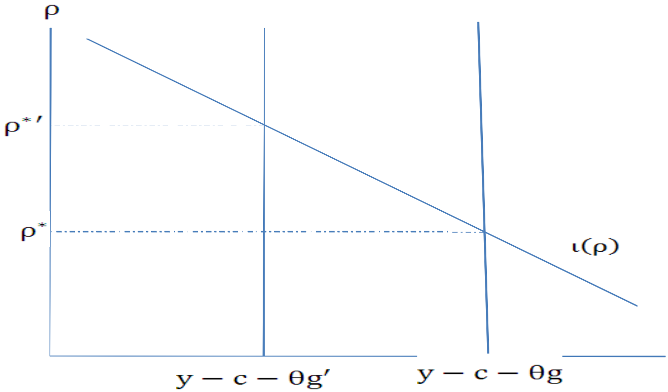
Adjustment Cost of Investment and Stabilization Policy

- Supply side conditions still determine GDP and the wage rate. Hence, fiscal policy and monetary policy has no impact on GDP and the wage rate.
  1. As an increase in government expenditure cannot change output, it reduces private expenditure.
  2. Money supply simply raises the price level. Hence, money is still neutral, too.
- The main difference is that fiscal policy can change the real interest rate. Because an increase in government expenditure reduces fund for the private investment. Hence, the real interest rate increases. To understand this, we can rewrite

$$
\begin{aligned}
\iota(\rho) &= y - c - \theta \hat{g} \\
&= y - c - \hat{g} + (1-\theta)\hat{g} \\
&= y - \tau^n + tr^h - c = s^g
\end{aligned}
$$

Adjustment Cost of Investment and Stabilization Policy

g’>g

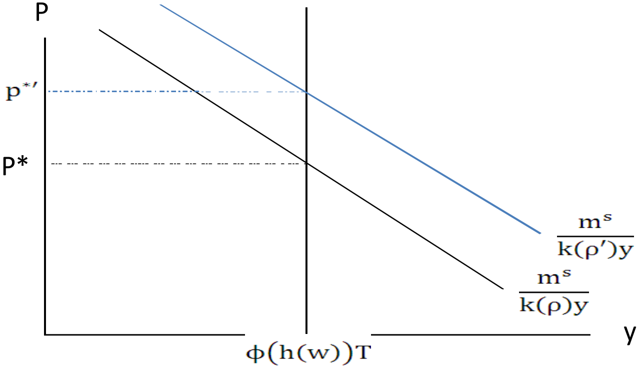


Adjustment Cost of Investment and Stabilization Policy

- Since a rise in government expenditure raises the interest rate, it raises the opportunity cost of money holding. Therefore, it reduces money demand. It means that money supply becomes larger than money demand. Hence, the price of money, $\frac{1}{P}$, goes down. In another word, we have inflation.

$$\rho \uparrow \Rightarrow k(\rho) \downarrow \Rightarrow k(\rho) y \downarrow \Rightarrow \frac{m^s}{P} \downarrow \Rightarrow P \uparrow$$

$$\rho' > \rho$$



Adjustment Cost of Investment and Stabilization Policy

Theorem

A permanent increase in public expenditure (which only affect demand side) will be offset by a permanent reduction of private expenditure in the long run. When the adjustment of investment is slow, it also raises the real interest rate and causes inflation in the long run. An increase in money supply causes inflation, but does not affect real variables in the long run.

- I define the short run as the period during which l differs from equilibrium working hours in the long run equilibrium $(h(w^*))$:

$$l \neq h(w^*)$$

where w^* is an equilibrium wage.

- We further assume that actually employed hours, e, are an increasing function of the price level:

$$l = e(P), \ e'(P) > 0$$

  - When an increase in demand raises output price, firms can make more profits by employing labor.

Stabilization Policy in the Short Run

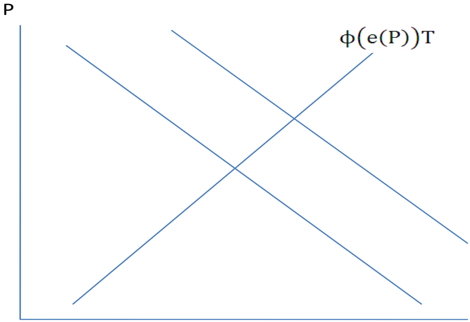
- Supply Side

$$\begin{aligned} y &= \phi\left(e\left(P\right)\right)T \\ w &= \phi'\left(e\left(P\right)\right)T \end{aligned}$$

- Demand Side

$$\begin{aligned} y &= c + \iota\left(\rho\right) + \theta\hat{g} \\ \frac{m^s}{P} &= k\left(\rho\right)y \end{aligned}$$

Aggregate Demand and Aggregate Supply in the Short Run

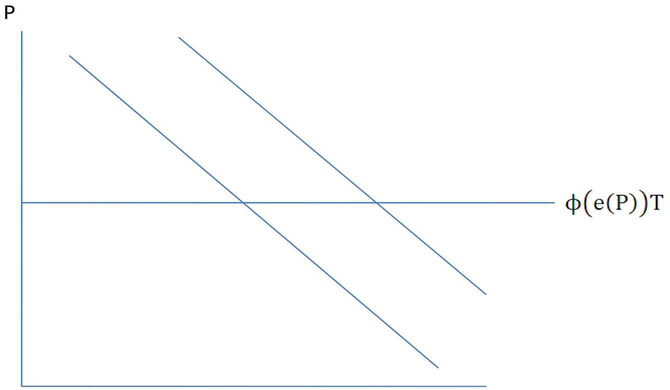


- Suppose P is given. Then the demand side two equations determine y and ρ.

$$\begin{aligned} IS &: \quad y = c + \iota(\rho) + \theta \hat{g} \\ LM &: \quad \frac{m^s}{P} = k(\rho)\, y \end{aligned}$$

Two equations describe IS curve and LM curve. IS is the abbreviation of Investment and Saving. LM is the abbreviation of Liquidity and Money.

Aggregate Demand and Aggregate Supply when P is given



Keynesian Cross

- Assume that P and ρ are given. The IS equation determines y.

$$y = c + i + \theta\hat{g}$$

  where $i = \iota(\rho)$.

- Suppose that

$$c = c\left[y + tr^h - \tau^n\right],\ c'[\cdot] \in (0.1)$$

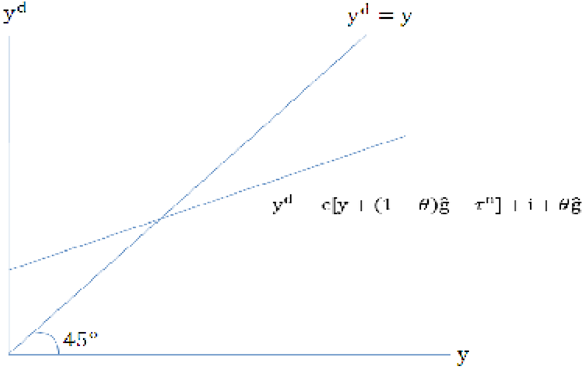
  where $tr^h = (1-\theta)\,\hat{g}$. This assumption implies that consumption depends only on the current disposable income.

- Define the planned expenditure, y^d as follows.

$$y^d = c\left[y + (1-\theta)\,\hat{g} - \tau^n\right] + i + \theta\hat{g}$$

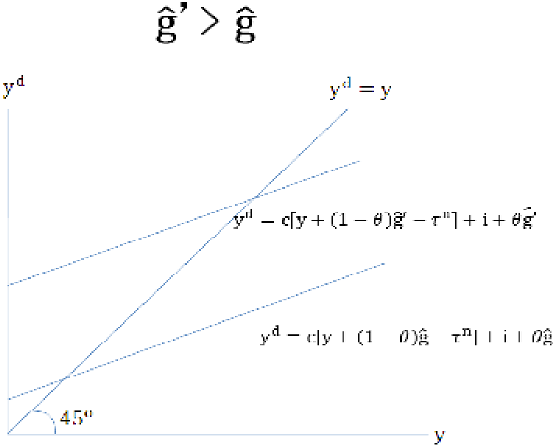
  Then IS equation is satisfied when $y = y^d$.

Cross



$_2$1.*png*

Cross$_g$



Keynesian Cross

- Example,

$$c = c_0 + c_1 * (y + (1-\theta)\,\hat{g} - \tau^n)$$

Then

$$\begin{aligned} y^d &= c_0 + c_1 * (y + (1-\theta)\,\hat{g} - \tau^n) + i + \theta\hat{g} \\ &= c_0 + c_1 * (y - \tau^n) + i + [c_1 * (1-\theta) + \theta]\,\hat{g} \end{aligned}$$

$$\begin{aligned} y &= y^d = c_0 + c_1 * (y - \tau^n) + i + [c_1 * (1-\theta) + \theta]\,\hat{g} \\ &= \frac{1}{1-c_1}\left\{c_0 - c_1 * \tau^n + i + [c_1 + (1-c_1)\,\theta]\,\hat{g}\right\} \end{aligned}$$

- Multiplier Effect

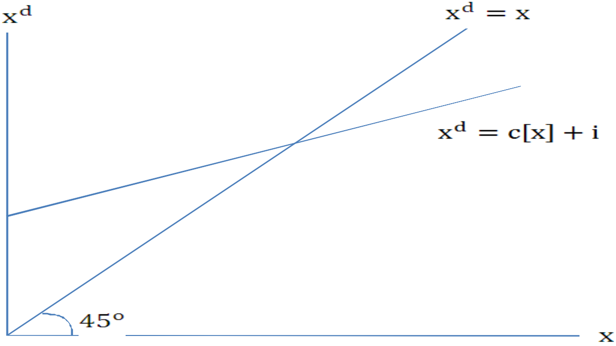
$$\frac{dy}{d\hat{g}} = \frac{c_1}{1-c_1} + \theta > \theta$$

- Note that we have assumed that government keeps a primary balance of zero, $\hat{g} = \tau^n$. In this case,

$$\begin{aligned} y^d &= c\left[y + (1-\theta)\hat{g} - \tau^n\right] + i + \theta\hat{g} \\ &= c\left[y + (1-\theta)\hat{g} - \hat{g}\right] + i + \theta\hat{g} \\ &= c\left[y - \theta\hat{g}\right] + i + \theta\hat{g} \\ x^d &= c\left[x\right] + i \end{aligned}$$

where $x = y - \theta\hat{g}$ and $x^d = y^d - \theta\hat{g}$, where x is the disposal income and x^d is the planed private expenditure. It means that goods market clearing condition is satisfied when $x = x^d$.

Keynesian Cross



Keynesian Cross

- Because $x = y - \theta\hat{g}$,

$$y = \theta\hat{g} + x$$

- Multiplier Effect: (because x is independent of $\hat{g}$)

$$\frac{dy}{dg} = \theta$$

IS Curve

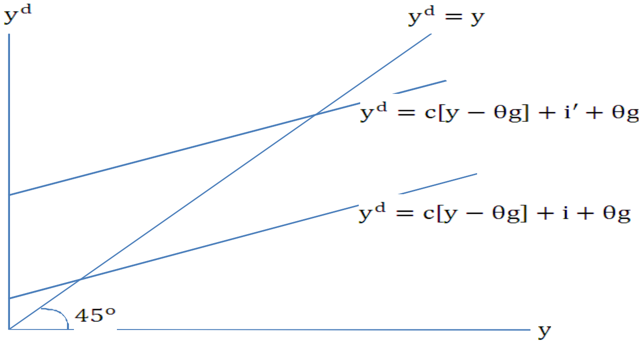
- Suppose that P is given. Then IS equation determines the relationship between y and ρ.

$$y = c + \iota(\rho) + \theta\hat{g}$$

- In order to understand the relationship, note that Keynsian Cross predicts that an increase in i results in an increase in y.

IS Curve

i’>i



IS Curve

- Note also that

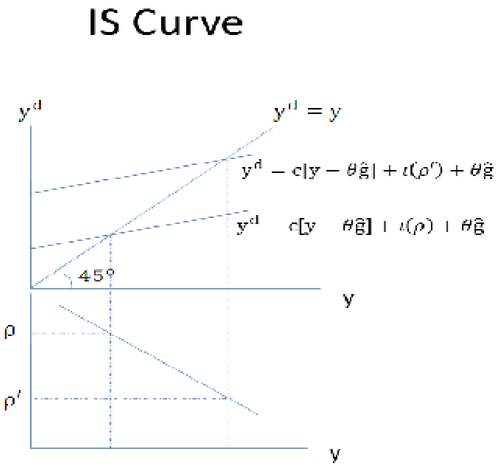
$$i = \iota' (\rho) < 0, \ \rho \uparrow \Rightarrow i \downarrow$$

- Hence

$$\rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow$$

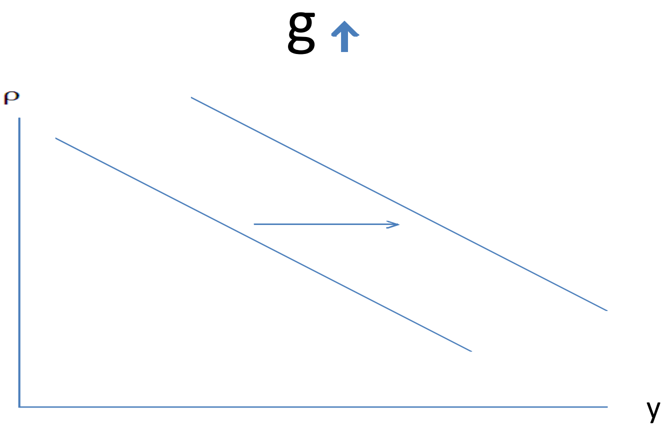
This relationship can be depicted by IS curve.

IS Curve


IS Curve
y^d
$y^d = y$
$y^d = c[y - \theta\hat{g}] + i(\rho') + \theta\hat{g}$
$y^d = c[y - \theta\hat{g}] + i(\rho) + \theta\hat{g}$
45°
y
ρ
ρ'
y

IS Curve

- A shift in IS curve. Note that for any given ρ, $\frac{dy}{dg} > 0$. Hence, an increase in $\hat{g}$ shifts the IS curve to the left.

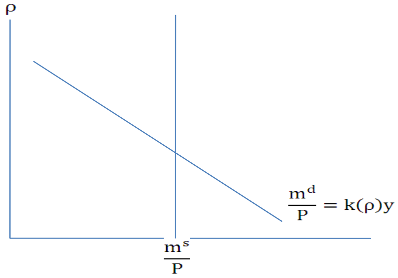


Money Market

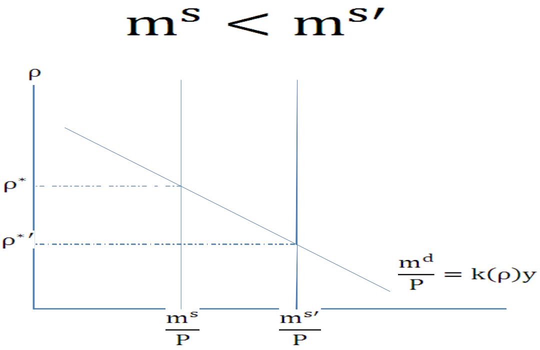
- When the price P is given, money market determines interest rate ρ.

$$LM : \frac{m^s}{P} = \frac{m^d}{P} = k(\rho)\, y$$

Money Market in the Short Run



- In the short run, an increase in money supply lowers interest rate.



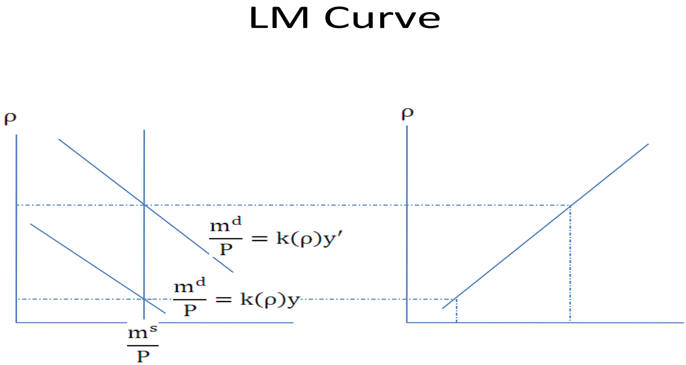
LM Curve

- For example, suppose that $k(\rho) = \frac{k}{\rho}$. Then

$$\frac{m^s}{P} = k(\rho)\, y = \frac{ky}{\rho}$$
$$\rho = \frac{kPy}{m^s}$$

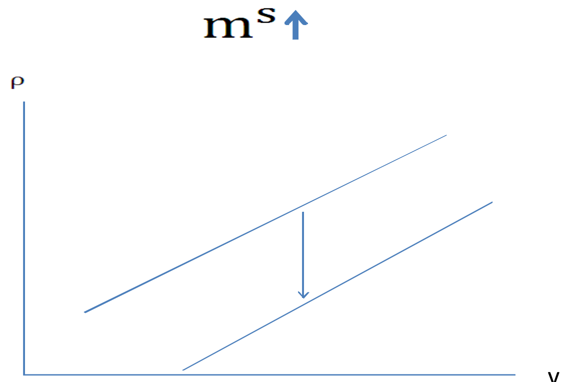
Hence, there is a negative relationship between m^s and ρ.

- When P is given, LM equation also suggest a positive relationship between y and ρ. This relationship is described by LM curve.



LM Curve

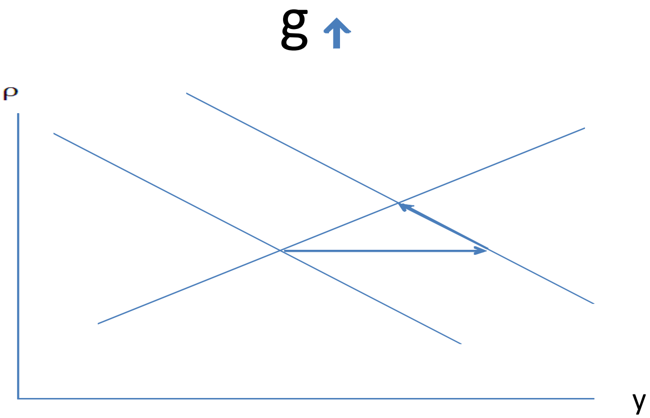
- A shift in LM curve: Because $\frac{d\rho}{dm^s} < 0$, an increase in m^s shifts LM curve down.



IS-LM analysis

- The impacts of a fiscal policy

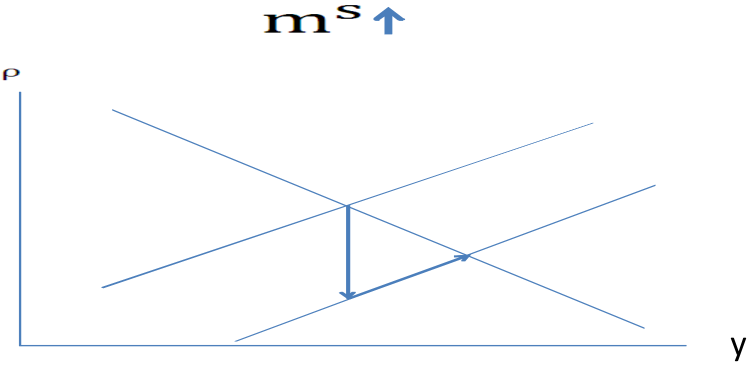
$$\hat{g} \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow \ldots$$



IS-LM analysis

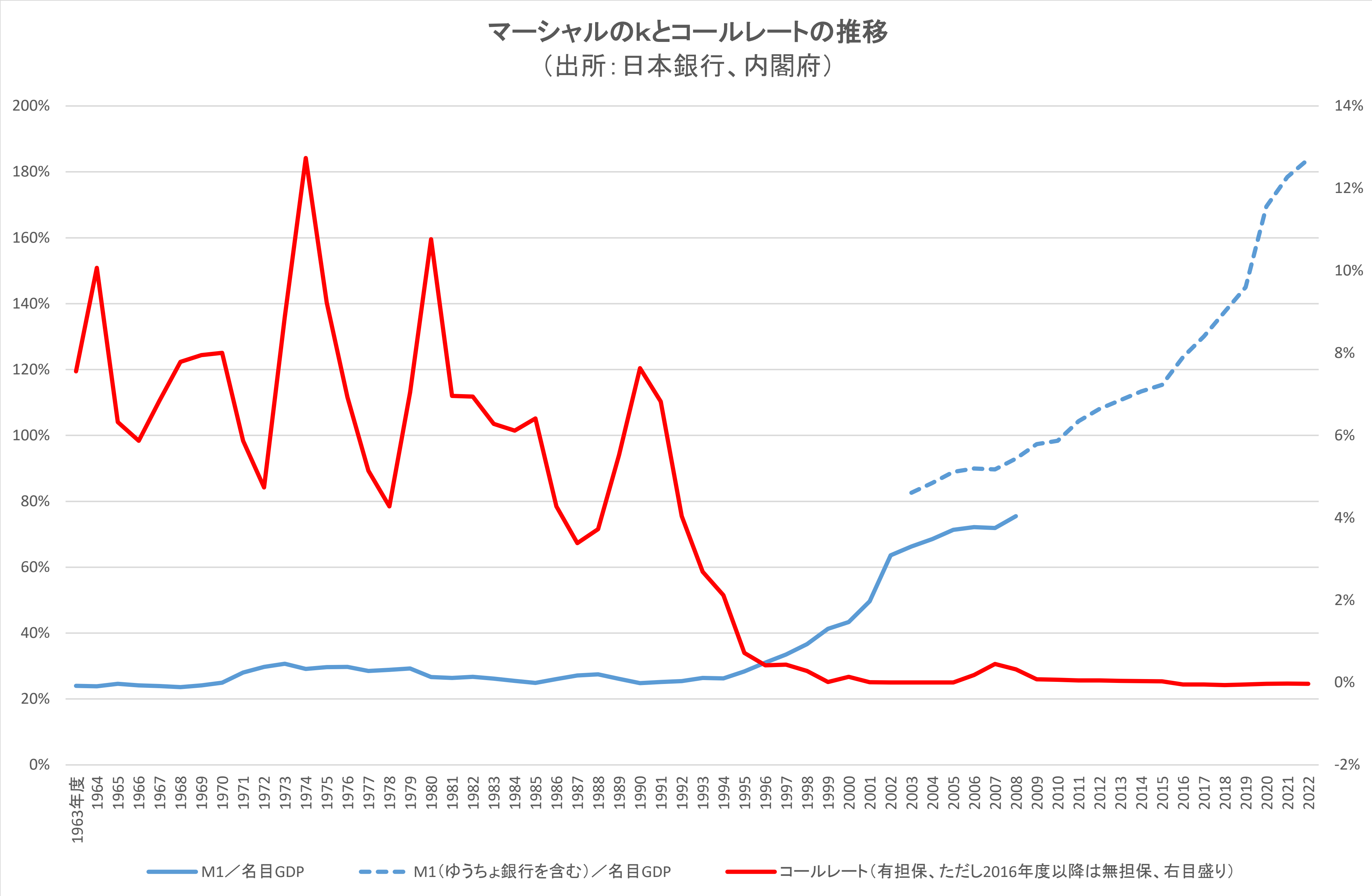
- The impacts of a monetary policy

$$m^s \uparrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow \Rightarrow y \uparrow \Rightarrow m^d \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow \ldots$$

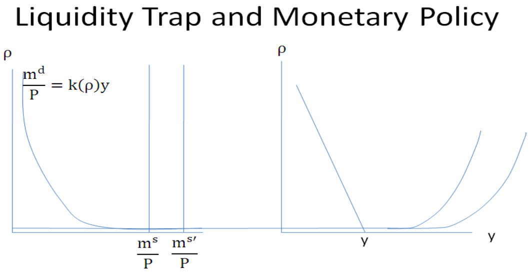


IS-LM analysis

- **Liquidity Trap:** When nominal interest rate is near 0, money demand can be infinite, $k(0) = \infty$. In this case, an increase in money supply cannot reduce nominal interest rate furthermore. Hence, LM curve becomes flat. This situation is called liquidity trap.

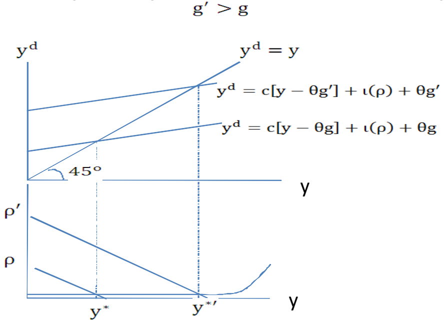


マーシャルのkとコールレートの推移
（出所：日本銀行、内閣府）
200%
180%
160%
140%
120%
100%
80%
60%
40%
20%
0%
14%
12%
10%
8%
6%
4%
2%
0%
-2%
1963年度
2022
M1／名目GDP
M1（ゆうちょ銀行を含む）／名目GDP
コールレート（有担保、ただし2016年度以降は無担保、右目盛り）



- If the interest rate is 0, monetary policy has no impacts on real economy. Then what is the purpose of zero nominal interest rate policy?

Liquidity Trap and Fiscal Policy



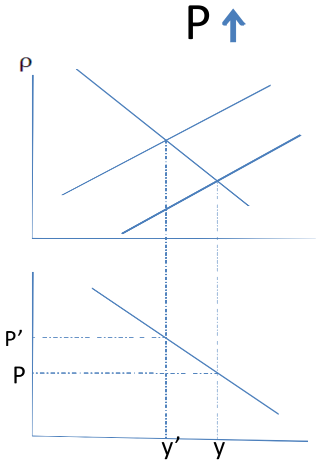
- If the interest rate is 0, a fiscal policy has large impacts on real economy. Then should we have recommended an expansionary fiscal policy during the zero nominal interest rate policy?

AS-AD analysis

- Note that
$$\rho = \frac{kPy}{m^s} \Rightarrow \frac{d\rho}{dP} > 0.$$
- Hence,
$$P \uparrow \Rightarrow \rho \uparrow \Rightarrow i \downarrow \Rightarrow y \downarrow \Rightarrow m^d \downarrow \Rightarrow \rho \downarrow \Rightarrow i \uparrow \Rightarrow y \uparrow \ldots$$

  It shows that the overall relationship between P and y is negative. This relationship is depicted by AD curve.

AS-AD analysis



AS-AD analysis

- A shift in AD curve. Note that for any given P, IS-LM analysis suggests that

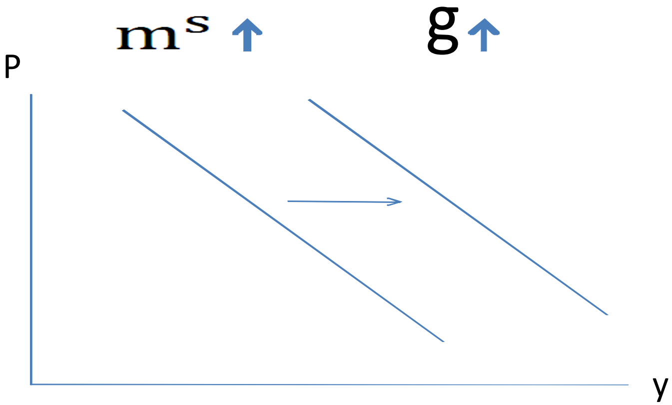
$$\hat{g} \uparrow \Rightarrow y \uparrow, \;\; m^s \uparrow \Rightarrow y \uparrow$$

  Hence, increases in government expenditure or/and in money supply shifts AD curve to the right.

AS-AD analysis

Theorem

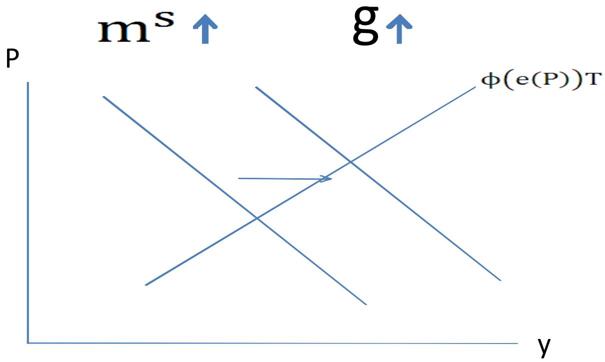
An increase in government expenditure or money supply raises aggregate demand in general.



AS-AD analysis

Theorem

An increase in government expenditure or money supply causes inflation and raises GDP in the short run.



The Short Run Aggregate Supply Curve

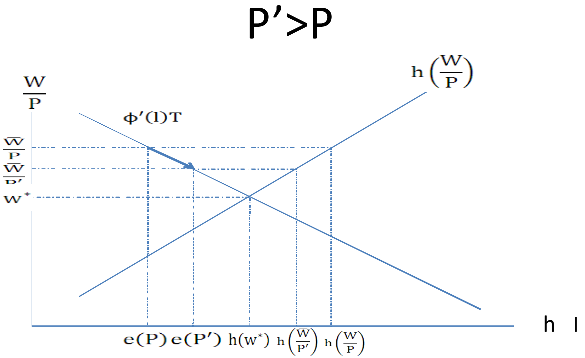
- As I defined before, the short run is the period the number of employed workers differs from its long run equilibrium level. I also assume that the number of employed workers is increasing in price level in the short run.
  1. Why does it differ from the equilibrium level in the short run?
  2. How can I derive a positive employment function?.
- I introduce two prototype models, which illustrate their main idea.

The Short Run Aggregate Supply Curve

- **Sticky Nominal Wage Model:**
- Let me assume that the nominal wage is fixed in the short run: $W = \bar{W}$. There might be several reasons for this assumption. For example, the bargaining process with labor union may prevent a reduction of nominal wage. For several contractual reasons, it may be difficult to change the nominal wage quickly.
- Since labor demand curve is a decreasing in the real wage, if the nominal wage is rigid, labor demand must be an increasing in P:

$$P \uparrow \Rightarrow w = \frac{\bar{W}}{P} \downarrow \Rightarrow \phi'(l)\,T \downarrow \Rightarrow l \uparrow$$

  Note that $\phi'(l_e)$ is decreasing in l_e.
- Note that because lowering nominal wage is much more difficult than increasing it, firms have more difficulties in adapting to the deflation. This is one of a reason that economists prefer mild inflation to deflation and no inflation.


P'>P
h(W/P)
W/P
φ'(l)T
W̄/P
W̄/P'
w*
h l
e(P) e(P') h(w*) h(W̄/P') h(W̄/P)

The Short Run Aggregate Supply Curve

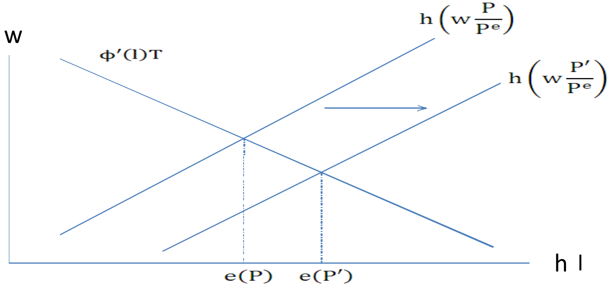
- **Imperfect Information Model:** Another possible reason on an increasing aggregate supply curve is the imperfection of information. There are several variants of imperfect information model. I describe a worker-misperception model to convey its main intuition.
- Assume that firms observe output price P, but workers cannot. Hence, workers must make their inferences about price. Workers' expected price is denoted as P^e. Then workers respond to $\frac{W}{P^e}$.

$$h\left(\frac{W}{P^e}\right) = h\left(\frac{W}{P}\frac{P}{P^e}\right) = h\left(w\frac{P}{P^e}\right)$$

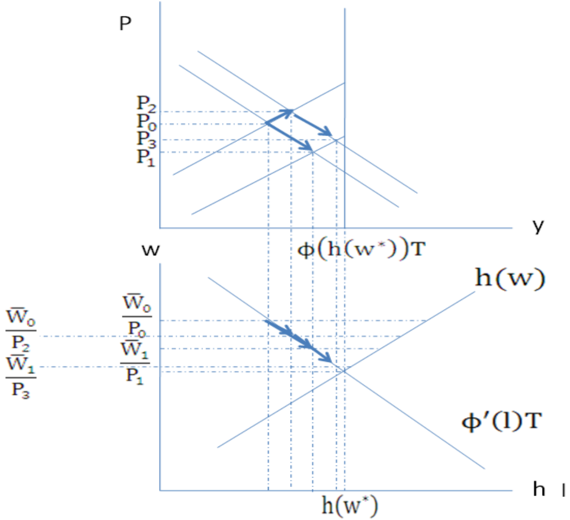
- Suppose that the overall price level P goes up. However, workers do not know a change in aggregate price. Hence, P^e stays the same. Hence, $\frac{P}{P^e}$ goes up and supply curve shifts to right.

The Short Run Aggregate Supply Curve

P'>P

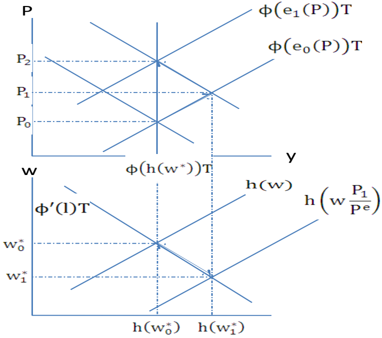


Adjustment under a Sticky Nominal Wage Model



From the Short Run to Long Run

Adjustment under a Worker-Misperception Model



Discussions

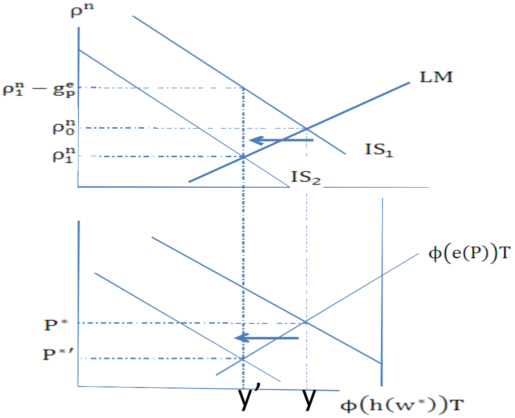
- Both models derives the short run aggregate supply curve. However, policy implication may change.
  1. Sticky nominal wage model brings unemployment; the imperfect information model does not. When nominal wage is sticky, there are workers who are willing to work with lower wage. When information is imperfect, all workers and firms agree on the market price given their perception.
  2. Policy implication differs. Because there is unemployed workers under sticky nominal wage model, an increase in demand can increase GDP by employing more workers. However, if the imperfect information model is correct, there are no unemployed workers. If active stabilization policy itself brings the uncertain movement of price, it may increase workers' further misperception. It reduces the welfare of an economy.

Deflationary Spirals

- Japanese economy stagnated more than 10 years. It is difficult to believe that only sticky price and noisy information can explain such a long term stagnation.
- There is one possibility that a short run deviation can be longer than one predicted by the above mechanism: DEFLATIONARY SPIRALS. Suppose that people start to expect a deflation, $g_P^e < 0$. Because investment decisions depend on the real interest rate, IS equation changes from IS_1 to IS_2.

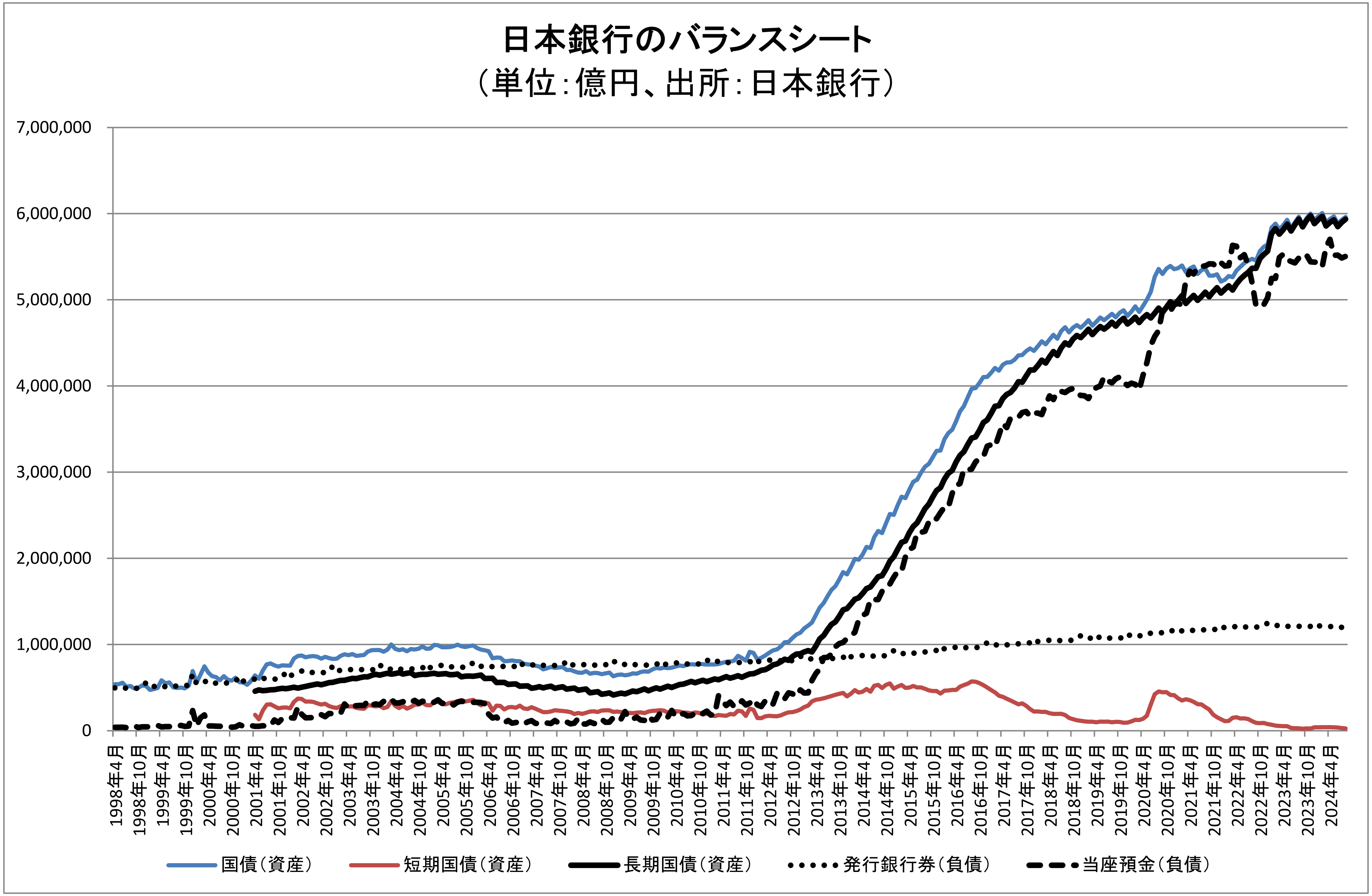
$$
\begin{aligned}
IS_1 &: \quad y = c + \iota\left(\rho^n + \delta\right) + \theta \hat{g} \\
IS_2 &: \quad y = c + \iota\left(\rho^n - g_P^e + \delta\right) + \theta \hat{g}
\end{aligned}
$$

DEFLATIONARY SPIRALS



Zero Interest Rate Policy

1. The Zero Interest Rate Policy (ZIRP) is a monetary policy in which the Bank of Japan (BOJ) lowers its policy rate to around 0% to reduce borrowing costs for firms and households and to stimulate economic activity.
    1. It was first introduced in 1999, and although Japan occasionally exited from it, the policy effectively continued until 2024.
    2. Later, the BOJ combined the ZIRP with Quantitative Easing (QE) and the Negative Interest Rate Policy (NIRP) to further strengthen its monetary stimulus.
2. Main objectives:
    1. Encourage investment and consumption by maintaining extremely low interest rates.
    2. Achieve economic recovery and end deflation.



日本銀行のバランスシート
（単位：億円、出所：日本銀行）
7,000,000
6,000,000
5,000,000
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3,000,000
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1998年10月
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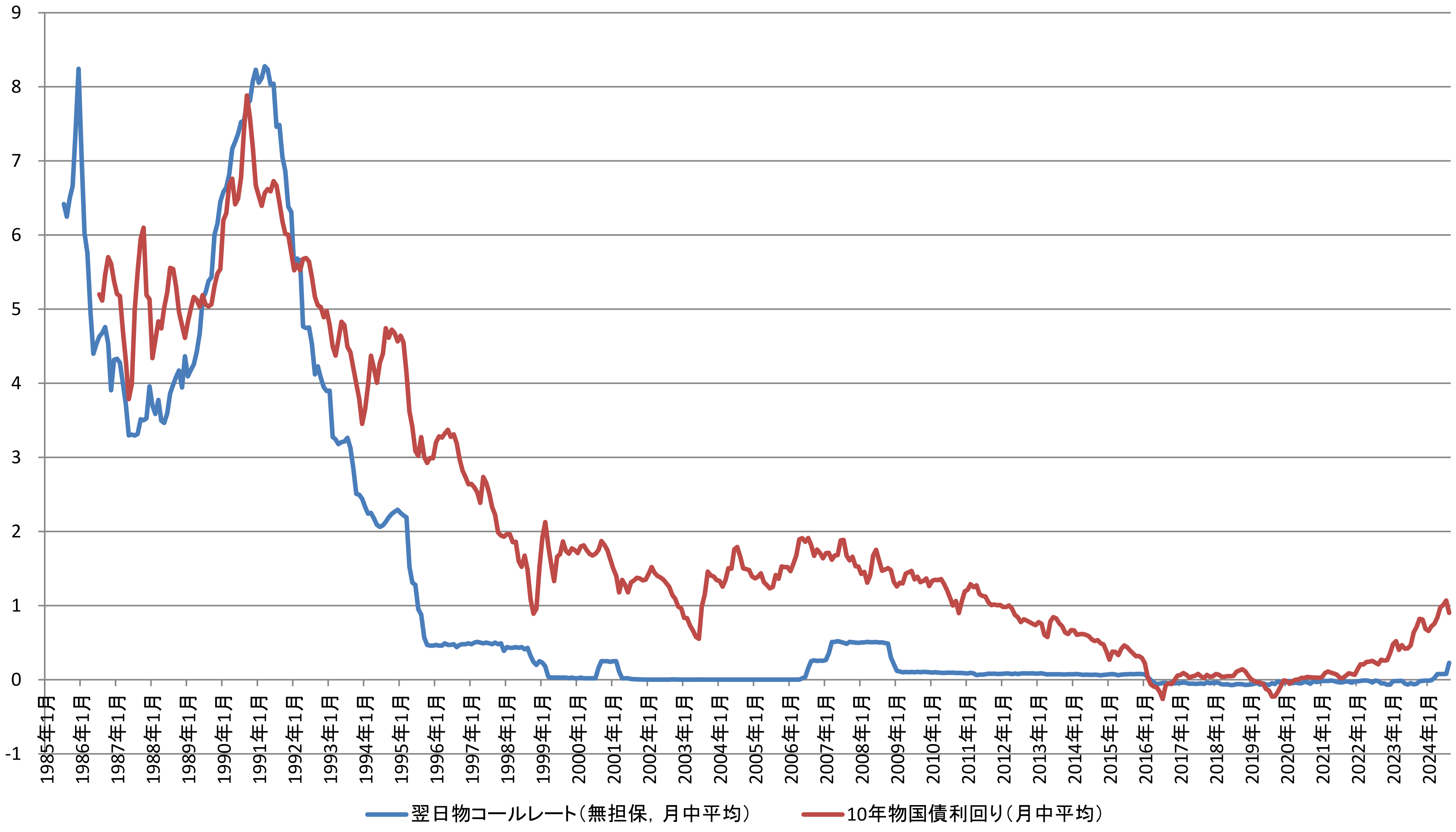
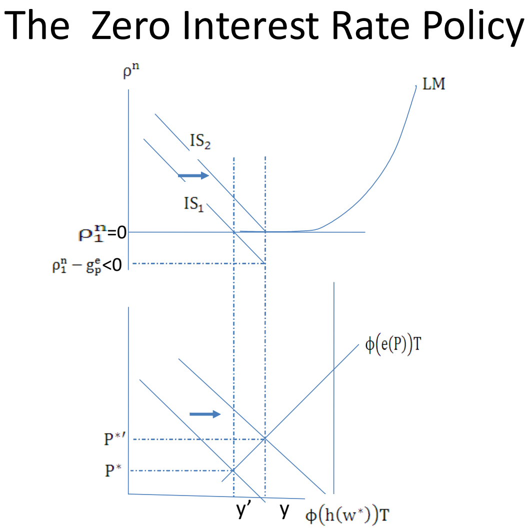
2022年4月
2022年10月
2023年4月
2023年10月
2024年4月
国債（資産）
短期国債（資産）
長期国債（資産）
発行銀行券（負債）
当座預金（負債）

日本の債券市場における長短金利の動向

（単位：%、出所：日本銀行、財務省）




The Zero Interest Rate Policy
ρn
LM
IS2
IS1
ρ1n=0
ρ1n − gpe<0
ϕ(e(P))T
P*′
P*
y'
y
ϕ(h(w*))T

Can Bank of Japan Influence the Expectation under zero Interest Rate?

- We have explained that the bank of Japan can potentially influence the expected inflation rate by committing future money supply: $g^e_{P,t} = \frac{E[\ln M_{t+1}] - \ln M_t + \gamma g^e_{P,t+1}}{1+\gamma}$. We have also discussed that it is difficult to control future money supply. Then whether can Bank of Japan control the expected inflation rate?
- Quantitative and Qualitative Monetary Easing (QQE)
    - Introduced: April 2013 by Governor Haruhiko Kuroda
    - Two pillars:
        1. Commitment to a 2% inflation target
        2. Expansion of the monetary base through large-scale bond purchases
    - Outcome: Limited impact on ending deflation
    - In January 2016, BOJ introduced a –0.1% negative interest rate (effective from February 16, 2016)

Long Run Cost of Expansionary Fiscal and Monetary Policies

- AD-AS model suggests that as far as there is no inflation, there is no cost of expansionary fiscal and monetary policies.
- Remenber that AD-AS model ignores the interaction between now and future.
  1. An increase in I_t does not increase K_{t+1}.
  2. $r^e = E\left[MPK_{t+1}\right]$ does not influence I_t.
  3. θ is constant over time.
  4. The future income, $\left\{ \begin{array}{c} (1+\rho^a)\, a_{t+x} + \left(\frac{m^s}{P}\right)_{t+x} + w_{t+x} h\left(w_{t+x}\right) \\ -\tau_{t+x} + tr_{t+x} + tr^h_{t+x} \end{array} \right\}_{x>t}$, does not influence c_t.
  5. $b_{+1} = (1+\rho)\, b$.
- How do the relaxations of these assumptions affect the results in the long run?

Long Run Cost of Expansionary Fiscal and Monetary Policies

- When I increases K_{t+1} and $r^e = E\left[MPK_{t+1}\right]$ influences I_t
  - Expansionary monetary policy lowers real interest rate and stimulate investment, even if there are no large investment opportunities. Because MPK declines as more capital is accumulated, the returns from investment would be lower in the long run. In fact we observe an increase in $\frac{K}{Y}$ and the declining MPK after 1990 (e.g., Fukao (2012)). Ando (2002), Hayashi (2006) and Saito (2008) claim that there are the indications of over-investment.
  - Because continuous investment without promising investment opportunities enforce us to misallocate resources to unproductive use, it is likely to reduce aggregate productivity T. In facts, Nishimura et al. (2005), Cabarello et al. (2008) and Kwon et all. (2010) found that the indication of misallocation.
  - Note that our long run model suggests that the growth rate of T enhances sustainable growth.

Long Run Cost of Expansionary Fiscal and Monetary Policies

- θ is constant over time.
  - Although our current model does not take into account government investment, if a part of government expenditure is used as investment and the marginal productivity of government capital declines, expansionary fiscal policy reduces the return from expansionary fiscal policy. This effect is likely to have a similar impact as a reduction in θ.
- The same debt per capita $b_{+1} = (1+\rho)\, b$ and The future income, $\left\{ (1+\rho^a)\, a_{t+x} + \left(\frac{m^s}{P}\right)_{t+x} + w_{t+x} h\left(w_{t+x}\right) - \tau_{t+x} + tr_{t+x} + tr^h_{t+x} \right\}_{x>t}$, does not influence c_t.
  1. When the current expansionary fiscal policy increases future debt, b_{t+1}, people may believe the future tax burden, τ_{t+1}, increases or the reduction of the future transfer, tr_{t+1}, such as pension. When we expect a reduction in the future income, it is likely for people to save more and to reduce current consumption, c_t.

Long Run Cost of Expansionary Fiscal and Monetary Policies

- If these effects are more important, government must think about different policies. For example,
  1. If deregulation increases increases T, it increases in $\frac{K}{N}$ and $\frac{Y}{N}$ under steady state ($k^* = \frac{K}{TN}$ is constant) and $w = \phi'(l)\, T$.
  2. If there is a secured pension scheme, household may feel that they can spend more today and increases c_t, by reducing saving at household sector.

Long Run Cost of Expansionary Fiscal and Monetary Policies

- So, even if there is no inflation, we cannot say that there is no cost of expansionary monetary and fiscal policies. These considerations suggest that the expansionary monetary and fiscal policy (short run policies) must be accompanied with the policy that can enhance the future productivity (long run policies).
- Before a jump into an conclusion, because the existence of unemployed workers is the main reason for the demand stabilization policy, we need more careful examination of Japanese labor market.

Assignment

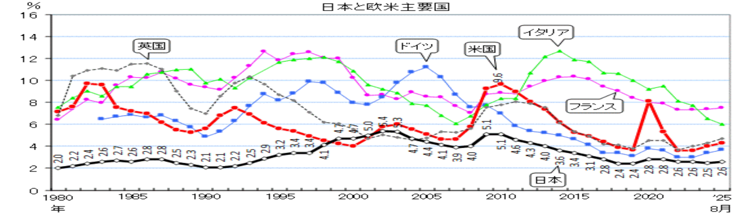
- Students must hand assignment 6 in at the next lecture.

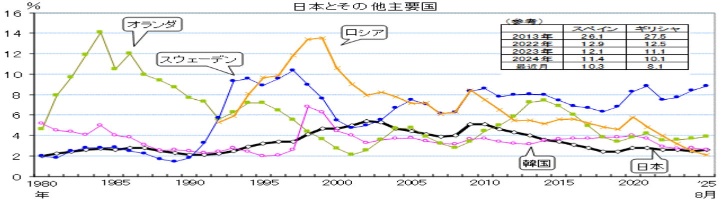
Stabilization Policy and Unemployment

- In order to understand the importance of stabilization policy, we need to understand why unemployment exists.
- When we look at unemployment rate in Japan, we can point out
  1. The unemployment exists even in the period of boom.
  2. Unemployment rate increases during a recession.
  3. Japanese unemployment rate is lower than other countries.

Stabilization Policy and Unemployment

失業率の推移(日本と主要国)





(数値%)

	日本	米国	韓国	英国	ドイツ	フランス	イタリア	ロシア	オランダ	スウェーデン
2022年	2.6	3.6	2.9	3.7	3.0	7.3	8.1	4.0	3.5	7.5
2023年	2.6	3.6	2.7	4.0	3.0	7.3	7.7	3.2	3.6	7.7
2024年	2.5	4.0	2.8	4.3	3.4	7.4	6.5	2.5	3.7	8.4
'25年8月	2.6	4.3	2.6	4.7	3.7	7.5	6.0	2.1	3.9	8.8
'25年8月以外の場合の月→				7月	7月		7月			

(注)最新の月次データを除き年平均。最新月は季節調整値。

(資料) 日本は総務省「労働力調査」、その他の国は、2015年までは世銀,WDI Online 2017.8.29、2018年以降の年・月は総務省「労働力調査」資料他による(フランス、オランダ、スウェーデン、スペイン、ギリシャはEUROSTAT,年次はLFS adjusted series、月次はHarmonised unemployment rateによる。ロシアはhttps://jp.tradingeconomics.com/による。

Stabilization Policy and Unemployment

- There are several reasons to believe that unemployed workers exist even in the long run. Economists call the unemployment rate in the long run *the natural rate of unemployment*. Unemployment in the short run can be seen the deviation of unemployment rate from the natural rate of unemployment.
- Unfortunately, the natural rate of unemployment may not be constant. Therefore, it is difficult to distinguish the short run deviation from the natural rate of unemployment.
- Depending on the estimation of the magnitude of the short run deviation, people have a different view on the effectiveness of stabilization policy.
- In this sense, we need to think about what determine the natural rate of unemployment.

The Natural Rate of Unemployment

- We derive the natural rate of unemployment by the framework of frictional unemployment.
  - Frictional unemployment occurs because finding a job takes time. It is not easy to find where a job offer is, what skill requirement of the job is and how much he expects to earn. It causes a temporal unemployment.
  - Frictional unemployment causes a serious problem when changing jobs requires changes in skills or locations. This type of unemployment can be separately called structural unemployment.

The Natural Rate of Unemployment

- Let me provide a theoretical model to explain the natural rate of unemployment:
  - Suppose that s_t fraction of employed workers are separated at year t.
  - Suppose that m_t fraction of unemployment workers meets a suitable job and leave from an unemployment pool.
- Assume that m_t is an increasing function of labor market tightness, Θ:

$$m_t = a_t q\left(\Theta_t\right), q'\left(\Theta_t\right) > 0,\ \Theta_t = \frac{v_t}{u_t}$$

where v is the efficiency of the matching function, v_t is the vacancy rate and u_t is the unemployment rate.

The Natural Rate of Unemployment

- The dynamics of the number of unemployed workers is

$$U_{t+1} = U_t + s_t E_t - a_t q\left(\Theta_t\right) U_t,$$

where U_t is the number of unemployed workers at date t and E_t is the number of employed workers at date t.

- As unemployed workers and employed workers consists of labor force,

$$N_t = E_t + U_t$$

where N denotes labor force. Hence

$$\begin{aligned} U_{t+1} &= U_t + s_t\left(N_t - U_t\right) - a_t q\left(\Theta_t\right) U_t \\ &= U_t + s_t N_t - \left(s_t + a_t q\left(\Theta_t\right)\right) U_t \end{aligned}$$

- Labor market tightness Θ_t can be influenced by short run fluctuation of business cycle, but the efficiency of matching function a_t are less likely to be influenced by business cycle. Therefore, the long run movement of matching function can be expressed by

$$m_t^l = a_t q\left(\Theta^*\right)$$

where Θ^* is the labor market tightness in the long run.

- Assume that

$$s_t = s_t^l + \varepsilon_t$$

where s_t^l is the long run separation rate and ε_t is error term.

- What might explain this long run trend of a_t and s_t^l?
  1. Firms always face changes in economic environment. The arrival of new technology may force firm to fire unskilled workers, firms may need to relocate their plants in foreign countries under the pressure of global competition. These changes may increase s_t^l.
  2. On the other hand, the arrival of new technology may create new types of jobs. But, it may require new skills. Or, in order for women to work in a job, they might need more flexible jobs. If these effects are important, it might increase mismatch, which will lower a_t.

The Natural Rate of Unemployment

- **Natural Rate of Unemployment:** I assume that the natural rate of unemployment is expressed by the steady state value of unemployment rate.
  - The steady state values of unemployed workers by the number of unemployed workers that equates the inflows into and outflows from unemployment pool.

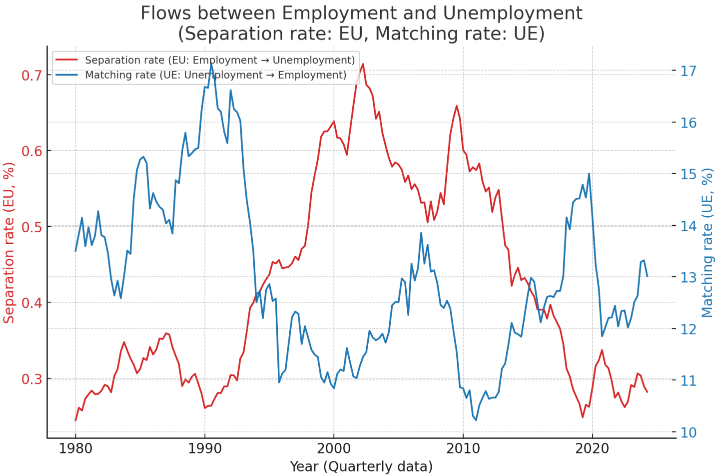
$$s_t^l N = \left(s_t^l + a_t q\left(\Theta^*\right)\right) U_t^n$$

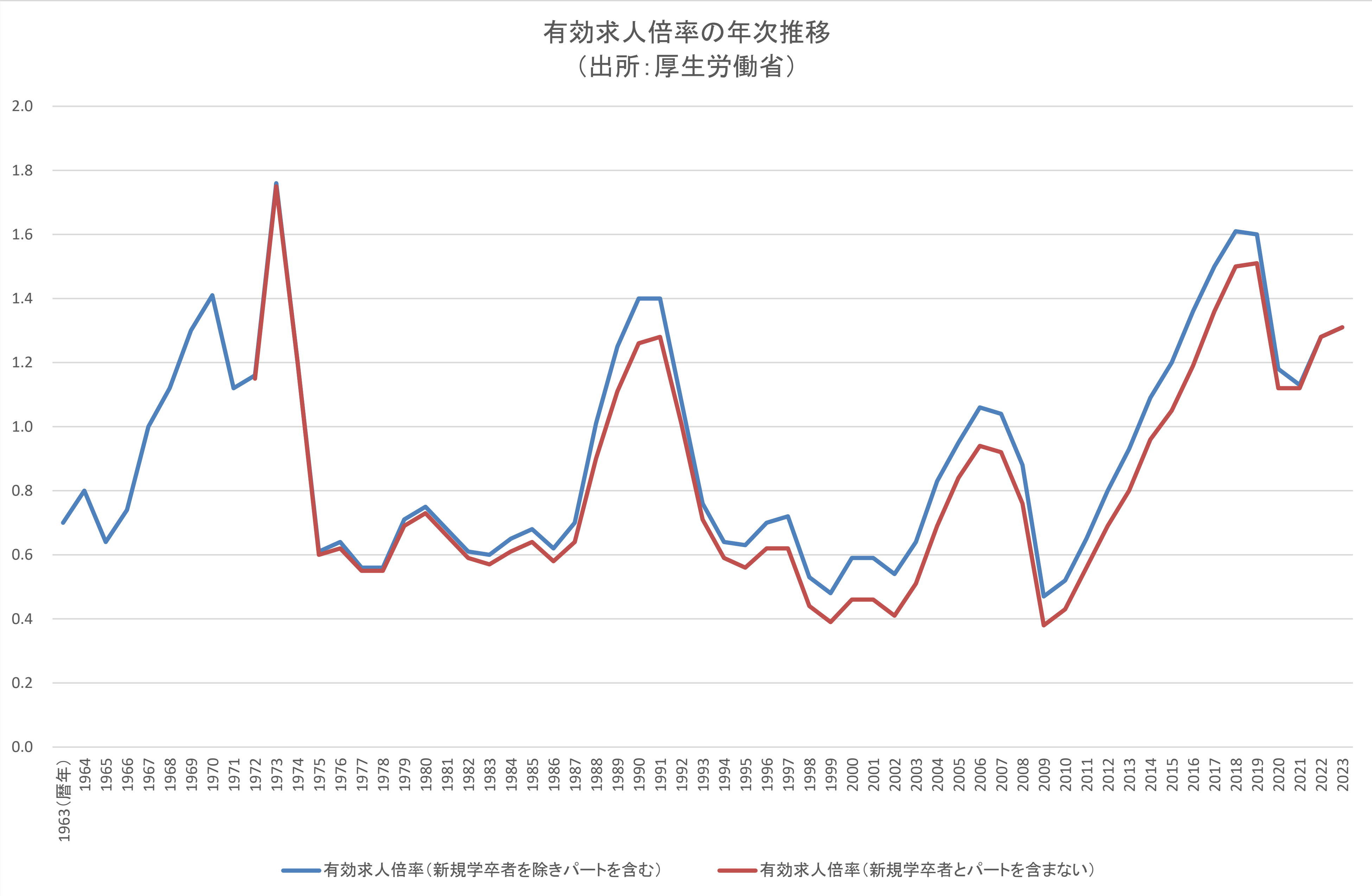
  - The natural rate of unemployment rate is

$$\begin{aligned} u_t^n &\equiv \frac{U_t^n}{N_t} \\ &= \frac{s_t^l}{s_t^l + a_t q\left(\Theta^*\right)} \end{aligned}$$

  - If matching probability is large, then unemployment rate is small: $a_t q\left(\Theta^*\right) \uparrow \Rightarrow u_t^n \downarrow$
  - If the separation rate is large, then unemployment rate is large: $s_t^l \uparrow \Rightarrow u_t^n \uparrow$

The Natural Rate of Unemployment





有効求人倍率の年次推移
（出所：厚生労働省）
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
1963（暦年）
1964
1965
1966
1967
1968
1969
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1971
1972
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2019
2020
2021
2022
2023
有効求人倍率（新規学卒者を除きパートを含む）
有効求人倍率（新規学卒者とパートを含まない）

The Natural Rate of Unemployment

- Assuming that a and s^l are constant, our model predicts the relationship between Θ and u.

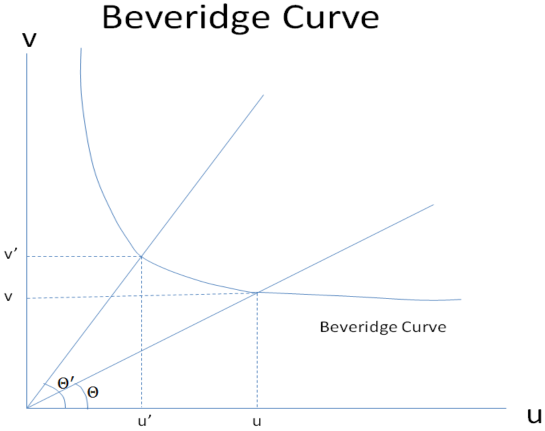
$$u = \frac{s^l}{s^l + aq(\Theta)}$$

  - An increasing in labor market tightness reduces the unemployment rate: $\Theta \uparrow \Rightarrow q(\Theta) \uparrow \Rightarrow u \downarrow$.

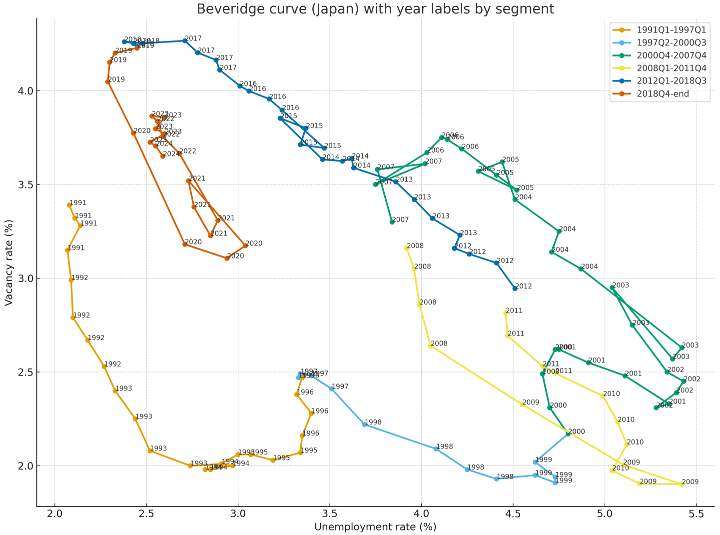
- This relationship brings theoretical predictions on the Beveridge curve around the steady state.

The Natural Rate of Unemployment

- Beveridge curve display the relationship between the vacancy rate and unemployment rate. Because u is a decrease in Θ, Beveridge curve is downward sloping.

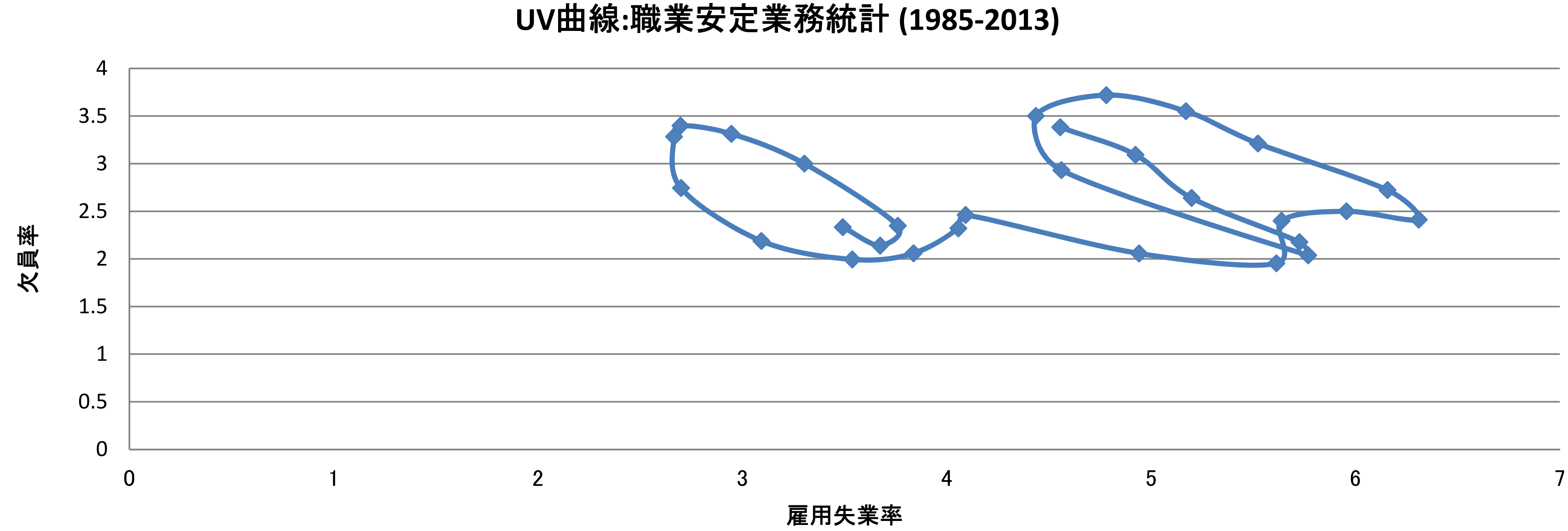


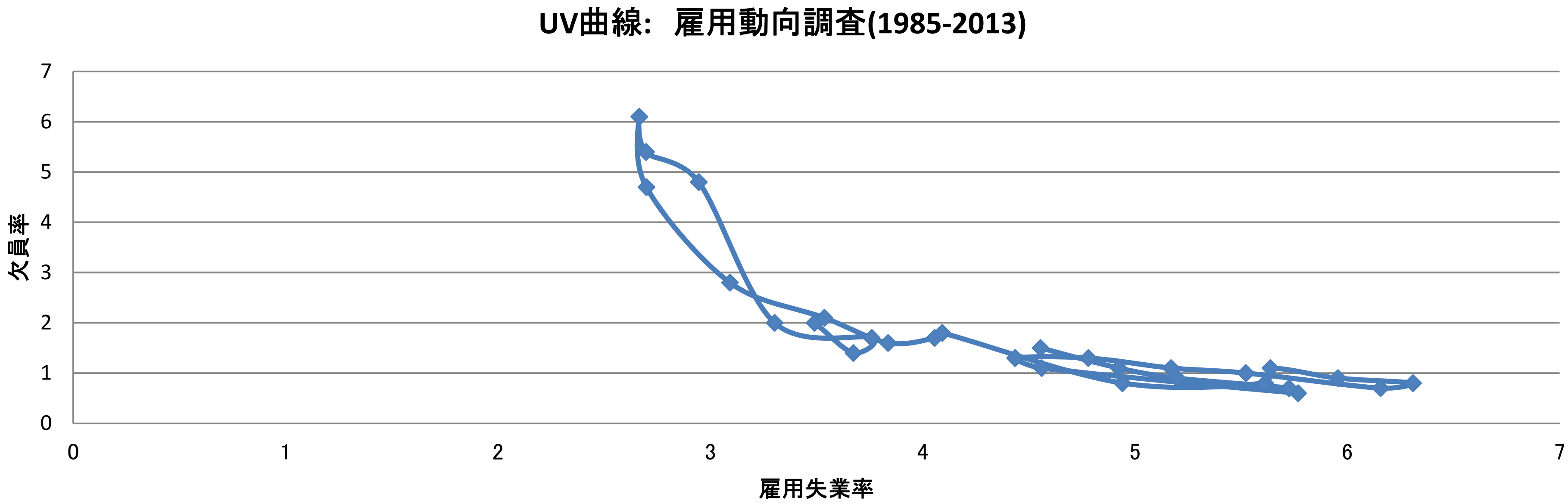
The Natural Rate of Unemployment



The Natural Rate of Unemployment

- In the short run, beveridge curve is downward sloping, but Japanese evidence suggests that often the location of this slope changes.
  - This might be seen as evidence of changing in the efficiency of the matching function or structural changes in separation rate.
- This reasoning is correct only if the data correctly captures Japanese economy.
  - Recently, people may find new jobs from internet. If this is important, official data cannot capture this effect.


UV曲線:職業安定業務統計 (1985-2013)
欠員率
雇用失業率
4
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7


UV曲線: 雇用動向調査(1985-2013)
欠員率
雇用失業率
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7

The Natural Rate of Unemployment

- Let us estimate the natural rate of unemployment.
- Because we can observe separation rate, matching rate and labor market tightness, from data, we can estimate s_t and a_t as follows.
- We estimate

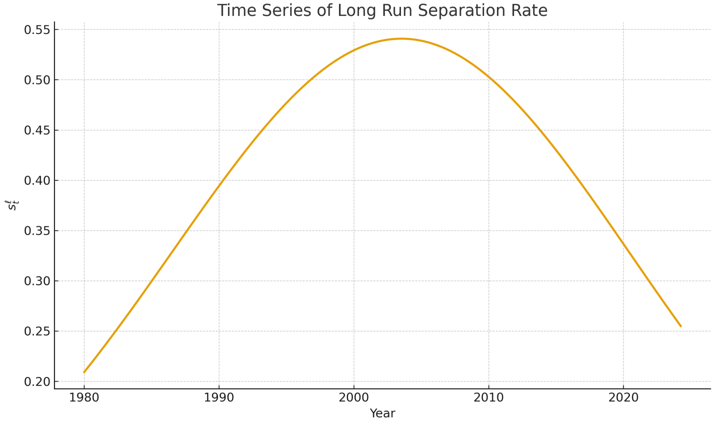
$$\hat{s}_t^l = \hat{\alpha}_0 + \hat{\alpha}_1 * t + \hat{\alpha}_2 * t^2 + \hat{\alpha}_3 * t^3$$

where $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$ are estimated by the following regression:

$$s_t = \alpha_0 + \alpha_1 * t + \alpha_2 * t^2 + \alpha_3 * t^3 + \varepsilon_t$$

where s_t is the observed separation rate and ε_t is an error term.

The Natural Rate of Unemployment



The Natural Rate of Unemployment

- Assume that $q(\Theta_t) = \Theta_t^{\beta_1}$. Then

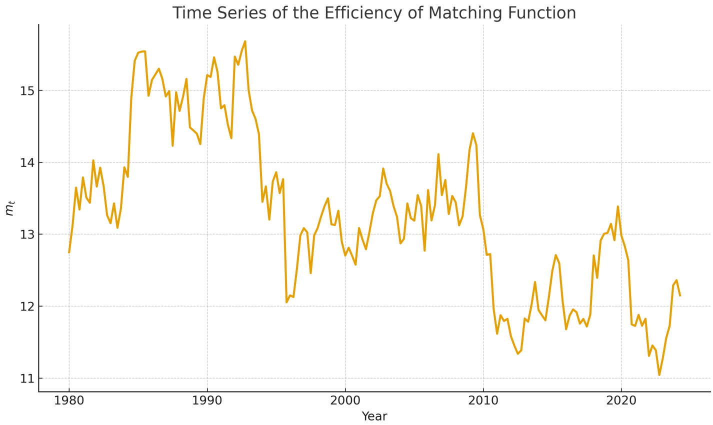
$$\hat{a}_t = \frac{m_t}{\Theta_t^{\hat{\beta}_1}}$$

where m_t is the observed matching rate and $\hat{\beta}_1$ is the estimated by the following regression.

$$\ln m_t = \beta_0 + \beta_1 \ln \Theta_t + u_t$$

where u_t is an error term.

The Natural Rate of Unemployment



The Natural Rate of Unemployment

- We define the natural rate of unemployment as follows

$$u_t^n = \frac{\hat{s}_t^l}{\hat{s}_t^l + \hat{a}_t \left(\Theta^*\right)^{\hat{\beta}_1}}$$

where Θ^* is the sample average of Θ_t .

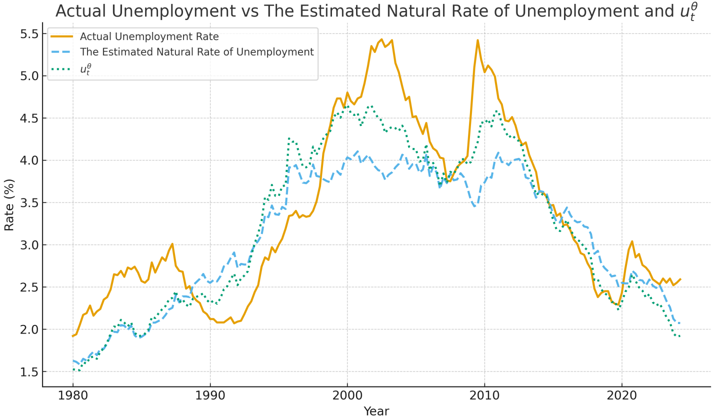
- Other Theoretical Predictions: u_t^θ, u_t^s and $u_t^{s-\theta}$

$$u_t^\theta = \frac{\hat{s}_t^l}{\hat{s}_t^l + \hat{a}_t \left(\Theta_t\right)^{\hat{\beta}_1}}$$

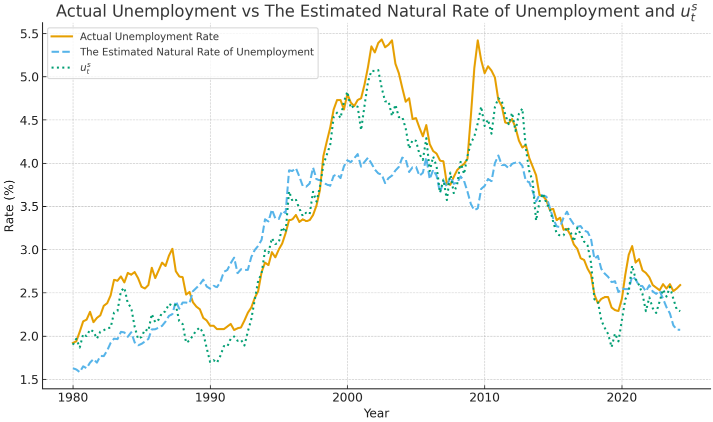
$$u_t^s = \frac{s_t}{s_t + \hat{a}_t \left(\Theta^*\right)^{\hat{\beta}_1}}$$

$$u_t^{s-\theta} = \frac{s_t}{s_t + \hat{a}_t \left(\Theta_t\right)^{\hat{\beta}_1}} = \frac{s_t}{s_t + m_t}$$

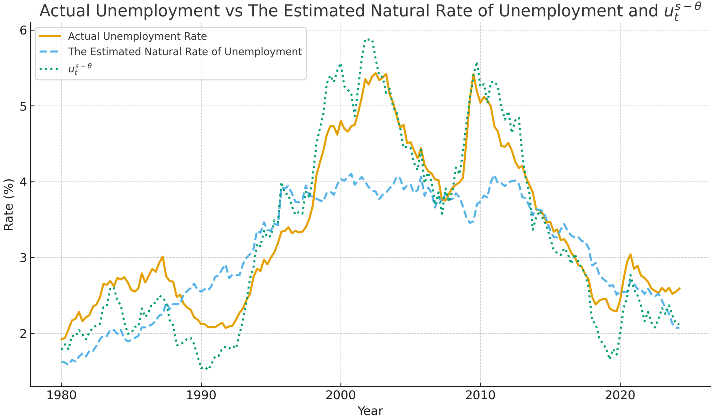
The Natural Rate of Unemployment



The Natural Rate of Unemployment



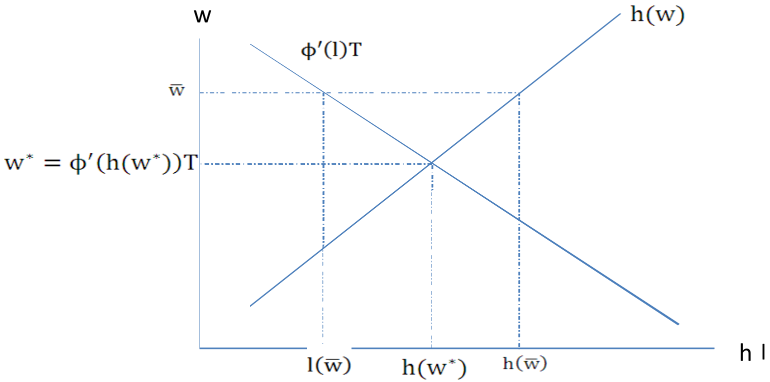
The Natural Rate of Unemployment



Wait Unemployment

- Even if m and s are constant, if Θ^* increases, an unemployment rate can be reduced. Why do not the vacancies increase in the long run even when there are unemployed workers?
- Sticky real wage can explain its reason. If real wage does not fall down to the equilibrium level, real wage cannot equate demand to supply. Therefore, we will observe excess supply of workers. Hence there is some rationing mechanism there. But what prevents real wage from falling down. This is called **wait unemployment**.
- Note that this is not sticky nominal wage. Hence, if a real wage is larger than an equilibrium wage, an increase in demand cannot help reducing unemployment.

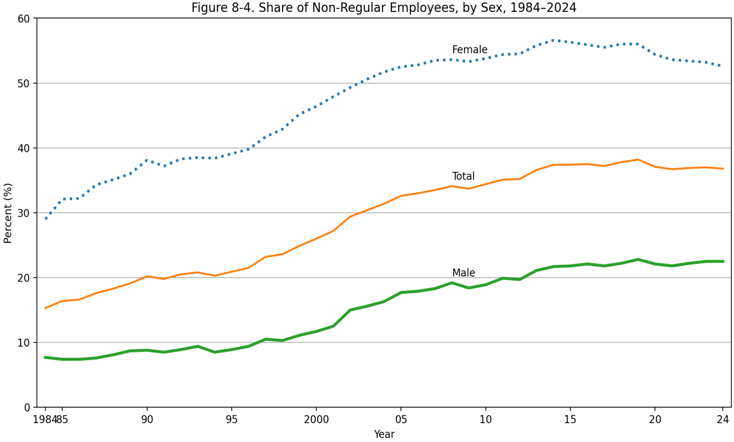
Wait Unemployment



Dual Labor Market

- Moreover, sticky real wage could explain a dual labor market in Japan.
- In Japan, there are significant differences in pay and benefits between regular employees and non-regular employees.
- The share of non-regular workers have two features.
  1. The share of non-regular female workers is much larger than that of male workers.
  2. Recently, the share of non-regular workers increase.

Dual Labor Market



Source: Statistics Bureau of Japan, Labour Force Survey. Up to 2001: Special Survey (February). 2002–2012: Detailed Tabulation (annual average). 2013 onward: Basic Tabulation (annual average).

Sticky Real Wage

- There are several theories to explain why a real wage does not fall down.
  1. Several protection for workers..For example, if a minimum wage increases proportional to an increase in price, a real wage may not decline.
  2. Insider-Outsider Model...Incumbent workers resist their wage from falling down.
  3. Efficiency wage theory...If high wage induces a high productivity, a firm prefers to keep high wage.

Efficiency Wage Theory

- There are several reasons why high wage brings high productivity.
  1. A high wage can increase workers' food consumption, which makes workers healthy and therefore productive.
  2. A higher wage can raise workers' incentive to work when managers cannot monitor it. When wage is higher than an equilibrium wage, workers in the firm receive rent. Since workers do not loose this rent, they will work harder in order to avoid the risk of being fired. This is called Shapiro-Stiglitz (1984) model. I will explain this theory later.)
  3. If able person has high reservation wage because he can receive better offer from others, but if a firm does not know who is able person, offering a high wage raises the average quality of applicants.
  4. A high wage prevents skilled workers from quitting a job. When wage is higher than an equilibrium wage, skilled workers receives rent. Therefore, they are less likely to quit the job.
  5. A high wage may induce workers' effort because people are likely to take reciprocal actions.

Shapiro-Stiglitz Model

- Assumptions:
  1. Agents can choose two efforts $e = \{e^*, 0\}$ where $e^* > 0$ means a high effort or 0 means a low effort.
  2. Agents must incur $C > 0$ for e^* and 0 for 0
  3. Output y is a function of effort, $y = e + u$ where u is a random variable.
  4. A firm pays monitoring cost ofMp to prevent shirking, where $M > 0$ is the parameter and $p \geq 0$ is the probability of finding shirkers.
  5. When an agent shirks and choose 0, s/he will be caught with probability p. If s/he was caught, s/he would be fired. If s/he was not caught or s/he made effort, s/he will get wage payment of w.

Shapiro-Stiglitz Model

- A firm's problem is

$$\begin{aligned} & \max_{p,w} \left\{ x^e - w - Mp \right\} \\ s.t. U &\leq w - C \quad (IR) \\ (1-p)\, w + pU &\leq w - C \quad (IC) \end{aligned}$$

where U is the utility of being unemployed workers or non-regular workers.

- Because of (IR) condition, $w > U$. Therefore, $(1-p)\, w + pU \geq U$. That means that if (IC) condition is satisfied, (IR) condition is always satisfied. Hence, we can ignore (IR) condition.

Shapiro-Stiglitz Model

- Note that

$$
\begin{aligned}
(1-p)\,w + pU &\leq w - C \\
w - (1-p)\,w &\geq pU + C \\
pw &\geq pU + C \\
w &\geq U + \frac{C}{p}
\end{aligned}
$$

- So the original problem can be rewritten as

$$
\min_{w,p} \{w + Mp\},
$$

$$
w \geq U + \frac{C}{p}
$$

- Note that a firm can increase its profits by reducing w. Hence,

$$w = U + \frac{C}{p}$$

- Hence, the original problem is equivalent to

$$\min_{p} \{w + Mp\},$$
$$w = U + \frac{C}{p}$$

Shapiro-Stiglitz Model

- FOC

$$M = \frac{C}{p^2}$$

- p

$$p = \left(\frac{C}{M}\right)^{\frac{1}{2}}$$

- w

$$\begin{aligned} w &= U + \frac{C}{\left(\frac{C}{M}\right)^{\frac{1}{2}}} \\ &= U + (CM)^{\frac{1}{2}} \end{aligned}$$

Shapiro-Stiglitz Model

- The result suggests that
  1. the wage is strictly larger than the utility of unemployed workers, $w > U$. That is, the workers are willing to accept the wage if they are employed.
  2. If the parameter of monitoring cost M, is larger, then the monitoring intensity, p is lower and the rent, $w - U$ is larger. (Rent and monitoring is substitute.)
     - This relation justifies why the large firm pay the higher wage by assuming the monitoring cost is the larger in the large firm.
  3. If the parameter of an effort cost C, is larger, then the rent, $w - U$ is larger.
     - We can also interpret as follows: If the costs of accumulating a firm specific skill are larger, the rent after accumulating skill is larger.

Shapiro-Stiglitz Model

- Shapiro-Stiglitz model is criticized because a firm can construct the same mechanism without a punishment of unemployment or non-regular workers. In this model, we can interpret w as the wage after promotion, U as the wage without promotion.

$$\begin{aligned} s.t. U &\leq w - C \quad (IR) \\ (1-p)\, w + pU &\leq w - C \quad (IC) \end{aligned}$$

- Thus, the deferred wage can be justified as one of mechanism to prevent shirking. But, the deferred wage requires enough positions to provide a high incentive.
- However, the declining growth rate of GDP and aging population makes firms difficult to prepare enough positions in a firm. This might be one of reasons to increase the share of non-regular workers.
- There are other explanations for increases in the share of non-regular workers such as a reduction in self-employed workers and an uncertainty in economic emvironment.

Free Entry and Unemployment

- Even if incumbents try to keep wage high, new firms can offer lower wage. Hence, Insider-outsider model and efficiency wage theories may not explain unemployment if there is a free entry.
- Question: if there is a free entry, is it possible to support unemployment?
  - Answer: Yes, if firms must incur sunk costs (= the pre-investment specific to the employed workers) before production. Examples of such costs are search costs, training costs or pre-committed rental price.
- In other words, I claim that, *if there is no minimum wage and entry barriers, and if there are no sunk costs such as search costs and training costs, there is no unemployed workers in the long run and in the short run.*

Free Entry and Unemployment

- In addition, if we cannot write an explicit contract, observed unemployment is not social desirable because of hold up problems.
- **Hold Up Problems:** When there is the pre-investment, the lack of commitment on the returns to investment cause several problems. When a firm makes investment specific to a particular workers before production, a firm must expect to receive enough return from the investment. But if we cannot write an explicit contract on the investment, after the investment, workers can threaten firms to leave. Expecting this possibility, a firm hesitates to makes enough investment, which brings unemployment.
- There are two reasons that we cannot write an explicit contract on the investment.
    1. A firm must invest before meet workers: search costs.
    2. It is impossible or difficult to write a complete contract: training costs.

Free Entry and Unemployment

- **A Version of Caballero and Hammour (1996)**.
  - A firm searches and trains a worker by C_f and produces Ah output, where h is workers' relation specific human capital.
  - The rental cost, training cost and search cost is sunk. But, after paying this cost, the worker can walk away.
- Suppose that if the worker walks away, the worker expects to receive U, but a firm cannot receive anything today. Therefore, the surplus from this match is
$$S = Ah - U$$

Free Entry and Unemployment

- Assume that the firm cannot write and commit an wage before investment and that the wage is determined by the bargaining of two parties.
- On one hand, the worker's wage w from this bargaining can be written by
$$w = \beta S + U.$$
If the worker decides to leave, the worker expects to get U. This is the outside option of workers. The addition to this value, the worker can receive β part of surplus. We call β the bargaining power of workers. Hence, if $\beta S > 0$, workers always prefer being employed.
- On the other hand, If the worker leaves, a firm does not receive anything. Therefore, the firm expected profit from this match would be
$$J = (1 - \beta) S.$$
That is, the firm can receive $(1 - \beta)$ portion of surplus from this match.

Free Entry and Unemployment

- Before the production, the firm searches and trains workers. The free entry condition implies that

$$C_f = J$$

- Finally, we explain what determines the worker's outside option, U. The worker can find the similar job with probability $e = 1 - u$ where e is the employment rate and u is unemployment rate. On the other hand, with probability u, the worker cannot find a similar job and receives unemployment benefits or wage from the secondary market. This reservation value is denoted by $z < w$.

$$U = (1 - u)\, w + uz = w - u\,(w - z)$$

where is the wage paid by the similar job. It shows that the outside option of workers is lower than w due to the existence of unemployment.

Free Entry and Unemployment

- **Equilibrium:** equilibrium consists of $\{S, J, U, w, u\}$ that satisfies
  - Definition of Surplus
  $$S = Ah - U$$
  - Sharing Rule
  $$\begin{aligned} w &= \beta S + U \\ J &= (1-\beta) S \end{aligned}$$
  - Free Entry Condition
  $$C_f = J$$
  - Worker's Outside Option
  $$U = w - u\left(w - z\right)$$

Free Entry and Unemployment

- Surplus: Substituting firm's profits into the free entry condition

$$
\begin{aligned}
C_f &= (1-\beta)\, S \\
S &= \frac{C_f}{1-\beta}
\end{aligned}
$$

- The outside option of workers

$$
\begin{aligned}
S &= Ah - U \\
U &= Ah - S = Ah - \frac{C_f}{1-\beta}
\end{aligned}
$$

  - This means that the worker's outside option is smaller when the bargaining power of workers is large. If the bargaining power of workers is large, firms cannot expect much profits in the market and firms are reluctant to enter the market. This must lower the outside option of workers.

Free Entry and Unemployment

- Wage payment

$$
\begin{aligned}
w &= \beta S + U \\
&= \beta S + Ah - S \\
&= Ah - (1-\beta) S \\
&= Ah - C_f
\end{aligned}
$$

  - The equation implies that the wage is independent of the bargaining power. When the bargaining power is large, because the worker can receive the large share of surplus, the wage is large. On the other hand, when the bargaining power of workers is large, the worker's outside option is small. In this model, two opposite effects are always cancelled out and w is independent of β.

Free Entry and Unemployment

- Note that the outside option of workers is

$$U = w - u(w - z).$$

It means that the outside option of workers is small when the wage payment in the market is small or the number of unemployed workers is large.

- Unemployment Rate

$$\begin{aligned} w - u(w - z) &= U \\ u(w - z) &= w - U \\ u &= \frac{\beta S}{w - z} = \frac{\beta \frac{C_f}{1-\beta}}{w - z} \\ &= \frac{\beta C_f}{(1-\beta)\left[Ah - C_f - z\right]} \end{aligned}$$

Free Entry and Unemployment

- Unemployment Rate

$$u = \frac{\beta C_f}{(1-\beta)(Ah - C_f - z)}$$

- Unemployment is positive when a specific investment exists, C_f. Because of this specific investment, firms hesitate to enter the market. Therefore, job creation is smaller and unemployment is larger.
- Unemployment is larger when the bargaining power of worker, β, is larger. If $\beta = 0$, no unemployment. Similar to insider and outside model, a strong bargaining power of incumbent workers may increase unemployment rate.

Free Entry and Unemployment

- Question: If a firm can optimally choose β, does it choose 0?
  - Answer: No if the accumulation of relation specific investment requires the effort of workers.
- Assume that in order to accumulate h, a worker must pay cost $\frac{C_w}{2}h^2$, then

$$\begin{aligned} & \max_h \left[w - \frac{C_w}{2} h^2 \right] \\ st.\ w &= \beta S + U \\ S &= Ah - U \end{aligned}$$

- Hence

$$\max_h \left[\beta \left(Ah - U \right) + U - \frac{C_w}{2} h^2 \right]$$

Free Entry and Unemployment

- h: First order condition implies

$$\begin{aligned} \beta A &= C_w h \\ h &= \frac{\beta A}{C_w} \end{aligned}$$

- If $\beta = 0$, $h = 0$. In this case, the profit of the firm $J = (1 - \beta) S = Ah - U = -U < 0$. Therefore, the firm must choose $\beta > 0$.

Free Entry and Unemployment

- Wage payment: Because the wage payment is expressed as

$$w = \beta S + U$$

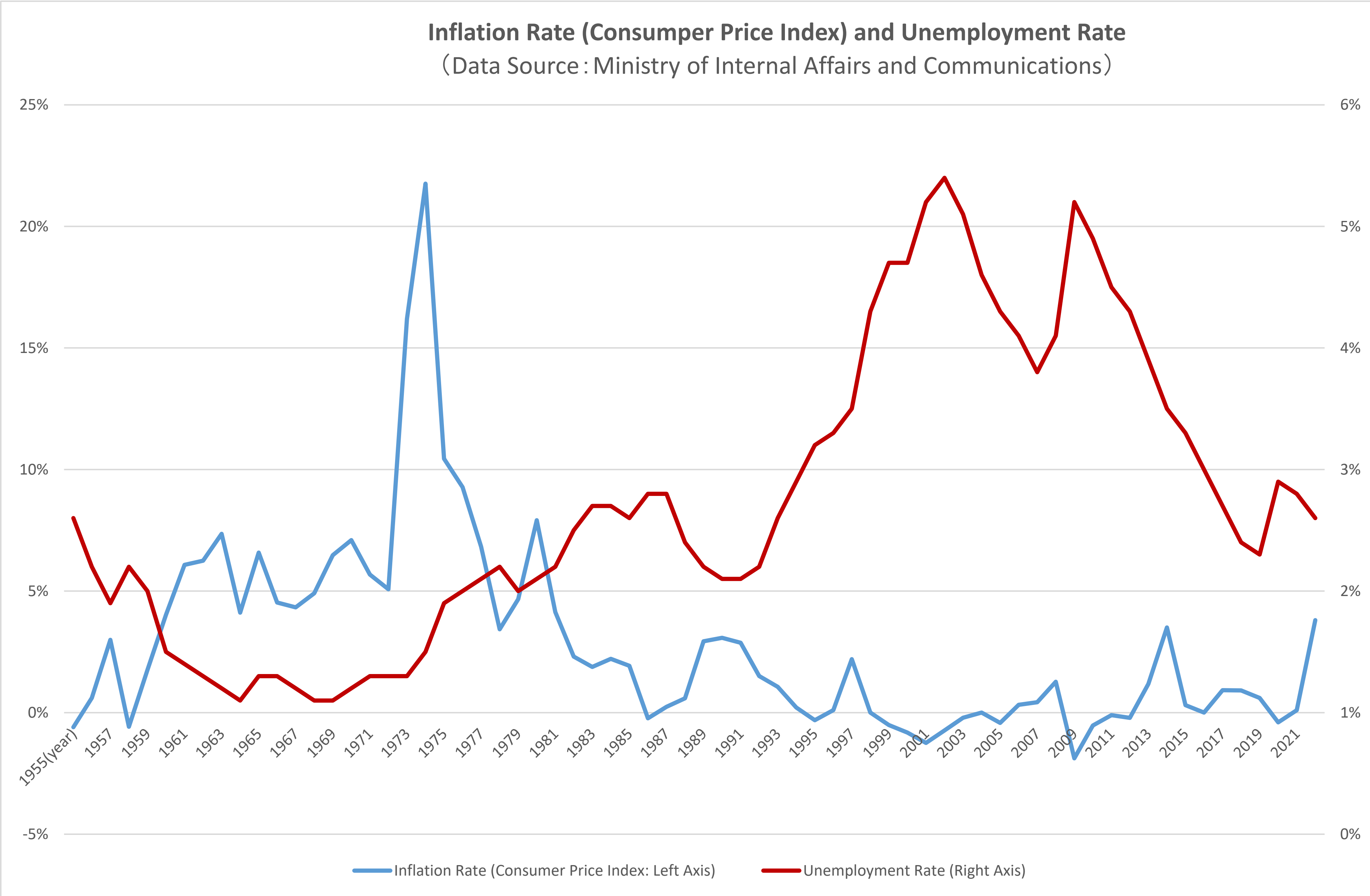
and profit 0 condition implies

$$S = \frac{C_f}{1-\beta} > 0.$$

the positive bargaining power $\beta > 0$ implies, the wage payment is larger than outside option, $w > U$. This is an essence of efficiency wage theory. Note that $C_f > 0$ is required for the positive surplus. That is, the efficiency wage theory also requires a firm's specific investment.

Phillips Curve

- In the short run, we often observe a negative relationship between an inflation rate and an unemployment rate, which is called **Phillips curve**.
- Phillips curve shows a trade-off between high inflation and high unemployment rate in the short run. When government wishes to kill high inflation, government needs to accept a high unemployment rate. When government wants to reduce the unemployment rate, they must accept a high inflation rate.
- Assuming that Phillips curve is stable in the short run, government and Central Bank choose the optimal inflation rate and unemployment rate.
- Unfortunately, Phillips curve cannot be stable in the long run. The natural rate of unemployment and the expected inflation rate may change in the long run.


Inflation Rate (Consumer Price Index) and Unemployment Rate
（Data Source：Ministry of Internal Affairs and Communications）
25%
20%
15%
10%
5%
0%
-5%
6%
5%
4%
3%
2%
1%
0%
1955(year)
1957
1959
1961
1963
1965
1967
1969
1971
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
2017
2019
2021
Inflation Rate (Consumer Price Index: Left Axis)
Unemployment Rate (Right Axis)

Unemployment in the Short Run

- Let us derive a Phillips curve.
- In the short run, the share of actually employed labor may deviate from the long run equilibrium level. Note that imperfect information model implies that the share of employed worker, e is a negative function of the aggregate price deflator over expected price deflator, $\frac{P}{P^e}$

$$e_t = e\left(\frac{P_t}{P_t^e}\right)$$

- Because unemployment rate can be defined by $u = 1 - e$, the unemployment rate is an decreasing function of $\frac{P_t}{P_t^e}$.

$$u_t = u\left(\frac{P_t}{P_t^e}\right)$$

- Assume that

$$u\left(\frac{P_t}{P_t^e}\right) = u^n - \alpha \ln \frac{P_t}{P_t^e}$$

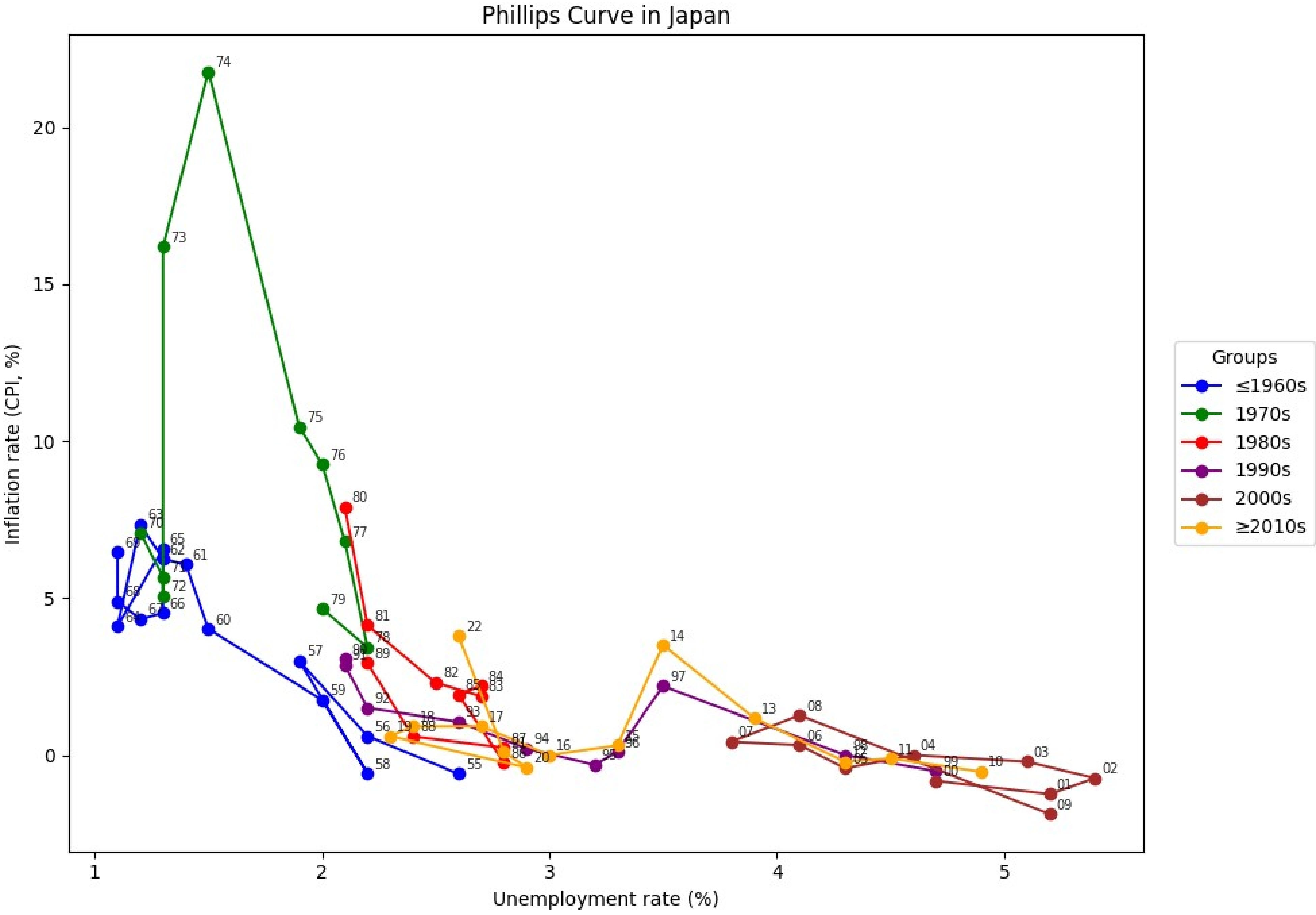
where u^n is the natural rate of unemployment.

Unemployment in the Short Run

- Then

$$\begin{aligned} u_t &= u^n - \alpha \left[\ln P_t - \ln P_{t-1} - \left(\ln P_t^e - \ln P_{t-1} \right) \right] \\ &= u^n - \alpha \left[g_{pt} - g_{pt}^e \right] \end{aligned} \quad (5)$$

  where $g_{pt} = \ln P_t - \ln P_{t-1}$ and $g_{pt}^e = \ln P_t^e - \ln P_{t-1}$.
- The equation $u_t = u^n - \alpha \left[g_{pt} - g_{pt}^e \right]$ is called Phillips curve.

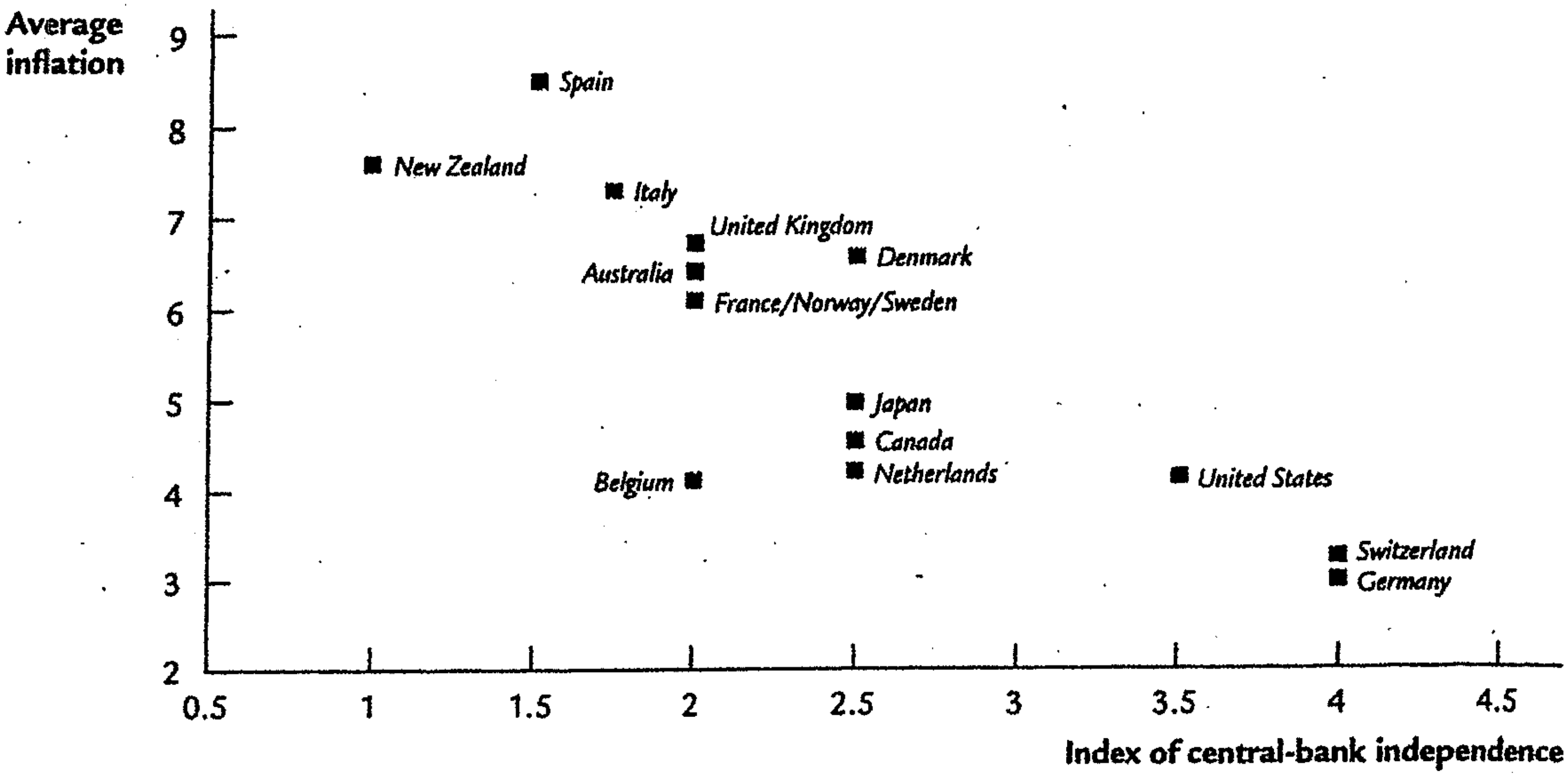

Phillips Curve in Japan
Inflation rate (CPI, %)
Unemployment rate (%)
Groups
≤1960s
1970s
1980s
1990s
2000s
≥2010s

- Japanese Phillips curve shows a negative relationship between an inflation rate and an unemployment rate before 1990, but almost no relationship after 1990.
- Even before 1990, the relationship changes over time: Phillips curve shifted to the right from 1960s to 1980s, and almost no relationship during 1970s.

Unemployment in the Short Run

- Even if active stabilization policy is desired, economists doubt the ability of government to conduct timely stabilization policy. Politicians have always a pressure from their supporter. Politician may want to have boom right before their election.
- In order to avoid this difficulty, many countries give central bank a discretion of monetary policy. In this way, monetary policy can be conducted without political intervention. There is a clear negative relationship between the independence of central banks and inflation rates.

figure 14-2



Inflation and Central-Bank Independence This scatterplot presents the international experience with central-bank independence. The evidence shows that more independent central banks tend to produce lower rates of inflation.

Source: Figure 1a, page 155, of Alberto Alesina and Lawrence H. Summers, "Central Bank Independence and Macroeconomic Performance: Some Comparative Evidence," *Journal of Money, Credit, and Banking* 25 (May 1993): 151–162. Average inflation is for the period 1955–1988.

- Even if a central bank has a reasonable discretion. There is another difficult problem: **commitment**.
- Note that Phillips curve, $u_t = u^n - \alpha \left[g_{pt} - g_{pt}^e \right]$, shows that there is a trade-off between inflation rate and unemployment rate. It means that a central bank cannot reduce both the inflation rate and the unemployment rate at the same time.
- Suppose that Bank of Japan wants to minimize the following loss function:

$$L(u, \pi) = u_t + \frac{\gamma}{2} g_{pt}^2 .$$

Suppose that Bank of Japan directly controls inflation, but not the unemployment rate. Hence, it chooses g_{pt} in order to minimize the loss function given

$$u_t = u^n - \alpha \left[g_{pt} - g_{pt}^e \right]$$

- Assume that the central bank announces $g_{pt} = 0$ and assume that public believes this announcement. Then

$$u_t = u^n - \alpha g_{pt}.$$

Substituting this equation into the loss function,

$$L(u, \pi) = u^n - \alpha g_{pt} + \frac{\gamma}{2} g_{pt}^2$$

Therefore, the first order condition implies

$$\alpha = \gamma g_{pt}$$

and the optimal choice is

$$g_{pt} = \frac{\alpha}{\gamma} > 0.$$

Hence, after announcing $g_{pt} = 0$, Bank of Japan always have an incentive to break the promise and to choose a positive inflation in order to reduce unemployment. This is called **time inconsistency** problem.

Unemployment in the Short Run

- Once, public knows that Bank of Japan cannot commit their announcement and its best strategy is $g_{pt} = \frac{\alpha}{\gamma}$. Hence, $g_{pt}^{e} = \frac{\alpha}{\gamma}$.

$$u_t = u^n - \alpha \left[g_{pt} - \frac{\alpha}{\gamma} \right],$$

and

$$L(u, \pi) = u^n - \alpha \left[g_{pt} - \frac{\alpha}{\gamma} \right] + \frac{\gamma}{2} g_{pt}^2.$$

In this case the best strategy for Bank of Japan is again

$$g_{pt} = \frac{\alpha}{\gamma}.$$

Therefore, this is consistent with Public's expectation and we can sustain this equilibrium. In this case,

$$u_t = u^n.$$

Hence an active stabilization policy eventually brings a positive inflation and the natural rate of unemployment.

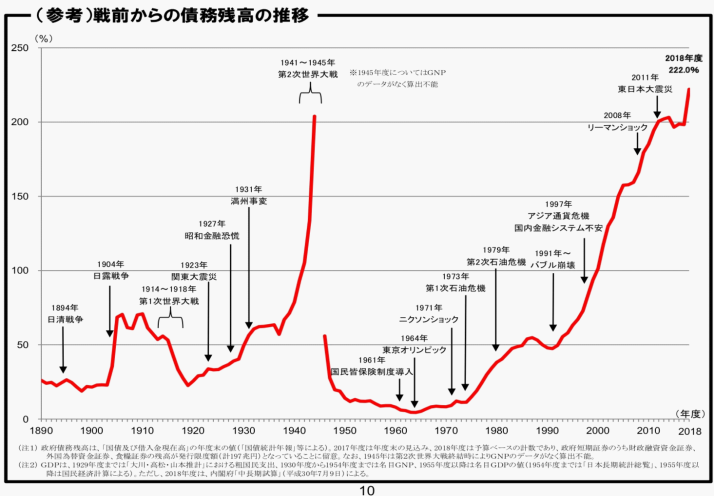
Unemployment in the Short Run

- Now consider passive stabilization policy. That is, Bank of Japan announces the rule of policy and commits the rule. Assume that this commitment is possible. In this example, assume that Bank of Japan announces $g_{pt} = 0$. Since there is a commitment device, public can believe that the inflation rate is 0: $g_{pt}^{e} = 0$. Then $u_t = u^n$.
- Note that a passive stabilization policy brings the better result for Bank of Japan. Because of time inconsistency problem, public cannot fully trust the bank's announcement unless they can actually commit the policy. Without commitment, large bank's discretionary power may make a worse situation.

Sustanability of Government Debt

- As the unemployment rate continues to decline and inflation remains positive, it is natural to ask why the Bank of Japan has not tightened its monetary policy more aggressively to bring inflation under control.
- The Bank of Japan has purchased a large amount of government debt. To combat inflation, it would need to sell government bonds and absorb liquidity from the market, thereby reducing the money supply.
- However, doing so would lower the prices of government bonds, making it more difficult for the government to finance its expenditures.
- For this reason, it is important to examine how monetary policy affects the sustainability of government debt.
- We begin by examining the current figures on government bonds and public expenditure.

Sustanability of Government Debt



Sustanability of Government Debt

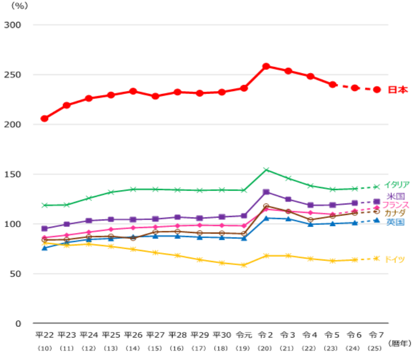
債務残高の国際比較（対GDP比）

○ 債務残高の対GDP比は、G７諸国のみならず、その他の諸外国と比べても突出した水準となっています。

＜全世界における順位（172ヵ国・地域中）＞

1	マカオ	0.0%	159	カナダ	107.7%
	⋮		160	フランス	109.7%
5	香港	6.3%		⋮	
	⋮		165	米国	119.0%
84	韓国	50.7%		⋮	
	⋮		168	イタリア	134.6%
111	ドイツ	62.9%		⋮	
	⋮		170	ギリシャ	165.2%
139	中国	82.0%	171	シンガポール	172.8%
	⋮		172	日本	240.0%
156	英国	100.4%			

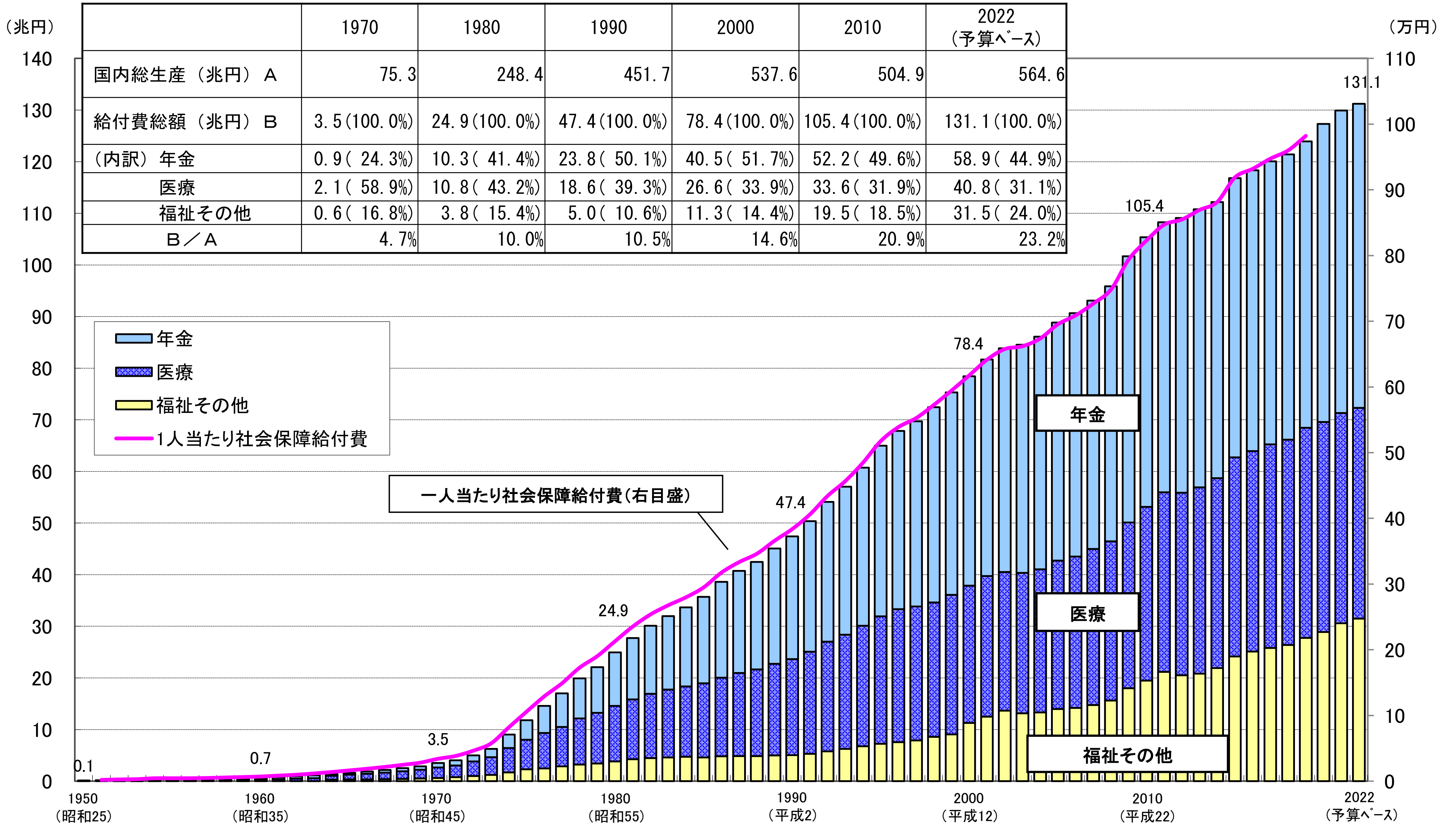
※ 数値は令和５年（2023年）の値。
2023年が推計値又は数値不明の国は除く。



(出所) IMF "World Economic Outlook" (2025年４月)
(注１) 数値は一般政府（中央政府、地方政府、社会保障基金を合わせたもの）ベース。
(注２) 右図の日本及びフランスは、2024年及び2025年が推計値。それ以外の国は、2025年が推計値。

社会保障給付費の推移

	1970	1980	1990	2000	2010	2022 （予算ベース）
国内総生産（兆円）Ａ	75.3	248.4	451.7	537.6	504.9	564.6
給付費総額（兆円）Ｂ	3.5(100.0%)	24.9(100.0%)	47.4(100.0%)	78.4(100.0%)	105.4(100.0%)	131.1(100.0%)
（内訳）年金	0.9(24.3%)	10.3(41.4%)	23.8(50.1%)	40.5(51.7%)	52.2(49.6%)	58.9(44.9%)
医療	2.1(58.9%)	10.8(43.2%)	18.6(39.3%)	26.6(33.9%)	33.6(31.9%)	40.8(31.1%)
福祉その他	0.6(16.8%)	3.8(15.4%)	5.0(10.6%)	11.3(14.4%)	19.5(18.5%)	31.5(24.0%)
Ｂ／Ａ	4.7%	10.0%	10.5%	14.6%	20.9%	23.2%

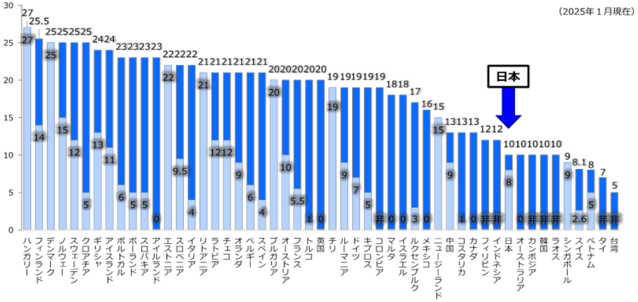


資料：国立社会保障・人口問題研究所「令和元年度社会保障費用統計」、2020～2022年度（予算ベース）は厚生労働省推計、
2022年度の国内総生産は「令和4年度の経済見通しと経済財政運営の基本的態度（令和4年1月17日閣議決定）」
（注）図中の数値は、1950,1960,1970,1980,1990,2000及び2010並びに2022年度（予算ベース）の社会保障給付費（兆円）である。

Sustainability of Government Debt

諸外国等における付加価値税率（標準税率及び食料品に対する適用税率）の比較

○　日本の標準税率は51ヵ国（OECD加盟国、EU、ASEAN+３（+台湾））中、42位（下から６番目）。



（注１）上記は、原則的な取扱いを示したもので、代表的な品目に対する税率のみを記載しており、品目によっては税率が変わることに留意が必要。
（注２）上記中、■が食料品に係る適用税率である。「０」と記載のある国は、食料品についてゼロ税率が適用される国である。「非」と記載のある国は、食料品が非課税対象となる国である。
なお、軽減税率・ゼロ税率の適用及び非課税対象とされる食料品の範囲は各国ごとに異なり、食料品の種類によっては上記以外の取扱いとなる場合がある。
（注３）日本については、10%（標準税率）のうち2.2%、８%（軽減税率）のうち1.76%は地方消費税（地方税）である。
（注４）EC指令においては、標準税率は15%以上とすることが求められている。
（注５）イタリア、スペイン、ポルトガル、スロベニア、ギリシャについては2024年７月１日現在の情報。
（注６）カナダについては、①連邦税としての付加価値税のみ課されている州、②連邦付加価値税に加えて、州税としての小売売上税も課されている州、③連邦・州共通の税としての付加価値税が課されている州が存在。
なお、表中では③の類型であるオンタリオ州の税率を記載（連邦・州共通の付加価値税13%（うち州税８%））。
（注７）米国では、連邦における付加価値税は存在しないが、州等による地方税として、事業者が消費者に販売する段階で課税を行う小売売上税が存在する場合がある（例：ニューヨーク市の場合 8.875%）。
（出典）OECD資料、欧州委員会及び各国政府ホームページ、IBFD等。

Sustanability of Government Debt

- In order to understand the sustanability of government debt, let us examine $g_{\frac{b}{y}}$. Note that

$$g_{\frac{b}{y}} \approx g_b - g_y$$

Hence, the growth rate of per capita debt is smaller than the growth rate of per capita GDP, government can manage debt without sovereign default.

Sustanability of Government Debt

- Note that when the population growth n is not zero, $b_{t+1} = \frac{1+\rho}{1+n} b_t$. Hence

$$
\begin{aligned}
b_{t+1} &= (1 + g_b)\, b_t \\
g_b &= \frac{(1+\rho)}{(1+n)} - 1
\end{aligned}
$$

$$
\begin{aligned}
g_b &= \frac{(1+\rho)}{(1+n)} - 1 \\
&= \frac{\rho - n}{1+n} \approx \rho - n
\end{aligned}
$$

Sustanability of Government Debt

- When the economy is on the steady state, we know that $g_y = g$, where g is the growth rate of the productivity. Hence

$$g_{\frac{b}{y}} \approx g_b - g_y \approx \rho - n - g.$$

- Note that $\rho = \rho^n - g_p^e$. Hence

$$g_{\frac{b}{y}} = \rho^n - g_p^e - n - g.$$

- This indicates that if

$$\rho^n < g_p^e + n + g$$

government can manage debt without sovereign default.

Financial Repression and Mild Inflation

- Financial repression refers to a set of policies that
  1. regulate financial markets in ways that keep nominal interest rates on government debt artificially low in real terms,
  2. channel private savings toward the purchase of government debt through various financial regulations.
- Reinhart and Sbrancia (2011) report that, as a result of financial repression and inflation, real interest rates on government debt remained negative for 25 to 50 percent of the years between 1945 and 1980 in many advanced economies, including the United Kingdom and the United States.
- This situation effectively amounted to a tax on savings (Inflation tax). Unlike ordinary forms of taxation, however, such implicit taxation was implemented by central banks without the need for parliamentary approval, making it far less visible to the public.

Financial Repression and Mild Inflation

- Is financial repression and mild inflation feasible?
    1. Financial repression was sustainable under the tight financial regulations introduced in response to financial crises.
    2. Today, when international financial transactions are relatively free, it remains uncertain how long a policy of maintaining negative real interest rates through financial repression and inflation can be sustained.

Financial Repression and Mild Inflation

- Is financial repression and mild inflation desirable?
  1. A decline in real interest rates may encourage excessive investment, which could lower long-term economic growth.
  2. Financial repression may weaken fiscal discipline, potentially leading to a continuous rise in public debt instead of a reduction.
  3. If confidence in the central bank erodes and hyperinflation occurs, restoring trust in the yen could cause severe economic disruption. To rebuild confidence, the government might need to use tax revenues to support the central bank, which would ultimately result in a future tax burden on citizens.

Lucas's Critique and Micro Foundation

- So far, I assume a consumption function and a money demand function. Because these functions are the results of individual behaviors, changes in environment may influence the properties of these functions.
- Lucas (1976) argues that as people make decisions based on their expectation on the future economic environment, the expected future policy change is likely to influence their expectation, and, therefore, their decisions. It means that the estimated parameters on consumption functions and money functions are likely to change when a government changes its policy. So we cannot use the estimated parameters for policy simulations.
- Following Lucas's critique, many macroeconomists pay more attention to the micro foundation of the consumer's behavior and firm's behavior, and derives consumption function, investment function and money demand function. I would like to review these discussions.

Lucas's Critique and Consumption Function

- I would like to show the idea of Lucas's critique by using a simple consumer's decision problem. Let me consider the following consumption function

$$c = \phi_0 + \phi_1 x_t + \varepsilon$$

  where x_t is disposable income.
- Once you obtain the parameters ϕ_1 and ϕ_2, one can conduct a policy simulation.
- Question: how much can we trust these estimated parameters? Lucas (1976) said that it might be good for a short run prediction. But it cannot be neither a basis of a policy evaluation nor a long run prediction.
- Let me demonstrate that Lucas's points here by using a simple example.

Lucas's Critique and Consumption Function

- Consider the following household's decision problem.

$$
\begin{aligned}
& \max_{c_t, c_{t+1}} \{u(c_t) + \beta u(c_{t+1})\} \\
s.t.\ a_{t+1} + c_t &= (1+\rho) a^* + w_t \\
a^* + c_{t+1} &= (1+\rho) a_{t+1} + w_{t+1} \\
where\ s_t^n &= a_{t+1} - a^*
\end{aligned}
$$

Here we assume that household must leave the same amount of asset a^* as a bequest for the next generation and household can perfectly predict future.

- Permanent income

$$
\begin{aligned}
a^* + c_{t+1} &= (1+\rho)\left((1+\rho)\,a^* + w_t - c_t\right) + w_{t+1} \\
\frac{a^* + c_{t+1}}{1+\rho} &= (1+\rho)\,a^* + w_t - c_t + \frac{w_{t+1}}{1+\rho} \\
c_t + \frac{c_{t+1}}{1+\rho} &= \rho a^* + w_t + a^* - \frac{a^*}{1+\rho} + \frac{w_{t+1}}{1+\rho} \\
&= \rho a^* + w_t + \frac{\rho a^* + w_{t+1}}{1+\rho} \\
&= x_t + \frac{x_{t+1}}{1+\rho} \\
&= x^p
\end{aligned}
$$

where $x_t = \rho a^* + w_t$ and $x_t^p = x_t + \frac{x_{t+1}}{1+\rho}$. The parameter x_t^p is called a permanent income or a life time income.

- The budget constraint implies that

$$c_{t+1} = (1+\rho)\left(x_t^p - c_t\right)$$

- Substituting the budget constraints into objective function, the original problem is rewritten as

$$\max_{c_t}\left\{u\left(c_t\right) + \beta u\left[(1+\rho)\left(x_t^p - c_t\right)\right]\right\}.$$

Lucas's Critique and Consumption Function

Lemma

Suppose that $y = \phi(x)$ *and* $z = f(y)$. *Then*

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx} = f'(y)\,\phi'(x)$$

Lucas's Critique and Consumption Function

- First Order Condition: Applying the lemma to $u(c_{t+1})$ and $c_{t+1} = (1+\rho)\left(x_t^p - c_t\right)$

$$0 = u'(c_t) - \beta(1+\rho)\, u'\left[c_{t+1}\right]$$

- For my simple analysis, assume that $u(c) = \gamma c - \frac{\eta}{2} c^2$

$$\begin{aligned}
\gamma - \eta c_t &= \beta(1+\rho)\left[\gamma - \eta c_{t+1}\right] \\
\gamma - \eta c_t &= \beta(1+\rho)\left[\gamma - \eta(1+\rho)\left(x_t^p - c_t\right)\right] \\
\eta\left[\beta(1+\rho)^2 + 1\right] c_t &= \gamma - \beta(1+\rho)\left[\gamma - \eta(1+\rho)\, x_t^p\right] \\
&= \left(1 - \beta(1+\rho)\right)\gamma + \beta\eta(1+\rho)^2 x_t^p \\
c_t &= \frac{\left(1 - \beta(1+\rho)\right)\gamma}{\eta\left[\beta(1+\rho)^2 + 1\right]} + \frac{\beta(1+\rho)^2}{\left[\beta(1+\rho)^2 + 1\right]} x_t^p
\end{aligned}$$

Lucas's Critique and Consumption Function

- Consumption Function:

$$c_t = A + Bx_t^p$$

where

$$\begin{aligned} A &= \frac{\gamma\left[1-\beta\left(1+\rho\right)\right]}{\eta\left[1+\beta\left(1+\rho\right)^2\right]} \\ B &= \frac{\beta\left(1+\rho\right)^2}{1+\beta\left(1+\rho\right)^2} \end{aligned}$$

The Alternative Derivation of Consumption Function with High School Math

- For my simple analysis, assume that $u(c) = \gamma c - \frac{\eta}{2}c^2$.

$$\begin{aligned}
& \beta u\left[(1+\rho)\left(x_t^p - c_t\right)\right] \\
= \; & \beta\gamma\left((1+\rho)\left(x_t^p - c_t\right)\right) - \frac{\beta\eta}{2}\left[(1+\rho)\left(x_t^p - c_t\right)\right]^2 \\
= \; & \beta\gamma(1+\rho)\left(x_t^p - c_t\right) - \frac{\beta\eta(1+\rho)^2}{2}\left(x_t^p - c_t\right)^2 \\
= \; & \beta\gamma(1+\rho)\left(x_t^p - c_t\right) - \frac{\eta(1+\rho)^2}{2}\left[\left(x_t^p\right)^2 - 2x_t^p c_t + c_t^2\right] \\
= \; & x^* + \beta\left[\eta(1+\rho)^2 x_t^p - \gamma(1+\rho)\right]c_t - \frac{\beta\eta(1+\rho)^2}{2}c_t^2
\end{aligned}$$

where $x^* = \beta\left[\gamma(1+\rho)x_t^p - \frac{\eta(1+\rho)^2}{2}\left(x_t^p\right)^2\right]$.

The Alternative Derivation of Consumption Function with High School Math

- The maximization Problem

$$\max_{c_t} \left\{ u\left(c_t\right) + \beta u\left[\left(1+\rho\right)\left(x_t^p - c_t\right)\right] \right\}.$$

$$= \max_{c_t} \left\{ \begin{array}{c} \gamma c_t - \frac{\eta}{2} c_t^2 + x^* + \\ \beta \left[\eta\left(1+\rho\right)^2 x_t^p - \gamma\left(1+\rho\right)\right] c_t - \frac{\beta\eta\left(1+\rho\right)^2}{2} c_t^2 \end{array} \right\}.$$

- The first order condition is

$$0 = \gamma - \eta c_t + \beta\left[\eta\left(1+\rho\right)^2 x_t^p - \gamma\left(1+\rho\right)\right] - \beta\eta\left(1+\rho\right)^2 c_t$$

The Alternative Derivation of Consumption Function with High School Math

- Consumption Function: From the first order condition,

$$\begin{aligned} & \eta\left[1+\beta(1+\rho)^2\right] c_t \\ = & \gamma[1-\beta(1+\rho)]+\beta\eta(1+\rho)^2 x_t^p . \end{aligned}$$

Hence

$$c_t = A + B x_t^p$$

where

$$\begin{aligned} A &= \frac{\gamma[1-\beta(1+\rho)]}{\eta\left[1+\beta(1+\rho)^2\right]} \\ B &= \frac{\beta(1+\rho)^2}{1+\beta(1+\rho)^2} \end{aligned}$$

Lucas's Critique and Consumption Function

- Let me assume that $\rho a^* + w_{t+1} = (1 - \tau_x) * (\rho a^* + w_t)$.

$$
\begin{aligned}
x_t^p &= \rho a^* + w_t + \frac{\rho a^* + w_{t+1}}{1+\rho} \\
&= \rho a^* + w_t + \frac{(1-\tau_x) * (\rho a^* + w_t)}{1+\rho} \\
&= \left[1 + \frac{1-\tau_x}{1+\rho}\right] \left[\rho a^* + w_t\right] \\
&= \left[\frac{2+\rho-\tau_x}{1+\rho}\right] x_t
\end{aligned}
$$

Lucas's Critique and Consumption Function

- The empirically testable equation is

$$\begin{aligned} c_t &= A + Bx_t^p = A + B\left[\frac{2+\rho-\tau_x}{1+\rho}\right]x_t, \\ &= \phi_0 + \phi_1 x_t + \varepsilon_t \end{aligned}$$

  where $\phi_0 + \varepsilon_t = A$ and $\phi_1 = B\left[\frac{2+\rho-\tau_x}{1+\rho}\right]$.

- Note that τ_x can change ϕ_1. Based on the estimated ϕ_0 and ϕ_1, a policy maker can simulate the impact of tax policy on aggregate consumption. But in fact, the future tax change affects ϕ_1 itself. Hence we cannot trust any estimations ϕ_1 for a basis of a policy evaluation. If we consider a general equilibrium effect, the results are more devastated. Since most likely policy changes will affect ρ, it changes the parameters ϕ_0 and ϕ_1. In other word, the parameters that are estimated from the reduced form estimation are not robust.

Lucas's Critique and Consumption Function

- After Lucas's critique, macroeconomists agree that it is important to examine the micro foundation of macroeconomics.
- Macro economists assume that changes in policy cannot influence preference and technology such as γ, β and η. Hence, once we identify γ, β and η, we may be able to conduct a policy simulation based on these parameters. For this purpose, macroeconomic model must be based on a reasonable micro foundation.
- Currently, most macro economists use the **dynamic stochastic general equilibrium model** as a foundation of macroeconomics.

Permanent Income Hypothesis

- The previous model derives current consumption as a function of a permanent income.
$$c_t = A + Bx_t^p$$
- The permanent income hypothesis has a strong implication: the temporal movement of income has little impact on consumption. Since the marginal benefit from consumption is decreasing, consumers prefer stable consumption to unstable consumption. Therefore, if it is possible, consumers have always incentive to smooth their consumption. When consumption depends on the permanent income, it is possible to smooth consumption by exchanging consumption today and tomorrow.
- Note that this implies that the even if a tax cut increases disposal income, it may fail to increase private consumption, and therefore, aggregate demand.

Permanent Income Hypothesis

- To see this implication formally, note the first order condition of the previous problem implies

$$\beta\left(1+\rho\right)\left[\gamma-\eta c_{t+1}\right]=\gamma-\eta c_t$$

Assume that $\rho_{t+1}=\rho$ is constant, which is satisfied by steady state. Then the equation implies that current consumption is the best predictor of the future consumption. Once we observe current consumption, we do not need any other information to predict c_t.

Permanent Income Hypothesis

- In particular, if $\beta = \frac{1}{1+\rho}$, then

$$c_{t+1} = c_t.$$

In this extreme case, the expected consumption is the same over time. That is, the predicted value of the future consumption is just current consumption, and a change in consumption is unpredictable.

$$\Delta c_t = 0$$

where $\Delta c_t = c_{t+1} - c_t$.

- Evidence: Hall (1978) tested this observation. He cannot reject the hypothesis that lagged values of either income and consumption cannot predict a change in consumption. This result supports the implication of the permanent income hypothesis. After Hall (1978), much empirical research was conducted for this issue. For example, Campbell and Mankiw (1989) did the following regression:

$$\begin{aligned} \Delta c_t &= \lambda \Delta x_t + v_t, \\ \textit{where } v_t &= (1-\lambda)\,\varepsilon_t, \end{aligned}$$

where ε_t is the change in consumers' prediction of their permanent income. Hence, if the permanent income hypothesis is right, λ is close to 0, and if the traditional theory is correct, λ is close to 1. They found $\lambda = 0.42$˜0.52. This result indicates that consumption partially responds to disposable income.

Permanent Income Hypothesis

- If a change in current income has an impact on a change in consumption, what explains this behavior.
  1. *Serially Correlated Income*: When today's income is highly correlated with the future income, consumers can easily predict the steam of the future income based on the current income. Remember that assuming $\rho a^* + w_{t+1} = (1 - \tau_x) * (\rho a^* + w_t)$, I show that
  $$x_t^p = \left[\frac{2 + \rho - \tau_x}{1 + \rho}\right] x_t.$$
  It shows that a rise in current income increases the permanent income, and therefore consumption.
  2. *Liquidity Constraint:* If some consumers cannot borrow enough money, they cannot buy consumption goods more than their income. Therefore, current disposable income limits consumption:
  $$c_t \leq x_t.$$
  If the optimal consumptions of many household are constrained by current income, and if current income increases, then obviously, consumers will increase their consumption.

Ricardian Equivalence and Government Debt

- One of the interesting application of the permanent income hypothesis is government debt. When government finances its expenditure by taxes, we don't see any multiplier effect even in the short run. What if government issues its bond. In this way, it does not increase tax burden today.
- However, if consumers care about their permanent income, they are worried not only about today's income, but tomorrow's income. Since today's debt can be seen as the future tax burden, it does not change their permanent income. Therefore, consumers do not change their consumption decision. This is called **Ricardian Equivalence**.

Ricardian Equivalence and Government Debt

- Let me formally analyze Ricardian Equivalence. Assume that population is 1 for a simple analysis. In order to finance government expenditure, $\hat{g}_t$, government either imposes a lump sum tax: τ_t or issues a bond, b_t. Assume that when government issues a bond at date t, it must pay back to consumers at date $t+1$. , the government's budget constraint is

$$\begin{aligned} b_{t+1} &= \hat{g}_t - \tau_t, \\ \hat{g}_{t+1} &= \tau_{t+1} - (1+\rho)\, b_{t+1}. \end{aligned}$$

- We can derive a government's intertemporal budget constraint.

$$\begin{aligned} \hat{g}_{t+1} &= \tau_{t+1} - (1+\rho)\,(\hat{g}_t - \tau_t) \\ \tau_t + \frac{\tau_{t+1}}{1+\rho} &= \hat{g}_t + \frac{\hat{g}_{t+1}}{1+\rho}. \end{aligned}$$

  This equation shows that the present value of tax revenue must be equal to the present value of government expenditure.

- Let $w_t^* = w_t - \tau_t$. Following the previous argument, it is shown that

$$
\begin{aligned}
c_t + \frac{c_{t+1}}{1+\rho} &= \rho_t^a a^* + w_t^* + \frac{\rho a^* + w_{t+1}^*}{1+\rho} \\
&= \rho a^* + w_t - \tau_t + \frac{\rho a^* + w_t - \tau_{t+1}}{1+\rho} \\
&= \rho_t a^* + w_t + \frac{\rho a^* + w_t}{1+\rho} - \left(\tau_t + \frac{\tau_{t+1}}{1+\rho}\right) \\
&= \rho_t a^* + w_t + \frac{\rho a^* + w_t}{1+\rho} - \left(\hat{g}_t + \frac{\hat{g}_{t+1}}{1+\rho}\right)
\end{aligned}
$$

- Note that the budget constraint does not depend on neither tax nor bond. That is, issuing bond does not change consumers' permanent income, therefore it does not change consumption.

Ricardian Equivalence and Government Debt

- *Deviation from Ricardian Equivalence:* In reality, Ricardian equivalence does not perfectly hold. There are several reasons that Ricardian Equivalence may not hold.
  1. *Liquidity Constraint:* If there is liquidity constraints, the reduction of tax can increase disposable income and raises consumption. More importantly, if a government faces liquidity constraint because of the fear of default, by definition, government cannot issue a bond to finance expenditure.
  2. *Transfer across generations:* If parents do not care about their children, parents can enjoy low tax today and enforce their children to pay for them. It means that if they care their children like themselves, Ricardian equivalence can still hold. However, if they do not care their children, an increase in government debt might induce their demand.
  3. *Distortional tax:* If a change in a tax influences the marginal benefit or cost of consumption, it affects consumer's decision and consumption. In particular, if government raises capital income tax, consumers are discouraged to save and increases consumption. In this case, obviously tax schedule matters.

Assignment

- Students must hand assignment 7 in at the next lecture.

Final Words

- Congratulation. This the end of this lecture. You are now standing in front of the door of a graduate level macroeconomics. It uses the dynamic stochastic general equilibrium model as a basic tool. If you find it interesting, see you at the graduate course.