Modern Macroeconomics I

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Abstract

This is my lecture note for modern macroeconomics I. You can distribute it with my permission. Since this is not edited yet, you may find many mistakes. Any comments are welcome.

1 Introduction -What is Macroeconomics

Macroeconomics is a study to explain the behavior of aggregate data such as GDP per capita, inflation rate and unemployment rate. Macroeconomists consider that these variables have a correlation with our happiness and make a judgement on the healthiness of our economy. If they show problems of the economy, macroeconomists want to know what causes these problems and how to cure them. To do so, we need to infer what is the structure of the economy that brings the movement of the observable data. Based on this inference, macroeconomists predict the impacts of changes in government' policies on the behavior of the aggregate data.

The next chapter I explains what is the three important data of macroeconomics: GDP, CPI and unemployment rate. These are the main variable which macroeconomists frequently use.

The chapter 3 develops a basic framework of macroeconomics. This model provides a unified framework to analyze economic growth and stabilization policy in the chapter 4 and 5.

The chapter 4 explains the neoclassical growth model. It shows that the neoclassical growth model systematically explains several stylized facts of the developed countries. Taking the neoclassical growth model as a starting point, I will ask different questions. What determines a long run growth rate of GDP per capita? Why some counties are rich, while others poor?

The chapter 5 discusses stabilization policy. Based on the neoclassical growth model I have developed in the Chapter 3, I introduce money and government in the model. I show that stabilization policy through monetary policy and fiscal policy does not change GDP per capita in the long run. Modifying the model, however, I argue that stabilization policy has an impact on GDP in the short run. A traditional IS-LM

model plays a central role in this world. I also argue that the impact of stabilization policy on unemployment and political difficulties of stability policy.

The chapter 6 develops a micro foundation of macroeconomics. Lucas (1976) insisted that simple reduced form estimation cannot be used as a foundation of a policy evaluation. If economic agents rationally predict future policy changes, the future policy changes affect agents' expectation and agents' behavior and their decision. Therefore, parameters estimated by reduced form estimation are a function of agents' expectation about the policy changes. Since changes in policy will change the parameters, we cannot use the estimated parameters in order to predict the impact of policy changes. Macroeconomists start studying a microstructure of macroeconomics on which policy changes cannot influence. Two decisions are important: consumption and investment. I will review several consumption decisions and investment decisions.

2 The Data of Macroeconomics

[1]This chapter explains three important aggregate data: GDP, CPI and unemployment rate. GDP measures the market value of economic activity, CPI measures the cost of living and an unemployment rate measures the degree of lack of jobs.

2.1 Gross Domestic Product

Gross Domestic Product (GDP) is the gross sum of the value added of each product measured by market prices in a country during a period. Gross Domestic Product (GDP) can be viewed as the total income of the whole economy. Other interpretation of GDP is the total expenditure on the economy's output of goods and services. For every transaction, buyers' expenditure is equivalent to sellers' income. Therefore, for the economy as whole, expenditure must equal income. National income accounting systematically measures GDP.

Five main features of GDP: I explain five main features of GDP.
The use of market value: GDP evaluates the value of goods and services by their market value since the prices of goods and services reflect how much consumers are willing to pay for them. Then GDP sums up the market value of goods and services in a country. For example, Suppose that a country produce 5 apples and 10 bananas, and the price of an apple is \$2 and the price of a banana is \$1. Then

$$GDP = 2 \times 5 + 10 \times 1 = 20$$

The value added: GDP is the sum of the value added of each product. The value added of a firm equals the value of the firm's output minus the value of intermediate

[1]This chapter is based on Mankiw (2000) Macroeconomics, Chapter 2.

product that the firm purchases. Since the value of intermediate goods is reflected by the market price of final goods. It roughly measures only the value of final goods. For example, a firm purchases an orange from a farmer by \$0.6 and sells an orange juice by \$1. Then the value added of the orange juice is \$0.4. If a farmer does not buy any intermediate goods, then the value added of an orange is \$0.6. Therefore the value added of two products equals

$$\$0.4 + \$0.6 = \$1.$$

This is the same as value of final goods, an orange juice. Since \$1 already reflects the value of an orange, \$0.6. Including the value of an orange in GDP becomes double counting of the value.

GDP vs. GNP: GDP measures the total income in a country not by residents of the country. Gross National Product (GNP) measures the total income earned by residents of the country. The difference is factor payments (wage, profits and rent) from abroad and factor payments to abroad:

$$\begin{aligned} GNP &= GDP \\ &\quad + factor\ payments\ from\ abroad \\ &\quad - factor\ payments\ to\ abroad. \end{aligned}$$

For example, if you stay in Japan, you work in China and receive wages, your contribution is GDP in China, but not GNP in China counts your contribution. Similarly GNP in Japan, but not GDP in Japan counts your contribution.

Flow vs. Stock: Economists distinguishes two different types of variables: flow and stock. A flow is a quantity measure per unit of time; a stock is a quantity measure at a given time. For example, an annual income is a flow variable. It tells how much you earn in a year. When you deposit the part of your income, the saving is also a flow variable. It shows how much money you save in a year. Your saving increases your wealth. The wealth is a stock variable. It tells how much you are able to spend at a given time.

Since GDP measures the total income earned during a period, such as a year, GDP is a flow variable. It measures how much monetary value is flowing into the economy during a certain period.

Gross vs. Net: GDP is gross sum of the value added. It does not subtract the depreciation of capital from the value added- the amount of capital (plants, equipment and residential structures) that wears out during the period. When you have a computer, you cannot expect that you use it forever. Eventually, the computer will be broken or obsolete. Therefore, the depreciation of the computer can be seen as cost of using computer. Hence, we need to subtract from the value added to estimate net value of the value added. There is no statistics like Net Domestic Product. But there is Net National Product (NNP):

$$NNP = GNP - Depreciation$$

Some details for computing GDP: Let me explain additional important details for computing GDP.
Used goods: the sale of used goods does not increase the additional value in a country. Therefore, the sale of used good is not included as part of GDP.
The treatment of inventories: When a firm produces a good, say milk, nobody can guarantee that it can sell all milk. If it cannot sell part of milk, two cases happen. Milk is spoiled and need to be thrown it away. Then it does not produce any value. So it is not counted as part of GDP. But if you can store milk and sell during the next period, it produces additional value during the next period. In this case, National Income Accounting system treats the inventory as the sale of goods to themselves during this period. Hence it is counted as part of GDP of this period. When milk is sold during the next period, it can be seen as the transfer of used goods from the firm to consumers. Hence it does not affect GDP of the next period. Because of this treatment of inventories, all goods produced are purchased by somebody. Therefore, total income always equals total expenditure in a country.
Imputations: When some goods are not sold in a market, they do not have market prices. If GDP includes these goods and services, we must estimate their value. Such an estimated value is called imputed value. Let me explain important imputation methods.

1. The Value of Housing: When you rent an apartment, you are paying a landlord for housing services. Hence the rent is part of GDP. When you own a house. You can enjoy the similar service, but you do not pay the rent. GDP includes the rent that house owners pay to themselves. So we need to estimate what the market rent for a house would be if it were rented.

2. Home Production and Durable Goods: In principle, we can also estimate other goods or services: cars and jewelry owned by households, and meals cooked at home etc. But the imputation of these goods or services is not made in practice. The value of these rental service and home production is left out from GDP.

3. Government Services: There is no market price for policy officers, fire fighters and so on. The national income accounts includes these values in GDP by valuing them at their cost.

4. Underground Economy: no imputation is made for the value of goods and services sold in the underground economy. An underground economy is the part of economy that people hide from government. Illegal drugs or prostitutes are examples.

Real GDP vs. Nominal GDP: Since GDP is measured by the market prices of

goods and services, GDP increases both when prices increases and when the quantities of outputs increases. If a high inflation increases GDP, GDP cannot be a suitable measure of economic well-being. In order to avoid the impact of inflation, economists separate real GDP from nominal GDP.

Nominal GDP uses current prices to measure the value of goods and services; real GDP uses a constant set of prices to measure the value of goods and services. In order to compute real GDP, economists choose the base year. Using base year prices economists measure the value of goods or services. Let me explain how to compute real GDP using example. Consider a country in which people produce only apples and bananas during 2000 and 2001. Let me choose 2000 is the base year. Then

$$\begin{aligned}\text{Real GDP in 2000} &= (\text{the price of apples in 2000}) \times (\text{the quantities of apples in 2000}) \\ &\quad +(\text{the price of bananas in 2000}) \times (\text{the quantities of bananas in 2000})\end{aligned}$$

$$\begin{aligned}\text{Real GDP in 2001} &= (\text{the price of apples in 2000}) \times (\text{the quantities of apples in 2001}) \\ &\quad +(\text{the price of bananas in 2000}) \times (\text{the quantities of bananas in 2001})\end{aligned}$$

In this way, an increase in price does not have any impact on real GDP. Hence this is the better measure of well-being.
GDP deflator: The ratio of nominal GDP to real GDP is called GDP deflator:

$$GDP\ Deflator = \frac{\text{Nominal GDP}}{\text{Real GDP}}$$

The GDP deflator captures the movement of overall level of prices in the economy.

International Comparison and International Prices: Different countries uses different currencies. In order to compare income across countries, which prices should we use? One way to convert prices is use of exchange rates. Exchange rates convert one currency to others. Using exchange rate we can express US$ value of apples in Japan.

But it is well known that exchange rates frequently change without any change in an economy. Investors speculate the movements of exchange rates to make money. This behavior affects the determinants of the exchange rates. We can hardly imagine that a change in the value of the US dollars from 130 yen to 120 yen in a month reflects a change in the fundamental value of Japanese yen.

The United Nations International Comparison Projects (ICP) developed international prices to make real GDP comparable across countries. They collect the price and expenditure of goods and services, which is converted to US dollars by exchange rate. Then for each goods they compute the weighted average of the price of the goods across countries by taking the country's share of expenditure as its weight. This is an international price of the good. Since Japanese consumes rice more than American, an change in demand for rice in Japan has more impact on the demand for

rice in the world. The weight is meant to capture this difference. Using international price Penn World Table reports their estimate of real GDP that is comparable across countries.

Purchasing-Power Parity: Purchasing-Power Parity (PPP) is the ratio of nominal GDP to real GDP measured by international prices

$$PPP = \frac{\text{Nominal GDP}}{\text{Real GDP measured by international prices}}$$

GDP per capita vs. GDP per worker: GDP per capita is GDP divided by total population:

$$\text{GDP per capita} = \frac{\text{GDP}}{\text{total population}}.$$

GDP per capita measures the average income of a country. It tells us how rich a country is.

Recently some researchers claimed that this measure systematically underestimates well-being of developing countries. Many industries are not developed in a developing countries: restaurants, maintenance of machine etc. The production of these goods are mainly done in household in developing countries. Since GDP cannot measure the value of home production, GDP per capita may underestimate well-being of developing countries.

Alternative measure is GDP per worker, which is GDP divided by the number of labor force.

$$\text{GDP per worker} = \frac{\text{GDP}}{\text{the number of labor force}}.$$

When some people are specialized in home production, they are out of labor force. Since GDP does not value home production, it may be reasonable to divide it by the number of labor force.

The Components of Expenditure: What GDP consists of? The demand for output (GDP), Y, can be divided into consumption of domestic goods and services, C^d, investment in domestic goods and services, I^d, government purchases of domestic goods and services, G^d, and exports of domestic goods and services, EX:

$$Y = C^d + I^d + G^d + EX$$

Now consumption, C, investment, I, and government expenditure, G can be divided into two components:

$$\begin{aligned} C &= C^d + C^f \\ I &= I^d + I^f \\ G &= G^d + G^f \end{aligned}$$

where C^f is consumption of foreign goods and services, I^f is investment in foreign goods and services and G^f is government purchases of foreign goods and services. Hence

$$\begin{aligned} Y &= C + I + G + EX - \left(C^f + I^f + G^f\right) \\ &= C + I + G + EX - IM \\ &= C + I + G + NX \end{aligned}$$

where IM and NX denotes import and net export, respectively.

Consumption is the goods and services bought by households. It can be divided into three subcategories: nondurable goods, durable goods and services. Investment consists of goods bought for future use. It is also divided into three subcategories: business fixed investment, residential fixed investment and inventory investment. Government expenditure is the goods and service bought by local and central governments. Net exports are the value of goods and services exported to other countries minus the value of goods and services that foreigners provide us.

2.2 The Consumer Price Index

Price of everything changes over time. That is, cost of living changes. An increase in the overall level of price is called inflation; a decrease in it is called deflation. In order to analyze change in overall cost of living, we need measure of a single index measuring the overall level of prices. Consumer price index (CPI) is the most commonly used measure of such index. This section explains how to compute CPI. How does CPI differ from GDP deflator.

CPI: Economists computes the price of a basket of goods and services purchased by a typical consumer. The CPI is the price of this basket of goods and services relative to the price of same basket in some base year. For example, consider a country in which typical consumers buy 5 apples and 10 bananas in a year during 2000 and 2001. Then the basket of goods consists of 5 apples and 10 bananas. Let me choose 2000 as the base year. The CPI can be defined as follows:

$$CPI \text{ in } 2000{=}1$$

$$CPI \text{ in } 2001 = \frac{5 \times \text{Price of apples in } 2001 + 10 \times \text{Price of bananas in } 2001}{5 \times \text{Price of apples in } 2000 + 10 \times \text{Price of bananas in } 2000}$$

CPI vs. the GDP Deflator: Both CPI and GDP deflator measures overall level of price. But there are three main differences.

1. The GDP deflator measures the price of all goods and services produced; CPI measures the prices of only the goods and services bought by consumers.

2. The GDP deflator includes only goods produced in a country. It excludes imported goods. CPI includes imported goods if consumers buy the goods.

3. The CPI assign fixed weights to the prices of different goods; the GDP deflator allows the basket of goods to change over time as the composition of GDP changes.

Despite these differences, CPI and GDP deflator shows a similar behavior.

2.3 The Unemployment Rate

The unemployment is another important index many macroeconomists are concerned about. It measures the percentages of people who want to work but who do not have jobs. This section explains how to estimate the unemployment rate.

Unemployment Rate: People are called unemployed when

1. They do not have a paid job.

2. They conducted a job seeking activity

3. If there is a job, they can do it soon (They are available).

On the other hand, people are called employed if they do have a paid job. The sum of employed workers and unemployed workers are called

labor force. The unemployment rate is the percentage of labor force that is unemployed

$$\text{unemployment rate} = \frac{\text{number of unemployed persons}}{\text{labor force}} \times 100$$

Labor Force Participation rate: The similar concept is the labor force participation rate. It is defined as the percentage of adult population (16 years and older) that is in the labor force:

$$\text{labor force participation rate} = \frac{\text{labor force}}{\text{adult population}} \times 100$$

If a person is 16 years old or more and he is neither employed not unemployed, he is not in the labor force. He could be a housekeeper, a student or somebody else.

3 The Basic Framework of Macroeconomics

This chapter provides a basic framework of macroeconomics. This model provides the starting points of asking a central question of macroeconomics. Using this framework, I will construct the neoclassical growth model in the chapter 4. Then I will ask the following questions. Why some countries are rich; others poor? What is the source of long run growth? In the chapter 5, I applied this model to the analysis of stabilization policy. Modifying the basic model, I derive the conditions stabilization policy works.

3.1 Model

In order to focus the behavior of aggregate data, macroeconomists ignore several heterogeneities. We know that different consumers have different taste, different firms produce different goods. Nonetheless, complicated models often do not provide any meaningful insight. Therefore, in order to simplify our analysis, we assume a representative firm and a representative consumer, and focus interactions of these two agents. Let's ignore government expenditure and net export.

The consumer owns all resources. In particular, macroeconomists classify two important resources: capital and labor. Capital includes machines, equipment, buildings and so on. Labor is the amount of time the consumer spends for production. The firm employs labor and capital and pay the wage and the rental price. A labor market and a capital market trade labor and capital, respectively.

When the firm produce output, it must be purchased by consumers (or firms) for consumption or investment.

The Firm's problem: One of the most important foundations of macroeconomics is the aggregate production function. It is a function from aggregate capital and labor to aggregate output:

$$Y_t = F(K_t, T_t L_t)$$

where Y_t is output on date t, K_t is physical capital stock on date t, L_t is labor input on date t and T_t is a labor augmenting technology. Note that an increase in T_t has the same impact as an increase in labor.

Of course in reality, we observe the variety of capital (computers, cars, and buildings etc.) and the variety of labor (skilled workers, unskilled workers and managers etc.) The above production function ignores this heterogeneities and resource allocations among these inputs. Then it focus the accumulation of aggregate capital stock and the movement of aggregate labor input.
Property of the Aggregate Production Function:

1. No production when no resources: $F(0, \cdot) = 0$ and $F(\cdot, 0)$.

2. An increase in capital and labor input: $F_1 > 0$ and $F_2 > 0$. This is intuitive. When the firm employs more capital or workers, it increases output.

3. Concave: $F_{11} < 0$, $F_{22} < 0$ and $(F_{12})^2 - F_{11}F_{22} < 0$ This means that the additional productivity of capital or labor is diminishing. Although an increase in the amount of computers always increases output, the extra output would be larger when the firm does not have computer than when the firm has many computers.

4. F is constant return to scale in K and L:

$$tF(K,L) = F(tK,tL), \ for \forall t > 0. \tag{1}$$

Why should the aggregate production function be constant return to scale? There are two reasons.

(a) whatever individual production functions, when these are aggregated, we can find a CRS production function that is consistent with the sum of individual production functions. For example, assume that an individual plant has a production function $y = f(l)$. Assume that a manager establishes the same n plants. Then the aggregate output Y is

$$\begin{aligned} Y &= yn \\ &= f(l)\, n \end{aligned}$$

Then define an aggregate production function production function F such that

$$Y = F(n, ln) \equiv f(l)\, n, \ \forall n > 0$$

Clearly this is constant return to scale in n and l/h. Note that $K = n$ and $L = ln$ for this economy. Since we can always replicate the same plant, the proportional increase in input can increase output proportionally. That is the essentially the property of CRS.

(b) It is consistent with a competitive market assumption. Given a constant return to scale production function, it is shown that

$$F(K,TL) = F_K(K,TL)\, K + F_L(K,TL)\, TL. \tag{2}$$

Differentiate both sides of equation (1) by t, then we get

$$F(K,TL) = F_K(tK,tTL)\, K + F_L(tK,tTL)\, TL.$$

Set $t = 1$. Then we get equation (2). It is shown later that

$$\begin{aligned} F_K(K,TL) &= r \\ F_L(K,TL)\, T &= w \end{aligned}$$

where r is the real rental price of capital and w is the real wage rate. Hence equation (2) implies that there is no economic profit:

$$F(K, TL) = rK + wL$$

When the market is competitive, more entrepreneurs will enter as long as economic profits are positive. You may think that in reality we can observe profits. That is because the concept of economic profit differs from usual accounting profit. Since three agents play a role in the firm: workers, owners of capital and owner of the firm, the firms revenue must be divided among wages, the return to capital and economic profit:

$$Y = wL + rK + \pi$$

where π is economic profit. But in reality, a firm's owner owns capital also. Hence, we cannot distinguish economic profits from return to capital. It means

$$Accounting\ profit = \pi + rK$$

Under a perfect competitive market, $\pi = 0$. Hence observable accounting profits is approximated by the return to capital.

5. Inada Conditions: these are a little bit technical conditions. It means that the marginal product of capital (or labor) tends to 0, when capital (or labor) goes infinite and that it tends to infinite, when capital (or labor) is 0.

$$\begin{aligned} \lim_{K \to 0} F_K &= \lim_{L \to 0} F_L = \infty, \\ \lim_{K \to \infty} F_K &= \lim_{L \to \infty} F_L = 0. \end{aligned}$$

The example of the aggregate production function: Cobb-Douglas Production Function

$$Y = AK^{\alpha} L^{(1-\alpha)}$$

When the production function is Cobb-Douglas, the value share of inputs in the value of output is constant:

$$\begin{aligned} \alpha &= \frac{F_K K}{Y} = \frac{rK}{Y}, \\ 1 - \alpha &= \frac{F_L L}{Y} = \frac{wL}{Y}. \end{aligned}$$

The firm's problem: The firm is assumed to be maximize economic profit, because it is the interests of the firm's owner..

$$\max_{K,L} \{F(K, TL) - rK - wL\}$$

The solutions of the maximization problem is characterized by the following the first order conditions:

$$\begin{aligned} F_K(K, TL) &= r \\ F_L(K, TL)\, T &= w \end{aligned}$$

Why? If $F_K(K, TL)$ is greater than r, the revenue from an additional increase in capital is greater than the cost. Hence, the firm has an incentive to rent more capital. If $F_K(K, TL)$ is less than r, the revenue from an additional increase in capital is less than the cost. Hence, the firm has an incentive to rent less capital. When $F_K(K, TL)$ equals r, the firm has no incentive to change its production plan.

Similarly, if $F_L(K, TL)\, T$ is greater than w, the revenue from an additional increase in labor is greater than the cost. Hence, the firm has an incentive to employ more workers. If $F_L(K, TL)\, T$ is less than w, the revenue from an additional increase in labor is less than the cost. Hence, the firm has an incentive to employ less workers. When $F_L(K, TL)\, T$ equals w, the firm has no incentive to change its production plan.

Consumer: Consumers decide how much they save, how much they work, given the following constraint:

$$NS_t + C_t = i_t A_t + w_t N_t$$

where NS_t is net saving, C_t is consumption, i_t is an interest rate, A_t is the consumer's asset, and N_t is population. I assume that the consumer supply all available time for work. It means that I ignore the value of leisure. In general net saving depends on interest rate, i, and income, $i_t A_t + w_t N_t$.

$$NS_t = NS(i_t, i_t A_t + w_t N_t)$$

I will specify the function later.

Rental Firm (Intermediation): Some rental firms (or consumers) borrow money from bank and purchase of investment goods. When the firm invests, the firm can rent it out. Therefore it expects r_t is a unit price of investment. But when the firm uses it, δ proportion of capital is depreciated, therefore, the real price is $r_t - \delta$. On the other hand, in order to purchase investment, the firm must borrow money from a financial market. The interest rate is i_t.

$$\max_{I_t} \{(r_t - \delta)(A_t + I_t) - i_t(A_t + I_t)\}$$

The optimal condition is

$$r_t - \delta = i_t,$$

If $r_t - \delta > i_t$, they invest. If $r_t - \delta < i_t$, they disinvest. On the equilibrium, $r_t - \delta = i_t$.

Capital Market: Demand for capital equals to supply for capital:

$$K_t = A_t$$

Labor Market: Demand for labor equals to supply for labor:

$$L_t = N_t$$

3.2 Equilibrium

Let me summarize equilibrium conditions

Definition 1 *Given* (A_t, N_t)*, A Market Equilibrium consists of* $(Y_t, K_t, L_t, C_t, NS_t, i_t, r_t, w_t)$ *which satisfies*

1. *A Firm's Profit Maximization and the Production Function determine* (Y_t, K_t, L_t)

$$Y_t = F(K_t, T_t L_t)$$

$$\begin{aligned} F_K(K_t, T_t L_t) &= r_t \\ F_L(K_t, T_t L_t) T_t &= w_t \end{aligned}$$

2. *A Consumer's Budget Constraint and Consumption Decision determine* $(C_t,\ NS_t)$

$$NS_t + C_t = i_t A_t + w_t N_t,$$

$$NS_t = NS(i_t, i_t A_t + w_t N_t, A_t)$$

3. *An Arbitrage Condition (A Rental Firm's Investment Decision) determines* i_t

$$r_t - \delta = i_t,$$

4. *A Capital Market Clearing Condition determines* r_t

$$K_t = A_t$$

5. *A Labor Market Clearing Condition determines* w_t

$$L_t = N_t$$

Walras Law and Goods Market: What happens to the goods market? Note that consumers' budget constraint is binding. Each consumer chooses consumption decision to bind her budget constraint. Otherwise, she can increase her utility by enjoying more consumption without violating her constraints. Since an individual budget constraint is always binding, an aggregate budget constraint is also binding. This is called, *Walras Law*. I would like to show that when the aggregate budget constraint is binding and demand equals supply at all resource markets, then demand also equals supply at goods market.

First, I would like to show that capital market clearing condition, $K = A$ can be expressed by a flow term condition: gross investment equals gross saving. Second, I will show that the flow term expression is equivalent to the goods market clearing condition.

Note that $NS_t = \dot{A}_t$, since net saving increase its asset. Since capital market clears at any t, $K_t = A_t$, $\dot{K}_t = \dot{A}_t$ By definition, $\dot{K}_t = I_t - \delta K_t$. Therefore,

$$NS_t = I_t - \delta K_t$$

Define gross saving S_t as

$$S_t \equiv NS_t + \delta A_t,$$

then

$$I_t = S_t.$$

This is the flow term expression of capital market clearing condition.

Next, I show that $I_t = S_t$ is equivalent to the goods market clearing condition. From the budget constraint, the arbitrage condition, the capital market clearing condition and the labor market clearing condition,

$$\begin{aligned} NS_t + C_t &= i_t A_t + w_t N_t \\ &= (r_t - \delta) K_t + w_t L_t \end{aligned}$$

Since the production function is constant return to scale, $Y_t = r_t K_t + w_t L_t$. Hence

$$\begin{aligned} Y_t &= NS_t + \delta A_t + C_t \\ &= S_t + C_t \\ &= I_t + C_t \end{aligned}$$

This is a goods market clearing condition: demand for output (Investment and Consumption) equals supply of output.

$$Y_t = I_t + C_t$$

That is, when the interest rate is chosen to clear capital market, it also clears goods market.

4 Economic Growth and Nation's Income

[2]Based on the previous model, I develop the neoclassical growth model. This model is a starting point of analyzing important two long run questions. What is the source of the long run growth? Why are some countries poor, when others are rich?
Basic Model: I would like to start analyze the neoclassical growth model. From the previous arguments, we know that

$$\dot{K}_t = NS\left(i_t, i_t A_t + w_t N_t\right)$$

For a simple analysis Solow assumed that

$$\begin{aligned} NS\left(i_t, i_t A_t + w_t N_t\right) &= S\left(i_t, i_t A_t + w_t N_t, \delta A_t\right) - \delta A_t \\ &= S\left(i_t, Y_t - \delta A_t, \delta A_t\right) - \delta A_t \\ &= sY_t - \delta K_t \\ &= sF\left(K_t, T_t N_t\right) - \delta K_t \end{aligned}$$

where s is gross savings rate. That is, Solow assumes that gross saving is proportional to gross income and it does not depend on interest rate. Hence

$$\dot{K}_t = sF\left(K_t, T_t N_t\right) - \delta K_t$$

Now we need to specify the movement of other two variables, T_t and N_t. Assume that

$$\begin{aligned} \dot{T}_t &= gT_t, \\ \dot{N}_t &= nN_t. \end{aligned}$$

That is, we assume that the growth rate of technological growth and population growth is constant.

Dynamics of capital stock per efficiency unit: The above equations have three dynamic equations, which is difficult solve. One way to analyze this model is to normalize every variable by per efficiency units. Because F is constant return to scale,

$$\begin{aligned} Y_t &= F\left(K_t, T_t N_t\right) \\ &= F\left(\frac{K_t}{T_t N_t}, 1\right) T_t N_t \end{aligned}$$

[2]This chapter is mainly based on David Romer (2001), Advanced Macroeconomics,The McGraw-Hill Companies, Inc.

Hence

$$\begin{aligned} y_t &= f(k_t), \\ where\ f(k_t) &= F(k_t, 1), \\ y_t &= \frac{Y_t}{T_t N_t}, \\ k_t &= \frac{K_t}{T_t N_t}. \end{aligned}$$

Now

$$\begin{aligned} \dot{k}_t &= \frac{d\left(\frac{K_t}{T_t N_t}\right)}{dt} \\ &= \frac{\dot{K}_t}{T_t N_t} - \frac{K_t\left(\dot{T}_t N_t + T_t \dot{N}_t\right)}{(T_t N_t)^2} \\ &= \frac{\dot{K}_t}{T_t N_t} - \frac{K_t}{T_t N_t}\left(\frac{\dot{T}_t}{T_t} + \frac{\dot{N}_t}{N_t}\right) \\ &= \frac{\dot{K}_t}{T_t N_t} - \frac{K_t}{T_t N_t}\left(\frac{\dot{T}_t}{T_t} + \frac{\dot{N}_t}{N_t}\right) \end{aligned}$$

Since $\dot{K}_t = sY_t - \delta K_t$, $\dot{T}_t = gT_t$ and $\dot{N}_t = nN_t$,

$$\begin{aligned} \dot{k}_t &= \frac{sY_t - \delta K_t}{T_t N_t} - k_t(g + n) \\ &= sy_t - (g + n + \delta) k_t \end{aligned}$$

Therefore we can summarize the dynamics of our macro economy by one dynamic equation:

$$\dot{k}_t = sf(k_t) - (g + n + \delta) k_t. \tag{3}$$

In order to analyze this dynamic equation, let me define the steady state.

Definition 2 *Steady state (or balanced growth) is the economic condition on which* $\{(c_t^*, y_t^*, k_t^*)\}$*satisfies*

$$\dot{c}_t^* = \dot{y}_t^* = \dot{k}_t^* = 0$$

where $c_t = \frac{C_t}{T_t N_t}$, $y_t = \frac{Y_t}{T_t N_t}$, *and* $k_t = \frac{K_t}{T_t N_t}$.

Therefore, on the steady state k^* must satisfy

$$sf(k^*) = (g + n + \delta) k^* \tag{4}$$

Look Figure 1. It shows two important properties of the economy.

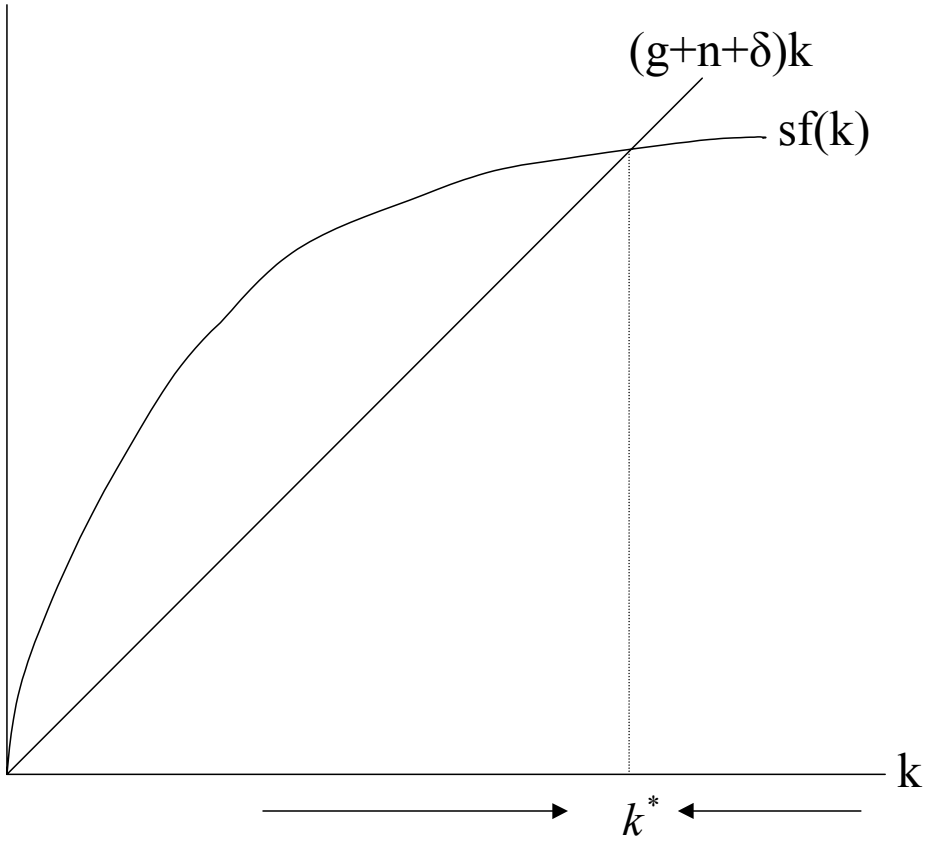

Figure 1: The dynamics of neoclassical growth model

1. There exists a unique steady state.

2. For any initial value of k_0, economy must converge to the steady state.

Steady State Analysis: Let me analyze the impact of saving ratio on the steady state on k^*

$$f(k^*)\,ds + sf'(k^*)\,dk^* = (g+n+\delta)\,dk^*$$

$$\begin{aligned}\frac{dk^*}{ds} &= \frac{f(k^*)}{(g+n+\delta) - sf'(k^*)} \\ &= \frac{f(k^*)}{s\left[\frac{f(k^*)}{k^*} - f'(k^*)\right]} > 0\end{aligned}$$

Since an increase in s increases k^* and therefore y^* and $\frac{Y}{N}$. Note that an increase in saving ratio increases the level of per capita GDP, but not a long run growth rate. That is, s has a level effect, but not a growth effect.

Kaldor's Stylized Fact (1963) and the Neoclassical Growth Model on the Steady State: The neoclassical growth theory predicts that eventually economy must converges to the steady state. So we expect that the behavior of real economy can be approximated by the behavior on the steady state. Let me examine empirical relevance of neoclassical growth model. Kaldor (1963) pointed out 6 stylized facts of economic growth. These facts are repeatedly observed by aggregate data of OECD countries. I would like to examine How the neoclassical growth model explains these stylized facts.

1. *The growth rate of GDP per capita is nearly constant:*

$$\frac{\frac{d\left(\frac{Y_t}{N_t}\right)}{dt}}{\frac{Y_t}{N_t}} = \frac{\frac{d(y_t^* T_t)}{dt}}{y_t^* T_t} = \frac{y_t^* \frac{d(T_t)}{dt}}{y_t^* T_t} = g$$

   The neoclassical growth model predicts that the long run growth rate depends only on the technological progress and the technological progress must be constant.

2. *The growth rate of capital per capita is nearly constant:.*

$$\frac{\frac{d\left(\frac{K_t}{N_t}\right)}{dt}}{\frac{K_t}{N_t}} = \frac{\frac{d(k_t^* T_t)}{dt}}{k_t^* T_t} = \frac{k_t^* \frac{d(T_t)}{dt}}{k_t^* T_t} = g$$

3. *The rate of return to capital is nearly constant:*

$$\frac{dY_t}{dK_t}|_{k_t=k_t^*} = f'(k_t^*) = const$$

4. *The ratio of physical capital to output is nearly constant:*

$$\frac{K_t}{Y_t} = \frac{k_t^* T_t N_t}{y_t^* T_t N_t} = \frac{k_t^*}{y_t^*} = const$$

5. *The shares of labor and physical capital are nearly constant:* Note that the marginal productivity of capital and labor is

$$r_t = \frac{dY_t}{dK_t} = \frac{df(k_t) T_t N_t}{dK_t} = f'(k_t^*) \frac{T_t N_t}{T_t N_t} = f'(k_t^*)$$

$$\begin{aligned} w_t &= \frac{dY_t}{dL_t} = \frac{df(k_t) T_t N_t}{dL_t} \\ &= f(k_t) T_t - f'(k_t) \frac{K_t T_t}{(T_t N_t)^2} T_t N_t \\ &= [f(k_t) - f'(k_t) k_t] T_t. \end{aligned}$$

Hence

$$\frac{r_t K_t}{Y_t} = \frac{f'(k_t^*) k_t^*}{y_t^*} = \frac{f'(k_t^*) k_t^*}{f(k_t^*)} = const$$

$$\frac{w_t N_t}{Y_t} = \frac{[f(k_t^*) - f'(k_t^*) k_t^*] T_t N_t}{Y_t} = \frac{[f(k_t^*) - f'(k_t^*) k_t^*]}{f(k_t^*)} = const$$

6. *The growth rate of output per worker differs substantially across countries:*

$$\frac{\frac{d\left(\frac{Y_t}{N_t}\right)}{dt}}{\frac{Y_t}{N_t}} = \frac{\frac{d(y_t^* T_t)}{dt}}{y_t^* T_t} = \frac{y_t^* \frac{d(T_t)}{dt}}{y_t^* T_t} = g$$

Hence in order to fit the neoclassical growth model to data, g must differ among countries.

Estimation of Technological Progress-Growth Accounting-: The neoclassical growth model predicts that the long run growth rate depends only on the technological progress. But in the short run, the movement of growth rate can be decomposed into the contribution of capital accumulation and the contribution of technological

progress. Let me introduce the method to decompose the contribution of technological progress from the contribution of capital accumulation from data.

$$\begin{aligned} y_t &= f(k_t) \\ \dot{y}_t &= f'(k_t)\dot{k}_t \\ \frac{\dot{y}_t}{y_t} &= \frac{f'(k_t)k_t}{f(k_t)}\frac{\dot{k}_t}{k_t} \\ &= \frac{rK_t}{Y_t}\frac{\dot{k}_t}{k_t} \end{aligned}$$

$$\left(\frac{\frac{d\left(\frac{Y_t}{N_t}\right)}{dt}}{\frac{Y_t}{N_t}} - \frac{\dot{T}_t}{T_t}\right) = \frac{rK_t}{Y_t}\left(\frac{\frac{d\left(\frac{K_t}{N_t}\right)}{dt}}{\frac{K_t}{N_t}} - \frac{\dot{T}_t}{T_t}\right)$$

$$\begin{aligned} \frac{\frac{d\left(\frac{Y_t}{N_t}\right)}{dt}}{\frac{Y_t}{N_t}} &= \frac{rK_t}{Y_t}\frac{\frac{d\left(\frac{K_t}{N_t}\right)}{dt}}{\frac{K_t}{N_t}} + \left(1 - \frac{rK_t}{Y_t}\right)\frac{\dot{T}_t}{T_t} \\ &= \left(1 - \frac{w_t N_t}{Y_t}\right)\frac{\frac{d\left(\frac{K_t}{N_t}\right)}{dt}}{\frac{K_t}{N_t}} + \frac{w_t N_t}{Y_t}\frac{\dot{T}_t}{T_t} \end{aligned}$$

$$\begin{aligned} \frac{\frac{d\left(\frac{Y_t}{N_t}\right)}{dt}}{\frac{Y_t}{N_t}} &= \left(1 - \frac{w_t N_t}{Y_t}\right)\frac{\frac{d\left(\frac{K_t}{N_t}\right)}{dt}}{\frac{K_t}{N_t}} + R(t) \\ &\text{where } R(t) = \frac{w_t N_t}{Y_t}\frac{\dot{T}_t}{T_t} \end{aligned}$$

- $R(t)$ is called the Solow residual. It reflects that all sources of growth other than the contribution of capital accumulation via its private return. It is interpreted as a measure of the contribution of technological change. Typically $R(t)$ explains a large proportion of economic growth rate. This evidence is consistent with the hypothesis: reality stays around the steady state.

- Young (1994) finds that the high growth rate of Hong Kong, Singapore, South Korea, and Taiwan over the past three decades is almost entirely due to rising investment, increase in labor force participation, and increase in the level of education, but not to rapid technological progress.

- Productivity slow down puzzle. $R(t)$ became small after 1970s.

4.1 Neoclassical Growth Model and Income Differences

The neoclassical growth model explains the long run behavior of developed countries fairly well if we could assume that g is constant and differs across countries. Macroeconomists starts to extends the neoclassical growth model in order to explain the development facts. First, I would like to show what is the new stylized facts. The next, I would like to ask if the neoclassical growth model can explain these facts.

New Development Facts: Parente and Prescott (1993) pointed out four main stylized facts.

1. Income difference across countries is large.
2. Wealth distribution has shifted up.
3. Relative Income distribution does not show convergence.
4. There have been development miracles and disasters.

Durlauf and Quah (1998) also pointed out that

- Relative Income distribution across countries shows two peaks.

Can the neoclassical growth model explain a large income difference?: Let me first examine whether the neoclassical growth model explains the first stylized fact: a large income differences. Let me start with rough estimation. Suppose that T_t is the same across countries. Then I can show (1) that in order to explain a large income differences, required difference in capital are too large and (2) that attributing difference in output to difference in capital implies a huge variation in the rate of return on capital. Assume that $y_t = k^\alpha$. The following exercises make the problems clear.
Rough Estimation (Lucas 1990):

1. In order to explain a large income differences, required difference in capital are too large.

$$k_t = (y_t)^{\frac{1}{\alpha}}$$

$$\frac{\left(\frac{K_t}{N_t}\right)_{us} \frac{1}{T_t}}{\left(\frac{K_t}{N_t}\right)_{India} \frac{1}{T_t}} = \frac{\left(\left(\frac{Y_t}{N_t}\right)_{us} \frac{1}{T_t}\right)^{\frac{1}{\alpha}}}{\left(\left(\frac{Y_t}{N_t}\right)_{india} \frac{1}{T_t}\right)^{\frac{1}{\alpha}}}$$

$$\frac{\left(\frac{K_t}{N_t}\right)_{us}}{\left(\frac{K_t}{N_t}\right)_{India}} = \frac{\left(\left(\frac{Y_t}{N_t}\right)_{us}\right)^{\frac{1}{\alpha}}}{\left(\left(\frac{Y_t}{N_t}\right)_{india}\right)^{\frac{1}{\alpha}}}$$

Since $\left(\frac{Y_t}{N_t}\right)_{us} / \left(\frac{Y_t}{N_t}\right)_{India} = 10$ and $\alpha = \frac{1}{3}$, $\left(\frac{K_t}{N_t}\right)_{us} / \left(\frac{K_t}{N_t}\right)_{India} = 1000$.

2. Attributing difference in output to difference in capital implies a huge variation in the rate of return on capital.

$$\begin{aligned} MRK &= f'(k_t) \\ &= \alpha k_t^{a-1} \\ &= \alpha (y_t)^{\frac{\alpha-1}{\alpha}} \end{aligned}$$

$$\begin{aligned} \frac{MRK_{us}}{MRK_{India}} &= \frac{\alpha \left(\left(\frac{Y_t}{N_t}\right)_{us} \frac{1}{T_t} \right)^{\frac{\alpha-1}{\alpha}}}{\alpha \left(\left(\frac{Y_t}{N_t}\right)_{India} \frac{1}{T_t} \right)^{\frac{\alpha-1}{\alpha}}} \\ &= \frac{\left(\left(\frac{Y_t}{N_t}\right)_{us} \right)^{\frac{\alpha-1}{\alpha}}}{\left(\left(\frac{Y_t}{N_t}\right)_{India} \right)^{\frac{\alpha-1}{\alpha}}} \end{aligned}$$

Since $\left(\frac{Y_t}{N_t}\right)_{us} / \left(\frac{Y_t}{N_t}\right)_{India} = 10$ and $\alpha = \frac{1}{3}$, $\frac{MRK_{us}}{MRK_{India}} = \frac{1}{100}$.

Both of example indicate that α is too small to fit data.

Cross Country Regression (Mankiw, Romer and Weil (1992)): Assume that every country has the same production function, $f(k_t) = k_t^{\alpha}$ and that every country is on its steady state, then

$$\begin{aligned} s (k_t^*)^{\alpha} &= (g + n + \delta) k_t^* \\ (k_t^*)^{1-\alpha} &= \left(\frac{s}{g + n + \delta} \right) \\ k_t^* &= \left(\frac{s}{g + n + \delta} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

Hence

$$\begin{aligned} \frac{Y_t}{N_t T_t} &= (k_t^*)^{\alpha} \\ \frac{Y_t}{N_t} &= \left(\frac{s}{g + n + \delta} \right)^{\frac{\alpha}{1-\alpha}} T_t \\ &= \left(\frac{s}{g + n + \delta} \right)^{\frac{\alpha}{1-\alpha}} T_0 e^{gt} \end{aligned}$$

$$\log \frac{Y_t}{N_t} = \log T_0 + gt + \frac{\alpha}{1-\alpha} \log (s) - \frac{\alpha}{1-\alpha} \log (g + n + \delta)$$

Suppose that

$$\log T_0 = a + \varepsilon_i$$

where a is constant and ε_i is country specific shock. Suppose that $t = 0$. Then

$$\log \frac{Y_{i0}}{L_{i0}} = a + \frac{\alpha}{1-\alpha} \log (s_i) - \frac{\alpha}{1-\alpha} \log (g + n_i + \delta) + \varepsilon_i$$

Assume that g and δ is constant, 0.05, across countries and s and n is independent of ε_i.
Data: Summers and Heston dataset. Non-Oil countries, Non-countries except for grade D countries and small population countries, OECD countries.

1. n.. the average growth of working age population over 1960-1985, where working age is defined as 15 to 64.

2. s...the average share of real investment in real GDP over 1960-1985.

3. Y/N...real GDP in 1985 divided by the working age population in that period.

The following 4 results are obtained by them.

1. The coefficients on saving and population growth have predicted signs and 2 of 3 are significant.

2. The restriction that the coefficient of $\ln(s)$ and $\ln(g + n + \delta)$ is the same cannot be rejected.

3. High R^2.

4. The estimated α is much higher than 1/3.

Conclusion: Although the neoclassical growth model qualitatively shows correct relation, but quantitatively, α is too small to explain a huge income difference. In order to explain the large income differences, we need to differ T_t.

4.2 Knowledge Accumulation and the Source of Long Run Growth.

As I have shown, the long run growth rate is determined by g on the neoclassical growth model. It is natural to ask what determines g. This is the main question of endogenous growth model. One potential candidate is the accumulation of knowledge. There are several forms of the accumulation of knowledge: education, R&D and a basic science. But there is a common feature of knowledge: knowledge is nonrival. Once somebody invents new idea, others can imitate it. This property brings an implication for the long run property of knowledge accumulation.

Jones (1995) and (2002): Initially many economists consider the following knowledge accumulation function

$$\dot{T}_t = BN_t^T T_t \tag{5}$$

where N_t^T is the number of workers who works at a knowledge accumulation sector. This equation implicitly assume that everybody has an equal probability to invent new idea. That is, the larger the population of the knowledge accumulation sector, the higher the probability to find new invention. Because of externality, past knowledge T_t, has a positive impact on the creation of new knowledge, T_t.

The above equation implies that

$$g = BN_t^T. \tag{6}$$

The growth rate of knowledge is proportional to the level of population. That is, the model has a scale effect. Since a government can conduct a policy that increases researchers or educational attainments, this model implies that a policy can change economic growth rate. Jones (1995) criticize that this observation is against evidence in the OECD countries. Since world war II, we observe the number of scientists engaged in R&D has dramatically increased, but the growth rate of TFP is quite stable. This evidence is against the above theory. Jones (1995) proposed different specification:

$$\dot{T}_t = BN_t^T T_t^\beta \tag{7}$$

where $0 < \beta < 1$. The previous model implies that an knowledge accumulation is constant return to scale in T_t, but the new model is decreasing return to scale in T_t. This difference makes a large difference in the implication of model. To see this, the above model implies

$$g = \frac{BN_t^T}{T_t^{1-\beta}}. \tag{8}$$

On the steady state, g is constant. Equation (8) shows that an increase in the level of technology reduces growth rate. Hence, although an increase in population temporarily increases the growth rate of technology, since it also increases the level of technology, in the long run eventually the effect of population vanish.

The total differentiation of the above equation implies

$$g = \frac{n^T}{1-\beta},$$

where $n^T = \frac{\dot{N}_t^T}{N_t^T}$. Hence the new model implies that the growth rate is proportional to population growth at the knowledge accumulation sector. This observation is roughly consistent with experience at several OECD countries. However, typically the growth rate of population is taken as given in a model. It means that it is difficult for a government to affect the long run growth rate of the economy.

Given his model, Jones (2002) further investigates the source of economic growth. Jones (2002) decomposes n^T into two parts and investigates the main source of economic growth in the US after world war II. We can think that N_t^T consists of the share of workers at knowledge sector and the total employment: $N_t^T = h_t N_t$ where $h_t = \frac{N_t^T}{N_t}$. Therefore,

$$n^T = \frac{\dot{h}_t}{h_t} + \frac{\dot{N}_t}{N_t}.$$

That is, n^T is decomposed into a raise in the share of workers at knowledge accumulation sector and in total employment growth. Jones (2002) documented that a rise in educational attainment and research intensity can explain 80 % of recent U.S. growth; population growth explains less than 20 percent. Note that an increase in h_t cannot continue indefinitely since it is bounded by 1. Jones (2002) call current economic growth constant growth, but not balanced growth, since it is not sustainable. He predicts that sooner or later, the world growth rate must decrease to the level of population growth.

Of course, I am talking about a super long run now. Since the knowledge spillover goes beyond a country, $\frac{\dot{h}_t}{h_t}$ would continue to be positive since many developing countries would increases the proportion of scientists and engineers.

Kremer (1993): Kremer tested the implication of externality by using super long run data: one million B.C. to 1990. He assume two assumptions.

1. People's chance of being lucky or smart enough to invent new is independent of population. Therefore, the larger population the higher the growth rate of technology.

2. Population is limited by the available technology. Therefore, growth rate of population is proportional to the growth rate of technology.

Because of these two assumptions, we can show that population growth is higher when population is larger. Kremer (1993) uses data from one million B.C. to 1990 and show that robust positive correlation between population growth and the level of population. Let me analyze Kremer's intuition by using above model. Suppose that

$$Y_t = A\left(K_t\right)^{\alpha}\left(T_t\left(1-h\right)N_t\right)^{1-\alpha}$$
$$\dot{T}_t = BhN_tT_t^{\beta}$$

where $h = \frac{N_t^T}{N_t}$ is constant over time. The above model assume that workers have a choice to work at either the production sector or the knowledge accumulation sector, and that on the steady state the share of workers at the knowledge sector is constant.

Suppose that K_t is land and it is fixed, $K_t = K$. Since the most of human history, the main production is an agricultural sector, this is a plausible assumption.

He also makes the Malthusian assumption that population is limited by the available technology. When technology improved, population increased. Eventually, population catches up with the level of technology. Hence, the most of human history, a per capita income is the subsistence level:

$$\frac{Y_t}{N_t} = y,$$

where y is constant. Then

$$y = A\left(K\right)^{\alpha}\left(T_t\left(1-h\right)\right)^{1-\alpha}N_t^{-\alpha} \tag{9}$$

By the total differentiation with respect to t, we can derive

$$\begin{aligned} \alpha\frac{\dot{N}_t}{N_t} &= \left(1-\alpha\right)\frac{\dot{T}_t}{T_t} \\ &= \left(1-\alpha\right)BhN_tT_t^{\beta-1} \end{aligned} \tag{10}$$

Equation (9) also implies that

$$\begin{aligned} y &= A\left(K\right)^{\alpha}\left(T_t\left(1-h\right)\right)^{1-\alpha}N_t^{-\alpha} \\ T_t^{1-\alpha} &= \frac{yN_t^{\alpha}}{A\left(K\right)^{\alpha}\left(1-h\right)^{1-\alpha}} \\ T_t &= \left[\frac{y}{A\left(K\right)^{\alpha}\left(1-h\right)^{1-\alpha}}\right]^{\frac{1}{1-\alpha}}N_t^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Substituting this equation into equation (10),

$$\begin{aligned} \frac{\dot{N}_t}{N_t} &= \frac{\left(1-\alpha\right)}{\alpha}BhN_t\left(\left[\frac{y}{A\left(K\right)^{\alpha}\left(1-h\right)^{1-\alpha}}\right]^{\frac{1}{1-\alpha}}N_t^{\frac{\alpha}{1-\alpha}}\right)^{\beta-1}, \\ &= \frac{\left(1-\alpha\right)}{\alpha}Bh\left[\frac{A\left(K\right)^{\alpha}\left(1-h\right)^{1-\alpha}}{y}\right]^{\frac{1-\beta}{1-\alpha}}N_t^{1-\frac{\alpha(1-\beta)}{1-\alpha}}. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\dot{N}_t}{N_t} &= CN_t^{\theta}, \\ where\ C &= \frac{\left(1-\alpha\right)}{\alpha}Bh\left[\frac{A\left(K\right)^{\alpha}\left(1-h\right)^{1-\alpha}}{y}\right]^{\frac{1-\beta}{1-\alpha}}, \\ \theta &= 1-\frac{\alpha\left(1-\beta\right)}{1-\alpha}. \end{aligned}$$

That is population growth is an increasing function of the level of population. Using population data from 1 million B.C. to 1990, kremer tests this implication. He found a robust positive correlation between the growth rate of population and the level of population.

4.3 Knowledge Diffusion, Human Capital, and Institution

It may be plausible to think that the accumulation of knowledge could be the source of the long run growth, and would determine the long run growth rate of economic growth, $g = \frac{\dot{T}_t}{T_t}$. But does it provide a clear guidance for many developing countries? Does it explain the reason why some countries continue to be poor. As I have shown before, if T_t is the same across countries, it is difficult to reconcile the neoclassical growth model with data. So we need to assume that T_t differs across countries. If the knowledge accumulation is the only source of T_t, the obvious question arises: why the poor countries do not imitate the knowledge in the developing countries? In other word, why cannot they enjoy the externalities from riches?

One hypothesis is that knowledge diffusion demands the ability to absorb the knowledge. Jovanovic (1997) suggested that we need to distinguish two types of knowledge: technology and human capital. Technology is defined as the laws of physics that are relevant to a particular way of producing something. It is described in blueprints. Human capital is the knowledge of how to work the blueprints. Similarly, Takii (2000) emphasizes the difference between codifiable knowledge and tacit knowledge. Although codifiable knowledge can be easily transferred, the transfer of tacit knowledge demands human movement. That is, they cannot enjoy the externalities without the movement of knowledgeable people. Let me investigate how much the difference in human capital can explain income differences.

Can human capital explain the income difference?:
Hall and Jones (1999): Hall and Jones (1999) conducts the following exercise. Assume that a country i has the production function:

$$Y_i = K_i^{\alpha} \left(T_i N_i\right)^{(1-\alpha)}$$

where $T_i = A_i h_i$. The variable A_i is the unobserved productivity and h_i is the level of human capital. Then

$$\begin{aligned} 1 &= \left(\frac{K_i}{Y_i}\right)^{\alpha} \left(\frac{T_i N_i}{Y_i}\right)^{(1-\alpha)} \\ \left(\frac{Y_i}{N_i}\right)^{(1-\alpha)} &= \left(\frac{K_i}{Y_i}\right)^{\alpha} (T_i)^{(1-\alpha)} \\ \frac{Y_i}{N_i} &= \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} T_i \end{aligned}$$

Hence

$$\frac{Y_i}{N_i} = \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} A_i h_i$$

This decomposition attributes the difference in per capita income to the difference in capital output ratio, difference in human capital, and difference in productivity. Hall and Jones estimate each variable as follows.

1. $\frac{Y_i}{N_i}$...National income and labor force data are taken from Summers and Heston (1991).

2. They assume that

$$\begin{aligned} h_i &= \exp\left(0.134 \times E\right),\ if\ E \leq 4, \\ &= \exp\left(0.134 \times 4 + 0.101 \times (E-4)\right),\ if\ 4 < E \leq 8, \\ &= \exp\left(0.134 \times 4 + 0.101 \times 4 + 0.068 \times (E-8)\right),\ if\ E > 8, \end{aligned}$$

   where E is average educational attainment. The coefficients, 13.4, 10.1 and 6.8, are assumed by referring to previous research. Average educational attainment is measured in 1985 for the population aged 25 and over, as reported in Barro and Lee (1993).

3. Capital stock is estimated by the perpetual inventory method:

$$K_{t+1} = I_t + (1-\delta) K_t$$

   where δ is assumed to be 0.06. We can recursively estimate capital stock when we know the initial value. The initial year is assumed to be 1960 and the initial value is estimated by

$$K_{1960} = \frac{I_{1960}}{g+\delta}$$

   where g is the average geometric growth rate from 1960 to 1970.

4. The parameter, α, is assumed to be $\frac{1}{3}$.

5. The variable, A_i, is estimated by residual.

They summarize their results by comparing the average of five richest countries and the average of five poorest countries. Output per worker in the five richest countries is 31.7 times higher than output per worker in the five lowest countries. Capital intensity, human capital, productivity in the five richest countries are 1.8, 2.2 and 8.3 times lager than those in the five lowest countries, respectively. It shows that including human capital does not help much explaining a huge income differences.

Hendricks (2002): The above estimation implies that human capital measured by educational attainment cannot be the main factor of income differences. However, obviously, education is not only source of human capital. Clearly, school quality, on the job training, child-rearing, and prenatal care vary across countries. In order to estimate the impact of these unobserved quality differences, Hendricks use the wage data of immigrants in the US. Assuming that the wage difference in the same labor market reflects the difference in human capital, he estimate if human capital

difference can explain income differences across countries. In order to look at this point, assume that production function is

$$Y = K^{\alpha} (AhN)^{(1-\alpha)}$$

Then

$$w = MPL = (1-\alpha) K^{\alpha} (Ah)^{(1-\alpha)} N^{-\alpha}$$

Consider the wage for workers from country A and B. Assume that the worker $A's$ human capital is larger than $B's$: $h_A > h_B$,

$$\begin{aligned} \frac{w_A}{w_B} &= \frac{(1-\alpha) K^{\alpha} (Ah_A)^{(1-\alpha)} N^{-\alpha}}{(1-\alpha) K^{\alpha} (Ah_B)^{(1-\alpha)} N^{-\alpha}} \\ &= \frac{(h_A)^{(1-\alpha)}}{(h_B)^{(1-\alpha)}} = \left(\frac{h_A}{h_B}\right)^{(1-\alpha)}. \end{aligned}$$

So the difference in wage reflects the difference in human capital. Intuitively, if two persons competing in the same labor market, other conditions are the same. So the difference in wage must reflect the difference in human capital. He concludes that for countries below 40 percent of U.S. output per worker, less than half of the output gap relative to the United States is attributed to human and physical capital. Of course, in reality, we must consider several other effects. If there is discrimination against foreign workers, the wage difference is larger than the difference in human capital. If there is a country specific skill, the wage difference in the US labor market is larger than the difference in human capital at home country, because the immigrants may be more productive at home country. These factors strengthen his conclusion.

One problem of his approach is the selection bias. If the average ability of immigrants are higher than the average ability of workers in their home countries, then the wage difference is lower than average human capital difference between the US and their home country. After considering the selection bias, he concluded that the selection bias cannot change his result.

The main reason is that wage gap between the US citizens and immigrants in the US is much lower than income gap across countries. It implies that if they compete in the same labor market, workers in developing countries can earn more. That is, human capital is less likely the source of income differences.

What can we learn from this?: Researchers are still discussing the potential importance of human capital. Although evidence is not supportive, it is too early to dismiss the importance of it. As Jovanovic (1997) emphasizes, if technology specific human capital is important, large human capital itself may hamper technological adoption, since a firm must destroy old human capital to adopt new technology. It may slowdown the adoption of new technology, and therefore lower T_t. Similarly education may faster the adoption of technology as suggested by Nelson and Phelps

(1966). Both case education in a country affects the speed of technology adoption. Therefore,

$$\dot{A}_t = B(h) A_t \tag{11}$$

Then

$$T_t = A_t h = A_0 e^{B(h)t} h$$

If this is a correct model, the small variation of h may have a large impact on the level of technology and income differences.

Another possible impact of education is externality: if some body learns new knowledge, it may affect others. Since we estimated human capital, it is based on the labor market observation. Hence, it cannot capture externality. Therefore, the impact of education would be larger than it may look. These are still open questions.

Rent Seeking Activities and Institutional Arrangement: If both physical capital and human capital cannot explain the main part of income differences, what else can explain it? Many macroeconomists start to focus an institutional arrangement. In particular, the institutional arrangement to enhance productive activities and prevent unproductive (=rent-seeking) activities. Rent-seeking activities refer to any kinds of activities that merely reallocate the output without making any additional value. For example, crime, lobbying for tax benefits, and political corruption are the parts of rent-seeking activities. Let me summarize several arguments related to rent-seeking activities.

Misallocation of talent: Baumol (1990) investigated historical evidence and provided the following three hypotheses:

1. The social system, which determines the relative payoffs to different entrepreneurial activities, changes over time and across regions.

2. Entrepreneurial behavior changes according to variations in the social system.

3. The allocation of entrepreneurship between productive and unproductive activities has a large effect on the innovation of technology and dissemination of technological discoveries.

Murphy, Shleifer and Vishny (1991) provided one type of formal model that clarifies Baumal's hypotheses. It shows that 1) if a talented manager is misallocated to a declining industry, this reduces the growth rate and rent seeking rewards and also prevents a talented person from becoming an entrepreneur. They also provided evidence that countries with a higher proportion of engineering college majors grow faster, whereas countries with a higher proportion of students concentrating in law grow more slowly.

Social Infrastructure: Hall and Jones (1999) called the institutional arrangement to enhance productive activities and to prevent unproductive activities social infrastructure. They also provided evidence that the differences in human capital accumulation, physical capital accumulation, productivity and therefore output per worker are driven by differences in social infrastructure: institutions and government policies that provide incentives for individuals and firms in an economy. Olson (1996) investigated the importance of institution comparing West Germany and East Germany, and South Korea and North Korea after World War II. We can fairly assume that their fundamental was the same before the War. But we know that the market oriented countries are much successful than communist countries. The main difference would be explained by institutional arrangement.

Resistance to Technology Adoption: The concept of social infrastructure is too broad. It includes any institutional arrangements. It means that it cannot identify what sort of arrangement is most important. Parente and Prescott (1994, 1999) emphasize a particular types of rent-seeking activity: resistance to technology adoption. If the adoption of new technology is the source of long run growth and if people in many countries have the ability to adopt the technology, there must have the barrier to adopt new technology. Otherwise we cannot explain why some countries do not adopt new technology. The difference in barrier may explain the difference in income. Parente and Prescott (1994) show that the barrier to the adoption of new technology have substantial impact on income differences. Parente and Prescott (1999) argue that monopoly right can be one such barrier. When government protects a particular company or industry, new comers cannot enter the market with new technologies. It hampers the adoption of new technology.

The question is why an incumbent does not adopt new technology? Their model do not answer this question. One possibility is technology specific skill. If the incumbent has a large specific skill for old technology, they must destroy it by adopting new technology. If preventing new entrance is cheaper than adopting new technology, they may resist new technology.

5 Stabilization Policy

[3]An increase in nation's wealth is not the only target of macroeconomic policy. Every country experiences boom and recession. The main question is whether government should or can actively stabilize aggregate demand for economy. I would like to investigate how demand stabilization policy through fiscal and monetary policy affects

[3]This chapter is influenced by Mankiw (2000), Macroeconomics and Yoshikawa (1984), Makuro Keizaigaku Kenkyu (The Research in Modern Macroeconomics), Chapter 1 and 2.

real economy.

This was a central issue of policy discussions. There is little disagreement that government can play a role to increase the level of GDP; protecting property right, providing infrastructure and a good education system, and so on. But there is a little disagreement about the role of government to stabilize economy. A school of economists advocates that a government should actively stabilize demand for economy; others disagree with this opinion. However, the distance between two schools is much shorter than people commonly believe. I would like to provide the framework to understand these policy discussions.

For this purpose, I add money and government to the above basic model. Then I will show that both fiscal and monetary policy does not have any real impact on GDP. You will see that an increase in government expenditure simply increases the interest rate and reduces private investment (crowding out). An increase in money supply simply increases inflation. Hence, the government stabilization policy has no impact on economy. The main reason is that supply side determines GDP in the neoclassical growth model, and therefore, the demand side cannot change the level of GDP. Many economists view that this will happen in the long run.

Many economists view that real economy will deviate from this long run equilibrium in the short run. Then stabilization policy may have an short run impact on the real economy. In order to see this possibility, I will modify the above model. I assume that GDP can deviate from its equilibrium in the short run, and introduce the possibility that demand stabilization policy has a real impact. Then I examine how fiscal and monetary policy can change aggregate demand and the level of GDP. The famous IS-LM model plays a central role for this purpose.

Thirdly, I show several explanations that enforce the economy to deviate from the long run equilibrium. I show five prototype idea to bring this deviation and show how different explanation brings different policy implication.

Fourthly, I discuss stabilization policy and unemployment. In the long run, economy cannot eliminate natural rate of unemployment. This type of unemployment sustain in the long run because of search cost or stickiness of real wage. In order to lower this type of unemployment, government should consider other than stabilization policy. On the other hand stabilization policy has a short run impact on unemployment.

Finally, I discuss a political difficulty of stabilization policy. This occurs because government fails to make commitment on its policy. Because of lack of commitment, firms and consumers cannot trust government's policy. I show that how lack of commitment causes undesirable outcome.

5.1 Money Supply and Money Demand

What is money? Money is the stock of assets that can be readily used to make transactions. Money has three functions in our economy: a store of value, a unit of

account and a medium of exchange. First, money can be used to transfer purchasing power from the present to future (a store of value). If you have money today, you can buy goods or services in the future by using your money. Second, money provides the yardstick with which we measure the value of our transaction (a unit of account). The value of a computer is measured by money, like 150,000 yen. Third, money is a tool with which we use to buy or sell goods.

The Mechanism of Money Supply: Money supply includes both currency in the hands and deposits at banks that households can use on demand for transaction, such as checking account:

$$M^s = C + D \tag{12}$$

where M^s is money supply, C is currency and D is demand deposits. When you deposit your money, say 1000yen, in a bank, the bank can use \$1000 and make loans to somebody who needs fund and earn interest. However, banks are required to keep a proportion of deposit by law in that depositors can always withdraw money. The deposits that banks have received but have not lent out are called reserves, denoted by R. Bank of Japan regulates the reserve deposit ratio, denoted by rd. When the bank use $(1 - rd) \times 1000yen$ to make loans, $(1 - rd) \times 1000yen$ goes to public as currency. Since money supply is sum of currency and deposits, money supply is $(1 - rd) \times 1000yen + 1000yen$. In this way, the bank can increase money supply.

The total amount of YEN that is supplied by Bank of Japan is called the monetary base, denoted by B and the monetary base equals currency plus reserves

$$B = C + R \tag{13}$$

Using equation (12) and equation (13),

$$\frac{M^s}{B} = \frac{C + D}{C + R}$$

Hence

$$M^s = \frac{cd + 1}{cd + rd} B$$

where $cd = C/D$ and $rd = R/D$. The parameter $\frac{cd+1}{cd+rd}$ is called money multiplier, which is a decreasing function of rd. This equation implies that Bank of Japan can control money supply by changing monetary base and reserve-deposit ratio.

How does Bank of Japan control B, there are three instruments:

1. **Open-market operations:** Bank of Japan can sell or buy government bonds. When it buys bonds on the market, it pays yen for the bonds. Therefore, monetary base goes up. On the other hand, when it sells, it receives yen. Hence monetary base goes down.

2. **Operations at exchange market:** Bank of Japan can sell or buy dollars. When it buys dollars on the market, it pays yen for the dollars. Therefore, monetary base goes up. On the other hand, when it sells, it receives yen. Hence monetary base goes down.

3. **Change in the Discount Rate:** When commercial banks find that they do note have enough reserves, they can ask Bank of Japan to discount their bills. When Bank of Japan reduces the discount rate, commercial banks can easily borrow money from Bank of Japan and it increases monetary base.

Bank of Japan can also affect money supply by changing reserve requirements, which is the minimum reserve-deposit ratio. As we saw before, an increase in rd reduces M.

Although we assume that Bank of Japan can control money supply, there is a big discussion about it. In particular, nowadays we have several credit cards, which are alternative to traditional money. That is, we invented new money and it increases money supply. Therefore, without traditional money, we can make our transactions.

Money Demand: Why do people demand money? Economists still try to develop the foundation of money demand. There is the benefit and cost of holding money. The cost of holding money is that you miss interest. Instead of holding money, you can buy stocks or bonds. If you buy bonds, you can earn interest. But it you hold money in your pockets, you miss this opportunity. Therefore, the higher the interest rate, the smaller the demand for money. Then what is the benefit of holding money? It makes our transactions smoother. When you made a deal with your business partners, you must pay money. If you invest all your asset in stock markets, you cannot buy anything. Hence we expect that when we have more transactions, we must demand more money. Since transactions increase when real GDP is larger, we expect that the demand for money is an increasing function of real GDP.

The above discussions suggest the following money demand function.

$$\begin{aligned} \frac{M^d}{P} &= L(i, Y) \\ L_i &< 0,\ L_Y > 0 \end{aligned}$$

where M^d is the demand for money and P is prices. When prices is higher, we need more money for transactions. Hence $\frac{M^d}{P}$ can be seen as real money demand. The above equation implies that a decrease in the interest rate and an increase in real GDP raises the demand for money.

Note that this treatment of money demand is the matter of convenience. In order to analyze the impact of money in models, we must be more cautious. Who demand money, firms, consumers or government? What affects individual decisions on money demand? Without asking these questions, it is not clear how several policies affect money demand.

5.2 Stabilization Policy in the Long Run

Government Sector: Assume that government can control money supply, M^s after negotiation with Bank of Japan. Also assume that government must spend government expenditure, G, in order to provide services for consumers, but G does not affect production. Assume that government must finance G by a lump sum tax, τ, with which consumers must pay fixed cost, τ, to government Therefore

$$\tau = G$$

Equilibrium: I now add government to the original model. For the sake of simplicity, I focus static environment. The static model makes my point clearer, though it excludes several important discussions in Macroeconomics. For example, a temporal reduction in tax rate cannot be discussed in this static framework.

Definition 3 *Given* (A, N, M^s, G)*, a market equilibrium with government consists of* $(Y, K, L, C, NS, i, r, w, \tau, P)$ *which satisfies*

1. *A Firm's Profit Maximization and the Production Function determine* (Y, K, L)*:*

$$Y = F(K, TL)$$

$$\begin{aligned} F_K(K, TL) &= r \\ F_L(K, TL)\, T &= w \end{aligned}$$

2. *A Consumer's Budget Constraint and Consumption Decision determine* $(C,\ NS)$*:*

$$NS + C = iA + wN - \tau,$$

$$NS = NS(i, iA + wN - \tau)$$

3. *An Arbitrage Condition (A Rental Firm's Investment Decision) determines* i*:*

$$r - \delta = i,$$

4. *The Capital Market Clearing Condition determines* r*:*

$$K = A$$

5. *The Labor Market Clearing Condition determines* w*:*

$$L = N$$

6. Government's budget constraint determines τ:

$$G = \tau$$

7. The money market cleaning condition determines P:

$$\frac{M^s}{P} = L(i, Y) \left(\equiv \frac{M^d}{P} \right)$$

Let me classify these market equilibrium conditions for a clear analysis. Plug a capital market clearing condition, a labor market clearing condition and the arbitrage condition into both the aggregate production function and the firm's profit maximization conditions. Then we can summarize supply side conditions.

$$\begin{aligned} Y &= F(A, TN) \\ i &= F_K(A, TN) + \delta \\ w &= F_L(A, TN) T \end{aligned}$$

On the other hand, as I did before, plug the capital market clearing condition, the labor market clearing condition, the arbitrage condition, the government's budget constraint and constant return to scale production function into the budget constraint we can derive a goods market clearing condition.

Since $NS = \dot{A} = \dot{K} = I - \delta K$,

$$\begin{aligned} I &= NS(i, iA + wN - \tau) + \delta K \\ &= NS(i, Y - \delta K - G) + \delta K \\ &\equiv S(i, Y - G) \end{aligned}$$

Goods market clearing conditions and the money market clearing together provide demand side conditions. Hence, we can summarize market equilibrium conditions as follows

Proposition 4 *Given (A, N, M^s, G), a market equilibrium with government can be summarized by (Y, w, i, C, P) which satisfies*

1. Supply side conditions determine (Y, w, i):

$$\begin{aligned} Y &= F(A, TN) && (14) \\ i &= F_K(A, TN) - \delta && (15) \\ w &= F_L(A, TN) T && (16) \end{aligned}$$

2. Demand side conditions determine $(I,\ P)$*:*

$$I = S\left(i, Y - G\right) \tag{17}$$

$$\frac{M^s}{P} = L\left(i, Y\right) \tag{18}$$

Stabilization Policy vs. Supply Side Policy: Looking at supply side conditions, I can easily detect an important observation: a change in both a monetary policy, M^s, and a fiscal policy, G, cannot change the level of GDP, Y, the real interest rate, i, and the real wage rate[4]. Given population N and A, Y, r, and w are completely determined. Therefore, there is no room that a change in G and M^s affects Y, r and w.

This result does not depend on the special feature of this model; labor supply is given. Let me discuss only the effect on GDP. But obviously the impact on the interest rate and the wage is the similar. In general labor supply depends on the real wage w: $N\left(w\right)$. Plugging $K = A$ into the first order condition, $F_L\left(K, TL\right)T = w$, it is easy to see $F_L\left(A, TL\right)T = w$. This equation implies that labor demand is function of the real wage: $L\left(w\right)$. Since the real wage is adjusted so that demand for labor equals supply of labor, $L\left(w\right) = N\left(w\right)$, it determines the real wage. Let w^* denote the equilibrium real wage. Once the market determines real wage, labor supply and demand on the equilibrium are determined. Therefore, again GDP is determined:

$$Y = F\left(A, TN\left(w^*\right)\right).$$

This analysis suggests that if a government wishes to control GDP, the policy must have an impact on supply side. For example, a decrease in labor income tax may encourage workers to work. That is, if the supply of labor is a decreasing function of labor income tax, $N\left(\tau^l\right)$, $N'\left(\cdot\right) < 0$, where τ^l is the labor income tax rate., it can affect GDP:

$$Y = F\left(A, TN\left(\tau^l\right)\right).$$

More important policy would be a reduction of capital income tax. a decrease in capital income tax, τ^K, encourage saving, it increases the saving rate in the neoclassical growth model. We know that an increase in saving rate will increase GDP per capita in the long run:

$$Y = F\left(A\left(\tau^K\right), TN\right)$$

[4]Since this is a static model, a change in policy must be interpreted by permanent change in policy. The static model can not allow us to analyze temporal change in policy. This is a drawback of this simplification.

These policies emphasizes the importance of tax reduction. On the other hand, an increase in tax may raise output if government expenditure increases productivity, $T(G), T'(\cdot) > 0$. In this case output is a positive function of government expenditure:

$$Y = F(A, T(G) N).$$

The provision of adequate infrastructure and good education system will increase productivity and increase GDP in the long run.

Note that these fiscal policies emphasizes the importance of supply side, but not demand control. Since the supply side determines GDP in the neoclassical model, demand stabilization policy has no impact on GDP. Then what do fiscal and monetary policy do in this framework? The following theorem summarize the main results.

Theorem 5 *Since GDP, real interest rate and real wage rate are determined by supply side in the long run, only fiscal policies that affect supply side have an impact on them in the long run. That is, demand stabilization policies have no impact on them in the long run.*

Fiscal Policy and Crowding Out: What is the impact of the fiscal policy that only affect demand side in the long run? It depends on the property of saving function. An increase in G raises tax burden, τ. It reduces consumers' disposable income. Hence, it must reduce either consumption or saving. To see this, from the budget constraint,

$$\begin{aligned} NS + C &= iA + wN - \tau \\ &= (r - \delta) A + wN - \tau \\ &= Y - \delta A - \tau \end{aligned}$$

Therefore

$$S + C = Y - \tau.$$

Since supply side determines Y, an increase in τ must reduce S or C. If consumer save less due to the reduction of disposable income, equation (17) implies it reduces investment. Hence, it reduce either investment or consumption. Hence, in any case, it reduce private expenditure. Consumers might obtain the better public services, but they must suffer from less private expenditure. The reduction in private expenditure due to an increase in public expenditure is called *crowding out*.

Theorem 6 *A permanent increase in public expenditure (which only affect demand side) will be offset by a permanent reduction of private expenditure in the long run (crowding out).*

In fact, if consumers do not have any liquidity constraint, it will be shown that a permanent increase in lump sum tax simply reduce consumption by the same amount, and it does not affect saving. Therefore it does not affect investment. Therefore, it will not have a long run impact on GDP, either.

Monetary Policy and Inflation: The next question is the long run impact of monetary policy. Since GDP Y and the real interest rate i are determined by supply side, equation (18) implies that an increase in money supply, M^s, causes inflation (= an increase in the price index, P). All variables measured in physical units, such as output and relative prices, are called real variables. On the other hand, variables expressed in terms of money are called nominal variables. This result implies that money supply affects nominal variables, but it does not affect real variables. The irrelevance of money for real variables is called *the neutrality of money.*

Another way to look at this result is the separation of real variables and nominal variables. In the neoclassical growth model, all real variables are completely determined by real variables. Nominal variables, such as money supply, affects only nominal variables, such as the price level. This separation of real variables and nominal variables is called *the classical dichotomy.*

Theorem 7 *An increase in money supply cause inflation, but does not affect real variables in the long run.*

Let me investigate the impact of change in money supply further more. Assume that the money demand is proportional to real GDP, then

$$\frac{M^s}{P} = k\left(i\right) Y,$$

where $k\left(i\right)$ is called Marshall's k. Marshall's k measure the demand for money given income Y. Then we can derive *the quantity theory of money*:

$$M^s V = PY \tag{19}$$

where $V = \frac{1}{k(i)}$. The V is called the income velocity of money, which tells us the number of times money enters someone's' income during a given period of time. It measures the speed of transaction. Note that the income velocity of money is inverse of Marshall's k. If people wish to hold much money given income (= large k), money cannot move much (= small V). If people hold little money in hand, money can frequently move around.

Since the supply side determines i, i is given. We know that on the steady state, actually, i is constant over time. If i is constant, V is constant. Then equation (19) implies that money supply determines nominal GDP. Assume that V is constant, then using the formula of growth rate, equation (19) implies

$$g^M = g^P + g^Y$$

where g^i is growth rate of i. We know that $g^{\frac{Y}{N}} = g^T$ on the steady state. It implies that $g^Y = g^N + g^T =$ constant. The above equation implies that growth rate of money supply is inflation rate, g^P plus constant term.

One problem of this analysis is the assumption, i is constant over time. Although I omit inflation in the previous model for a simple analysis, an inclusion of inflation can V. Since I ignore inflation in the previous analysis, I ignored the difference between the real interest rate and the nominal interest rate. The real interest rate is defined as nominal interest rate minus the expected inflation rate:

$$i^r = i^n - \pi^e \tag{20}$$

where i^r is the real interest rate, i^n is the nominal interest rate and π^e is the expected inflation rate. The real interest rate is the return to investment. If the nominal interest rate of bond is high, you expect to have the high nominal return in the future. However, if you expect the high inflation rate, then the real value of the return will be low.

The problem is that money demand depends on the nominal interest rate but on the real interest rate. When you keep money in your pocket, you loose the interest rate. This interest rate is nominal. Hence the opportunity cost of keeping money is the nominal interest rate. Rearranging equation (20) implies

$$i^n = i^r + \pi^e$$

This is called Fisher equation. We know that the real interest rate is constant in the long run, however, and therefore the short run behavior of nominal interest rate would be affected by the expected inflation rate. Marshall's k is a decreasing function of i^n, and therefore the V is an increasing function of i^n.

$$M^s V\left(i^r + \pi^e\right) = PY,\ V' > 0.$$

Money supply increases price level. It might cause the higher expectation of inflation. If so, it increases the nominal interest rate and reduces demand for money. Since more money will move around in the economy. income velocity of money increases. It again increases price level. This additional effect amplify the impact of money supply.

Discussions: Friedman insists that (1) V is stable in the long run, (2) a change in M^s is observed before PY changes and (3) stable supply of M^s brings stable PY. Let me show two discussions about this statement.

First, Kaldor (1970) questioned the causality. As I said before, money supply is an endogenous variable. How much can government control money supply? This is a big question. During Xmas season, we can observe an increase in money supply and nominal GDP. However, it is difficult to believe that an increase in money supply during Xmas causes an increase in nominal GDP. Natural interpretation is opposite:

since people transact more during Xmas seasons, people demand more money. After Kaldor (1970), Sims (1973) found evidence that there is no systematic feedback from PY to M^s. This evidence support Friedman. Mehra (1978) found that M^s is not exogenous variable when he includes real income and the nominal interest rate in his equation. The causality between money supply and nominal GDP is still under discussions.

Second, in order to control money supply in practice, V has to be stable even in the short period. If Bank of Japan succeeded stable supply of money, the inflation rate would be stable, and therefore the expected inflation rate will be constant. Hence, if the real interest rate is stable even in the short period, nominal interest rate should be stable and V will be stable. Question arises: is the real interest rate stable in the short run?

Fama (1975) tested this hypothesis as follows. If i^r is constant, Fisher equation implies

$$E\left(\pi_{t+1}|\Omega_t\right) = a + i_t^n.$$

where $a = -i^r$ is constant and Ω_t is information set at date t. $E\left(\pi_{t+1}|\Omega_t\right)$ means that agents make prediction on the future inflation based on available information at date t. Define $u_t = \pi_{t+1} - E\left(\pi_{t+1}|\Omega_t\right)$. Then

$$\pi_{t+1} = a + i_t^n + u_t$$

Assume that $u_t = c\omega_t + \varepsilon_t$ where ε_t is normally distribution with the mean 0 and the variance σ^2. The variable ω_t is other available information at date t. We can derive the following testable equation

$$\pi_{t+1} = a + bi_t^n + c\omega_t + \varepsilon_t \tag{21}$$

The theory predicts $b = 1$ and $c = 0$. To see why c is 0, note that

$$\begin{aligned} E\left(u_t\right) &= E\left(\pi_{t+1} - E\left(\pi_{t+1}|\Omega_t\right)\right) \\ &= E\left(\pi_{t+1}\right) - E\left(\pi_{t+1}\right) = 0 \end{aligned}$$

Therefore

$$E\left(c\omega_t + \varepsilon_t\right) = cE\left(\omega_t\right) = 0$$

Therefore c should be 0.

Fama conducted regression such as equation (21) and supports the statement that the real interest rate is constant even in the short run. However, Nelson and Schwert (1977) use different variables for ω_t and rejected Fama's result. So the short run stability of the real interest rate is also under discussion.

5.3 Stabilization Policy under Slow Adjustment of Investment

I have shown that stabilization policy has no impact on GDP in the long run, because supplied endowment is fully used for production. So far I assumed that firms can instantaneously make any investment. Hence, investment is passive. Aggregate saving automatically determines aggregate investment. In the view of a long run, this may be a good proxy. However, the lack of active investment causes a big problem when you wish to analyze a short run problem. Therefore, this section can provides a bridge between a long run behavior of economy and a short run behavior of economy. I show that slow adjustment of investment does not change the major results in the long run.

I implicitly assume adjustment costs of investment in this section. When a manager decide to build an office building in Tokyo, this plan does not realize in a day. There is an adjustment period. If he decides to buy new computers in the office, he must wait a delivery lag, or you may incur extra cost to receive it today.

When I assume the existence of adjustment costs, the marginal benefit of investment $(r-\delta)$ does not equal the marginal cost (i). I assume the following investment function:

$$\begin{aligned} I &= I_n(r-\delta-i)+\delta K,\ I_n(0)=0,\ I_n'(\cdot)>0 \\ &\equiv I(r-i),\ I'(\cdot)>0 \end{aligned}$$

where $I_n(\cdot)$ is net investment function and $I(i)$ is gross investment function. The rental firm conduct a new investment plan as long as the marginal benefit is greater than the marginal cost. Since δ proportion of capital is depreciated, the firm must replace the depreciated capital stock. Therefore gross investment is sum of the net investment and the replacement of the depreciated capital stock. The gross investment function is a short cut expression of this relationship. It shows that gross investment is small when real interest rate is high.

Because $r-\delta \neq i$, the marginal product of capital determines the rental price but not the interest rate:

$$r = F_K(A, TN)$$

This condition implies that the firm can rent every available capital stock in the economy.

I also modify a labor market. A perfect elastic labor supply is a just simplification. It may be a good proxy in the long run. But it is not good assumption for the short run analysis. I assume the supply of labor is an increasing function of the wage rate: $N(w)$. However, as I discussed before, generalizing supply function does not change main results. Hence, this modification is not important one. Given these modifications, the equilibrium conditions under a slow adjustment of investment can be summarized as follows.

Proposition 8 *Given (A, N, M^s, G), a market equilibrium with government under a slow adjustment of investment can be summarized by (Y, w, r, i, P) which satisfies the following conditions*

1. *Supply side conditions determine (Y, w, r):*

$$Y = F(A, TN(w)) \tag{22}$$
$$r = F_K(A, TN(w)) \tag{23}$$
$$w = F_L(A, TN(w))\, T \tag{24}$$

2. *Demand side conditions determine (i, P):*

$$I(r - i) = S(i, Y - G) \tag{25}$$
$$\frac{M^s}{P} = L(i, Y) \tag{26}$$

The effect of fiscal policy and monetary policy is the similar to the previous model. Supply side conditions still determine GDP and the wage rate. Hence, fiscal policy and monetary policy has no impact on GDP and the wage rate. An increase in government expenditure raises the burden of tax. It reduces consumption and saving, and therefore investment. In this way, we still observe a crowding out effect of fiscal policy. Money supply simply raises the price level. Hence, money is still neutral, too.

The main difference is that fiscal policy can change the real interest rate. If an increase in tax reduces saving. It decreases supply of fund and raises real interest rate. Because the real interest rate increases, it discourages investment. This is a mechanism of the crowding out effect when we consider a slow adjustment of investment. Since a rise in government expenditure raises the real interest rate, it raises the opportunity cost of money holding. Therefore, it reduces money demand. It means that money supply becomes larger than money demand. In order to fix this gap price level must go up eventually. Hence, the previous results in the long run can be modified as follows.

Theorem 9 *A permanent increase in public expenditure (which only affect demand side) will be offset by a permanent reduction of private expenditure in the long run. When the adjustment of investment is slow, it also raises the real interest rate and causes inflation in the long run. An increase in money supply cause inflation, but does not affect real variables in the long run.*

Equipped with a new tool, I am ready to analyze the short run behavior of economy.

5.4 Stabilization Policy in the Short Run

I define the short run as the period during which the number of employed workers (E) differs from the amount of employed workers in the long run equilibrium ($N(w^*)$):

$$E \neq N(w^*)$$

where w^* is the equilibrium wage. Given this period, we expect that the number of employed workers are an increasing function of the price level:

$$E = E(P)$$

When E is equal to the equilibrium level of employment, supply side restricts employment level. Therefore, firms cannot employ more in order to react a rise in demand. Therefore, price must go up in order to adjust demand and supply. However, if employment is not restricted by supply side, a different story can happen. when a increase in demand raises output price, firms can make profits by employing more workers and raise its output.

I will discuss several reasons why the number of employed workers might differs from its long run equilibrium and how I can derive the above employment function in the next section. Without asking these reasons, I simply assume this employment function and look at its policy implication in this section.

Given the above employment function, I can summarize a market equilibrium with government in the short run as follows.

Proposition 10 *Given (A, N, M^s, G), a market equilibrium with government in the short run can be summarized by (Y, r, i, P) which satisfies the following conditions*

1. *Supply side conditions:*

$$Y = F(A, TE(P)) \tag{27}$$
$$r = F_K(A, TE(P)) \tag{28}$$

2. *Demand side conditions:*

$$I(r - i) = S(i, Y - G) \tag{29}$$
$$\frac{M^s}{P} = L(i, Y) \tag{30}$$

Although I changed only one condition, the impact of this change is dramatic. As you can see, only the supply side conditions cannot determine GDP anymore. However, consider the supper short run: price is fixed. This is the period during which firms can quickly produce output without waiting price adjustment. Then

demand side conditions fully determine Y and i given the fixed price. This is the situation IS-LM analysis play a role.

IS-LM Model and Aggregate Demand: First, I examine an economy in a super short run: price is fixed and analyze IS-LM model. After that I will derive aggregate demand curve by allowing the movement of price.

Given price, equation (29) (a goods market clearing condition) and equation (**??**) (a money market clearing condition) determines the interest rate and GDP. We call equation (29) IS curve and equation (30) LM curve. Let me first analyze IS curve. *IS Curve:* The left hand side of equation (29) is gross investment, which is a decreasing function of the interest rate. The larger the real interest rate the higher the cost of investment. Hence the high interest rate lowers investment. The right hand side is the gross saving, which is an increasing function of the real interest rate and GDP.

When the real interest rate becomes higher, investment decreases and saving increases. In order to satisfy IS curve, GDP should be lower. Otherwise the left hand side becomes smaller than the right hand side. Therefore, IS curve is declining on the plain of i and Y. In order to analyze this point mathematically, let me introduce the implicit function theorem.

Lemma 11 *(The implicit function theorem) Suppose that the relationship between the variables X and Y are expressed by a function F:*

$$0 = F(X, Y)$$

If $F_Y(X, Y) \neq 0$ for any Y, there exists a function $f : Y = f(X)$, which satisfies

$$0 = F(X, f(X)).$$

Moreover,

$$f'(x) \equiv \frac{dY}{dX} = -\frac{F_X(X, Y)}{F_Y(X, Y)}$$

Using the implicit function theorem, since

$$I'(r - i)\, dr - I'(r - i)\, di = S_1(i, Y - G)\, di + S_2(i, Y - G)\, d(Y - G),$$

the implicit function theorem implies that there exist a function $IS(\cdot)$ such that

$$i = IS(Y - G, r) \tag{31}$$

where

$$I(IS(Y - G, r)) = S(IS(Y - G, r), Y - G)$$

and

$$\begin{aligned} IS_1(Y - G, r) &= \frac{di}{d(Y - G)} \\ &= \frac{S_2(i, Y - G)}{-I'(r - i) - S_1(i, Y - G)} < 0. \end{aligned}$$

and

$$\begin{aligned} IS_2(Y-G,r) &= \frac{di}{dr} \\ &= \frac{I'(r-i)}{I'(r-i)+S_1(i,Y-G)} > 0. \end{aligned}$$

Note that an increase in government expenditure shift up IS curve on $i-Y$ plain:

$$\frac{di}{dG} = -IS_1(Y-G,r) > 0$$

An increase in government expenditure raises tax burden. It lowers disposable income, and therefore lowers saving. A reduction of saving makes asset more valuable, and therefore raises the real interest rate.

LM Curve: The next let me consider LM curve. Since price is given, the left hand side of LM curve is constant. The right hand side of LM curve shows the demand for money, which is a decreasing function of the interest rate and an increasing function of GDP. An increase in the interest rate raises the opportunity cost of holding money; an increase in GDP implies more transaction in the economy, and therefore people need more money. Hence, when the interest goes up, money demand goes down. In order to satisfy LM curve, GDP must increase. Therefore, LM curve is increasing on the plain of i and Y. Mathematically, since

$$d\frac{M^s}{P} = L_1(i,Y)\,di + L_2(i,Y)\,dY,$$

the implicit function theorem implies that there exist a function $LM(\cdot,\cdot)$ such that

$$i = LM\left(Y, \frac{M^s}{P}\right) \tag{32}$$

where

$$\frac{M^s}{P} = L\left(LM\left(Y, \frac{M^s}{P}\right), Y\right),$$

$$\begin{aligned} LM_1 &= \frac{di}{dY} \\ &= -\frac{L_2(i,Y)}{L_1(i,Y)} > 0 \end{aligned}$$

and

$$\begin{aligned} LM_2 &= \frac{di}{d\frac{M^s}{P}} \\ &= \frac{1}{L_1(i,Y)} < 0. \end{aligned}$$

The sign $LM_s < 0$ means that an increase in money supply shift down LM curve on $i - Y$ plain. An increase in money supply reduce the market value of money. Therefore it lowers the interest rate.
IS-LM Analysis: Using IS curve and LM curve, we can analyze the impact of fiscal policy and monetary policy during the period of constant price. Consider $i-Y$ plain. An increase in government expenditure shifts up IS curve. Therefore, it raises the real interest rate and GDP. On one hand a rise in government expenditure raises aggregate demand. On the other hand, because of crowding out effect, an increase in government expenditure raises the interest rate and lowers investment. However, the interest rate does not go up very much, because when it goes up, it lowers money demand. However, money supply is constant. Therefore, on the money market there exists the pressure which lowers the interest rate. Hence, in general, the first effect dominates the second effect, and therefore a increase in government expenditure raises aggregate demand in total. Since when price is constant, firms can quickly employ workers and meet the demand. It raises GDP.

Theorem 12 *When price is constant, an increase in government expenditure raises the real interest and GDP in general.*

The next I analyze the impact of monetary policy. A rise in money supply shift down LM curve on the $i - Y$ plain. Therefore, it lowers interest rate and increases GDP. Intuitive logic are as follows. An increase in money supply lowers market value of money, and therefore it lowers interest rate. The low interest rate encourage investment and it raises aggregate demand. When price is constant, a rise in aggregate demand is absorbed by a rise in GDP. A rise in GDP raises transaction in the economy and demands more money. This effect raises the interest rate. In general the first effect dominates the second effect. Therefore, an increase in money supply lowers the interest rate. This result can be summarized as follows.

Theorem 13 *When price is constant, an increase in money supply lowers the real interest and raises GDP in general.*

Aggregate Demand: Aggregate demand is the relationship between the quantity of output demanded and the aggregate price level. Combining equation (31) and (32) derives aggregate demand. Since on the equilibrium

$$IS\left(Y-G,r\right)=LM\left(Y,\frac{M^{s}}{P}\right),$$

equation

$$\begin{aligned} & IS_{1}\left(Y-G,r\right)dY-IS_{1}\left(Y-G,r\right)dG+IS_{2}\left(Y-G,r\right)dr \\ = \; & LM_{1}\left(Y,\frac{M^{s}}{P}\right)dY+LM_{2}\left(Y,\frac{M^{s}}{P}\right)d\frac{M^{s}}{P} \end{aligned}$$

must be satisfied. Therefore, the implicit function theorem implies that there exists a function Y^d

$$Y = Y^d\left(r, \frac{M^s}{P}, G\right) \tag{33}$$

where

$$IS\left(Y^d\left(r, \frac{M^s}{P}, G\right) - G\right) = LM\left(r, Y^d\left(\frac{M^s}{P}, G\right), \frac{M^s}{P}\right)$$

and

$$\begin{aligned} Y_1^d\left(r, \frac{M^s}{P}, G\right) &= \frac{dY}{dr} \\ &= \frac{-IS_2\left(Y-G, r\right)}{IS_1\left(Y-G, r\right) - LM_1\left(Y, \frac{M^s}{P}\right)} > 0 \end{aligned}$$

$$\begin{aligned} Y_2^d\left(r, \frac{M^s}{P}, G\right) &= \frac{dY}{d\frac{M^s}{P}} \\ &= \frac{LM_2\left(Y, \frac{M^s}{P}\right)}{IS_1\left(Y-G, r\right) - LM_1\left(Y, \frac{M^s}{P}\right)} > 0 \end{aligned}$$

and

$$\begin{aligned} Y_3^d\left(r, \frac{M^s}{P}, G\right) &= \frac{dY}{dG} \\ &= \frac{IS_1\left(Y-G, r\right)}{IS_1\left(Y-G, r\right) - LM_1\left(Y, \frac{M^s}{P}\right)} > 0. \end{aligned}$$

Substitute equation (28) into equation (33), we can derive

$$Y = Y^d\left(F_K\left(A, TE\left(P\right)\right), \frac{M^s}{P}, G\right) \tag{34}$$

Equation (34) shows the relationship between Y and P given A, M^s, T and G. This derives aggregate demand curve on the plain of P-Y:

$$\frac{dY}{dP} = Y_1^d\left(r, \frac{M^s}{P}, G\right) F_{KL}\left(A, TE\left(P\right)\right) E'\left(P\right) - Y_2^d\left(r, \frac{M^s}{P}, G\right) \frac{M^s}{P^2}.$$

Hence, in general we cannot determine the slope of aggregate demand curve. Typically, it is assumed that the second term dominates the first term. That is, it is considered that aggregate demand curve has negative slope on the plain of P-Y. .

Note that both $Y_2^d\left(r, \frac{M^s}{P}, G\right)$ and $Y_2^d\left(r, \frac{M^s}{P}, G\right)$ are positive. It means that an increase in money and government expenditure shift up aggregate demand curve on the plain of P-Y. This results can be summarized as follows.

Theorem 14 *An increase in government expenditure or money supply raises aggregate demand in general.*

Aggregate Demand and Aggregate Supply Analysis: Let me combine the derived aggregate demand curve with the aggregate supply curve of the economy. The aggregate supply is the relationship between aggregate output supplied and aggregate price level, and the aggregate supply curve is expressed by equation (27). It shows that the relationship between aggregate output supplied and aggregate price level is positive:

$$\frac{dY}{dP} = F_L\left(A, TE\left(P\right)\right) E'\left(P\right) > 0.$$

AD-AS Analysis: Consider $P - Y$ plain. We know that both fiscal policy and monetary policy shifts up aggregate demand curve. Since the aggregate supply curve is increasing on the $P - Y$ plain, both monetary policy and fiscal policy increases price and GDP. When aggregate demand goes up, firms try to react by employing more workers and producing more output. Eventually, labor market becomes tight and cost increases, and therefore supply cannot perfectly cover demand. Hence, price has to go up. However, when price goes up, it reduces the real money supply and raises the interest rate, and therefore lowers investment. This effect lowers aggregate demand. However, the first effect always stronger the second effect, and therefore GDP goes up.

One remark is that economy in the long run is the case when the aggregate supply curve is vertical on the $P - Y$ plain. If these results are summarized as follows.

Theorem 15 *An increase in government expenditure or money supply causes inflation and raises GDP in the short run.*

5.5 The Source of Slow Price Movement

I have shown that stabilization policy can increase GDP in the short run. As I defined before, the short run is the period the number of employed workers differs from its long run equilibrium level. I also argue that the number of employed workers are increasing in price level in the short run. Question is why it differs from the equilibrium level in the short run? How can I derive a positive employment function?. This section answers this question. Researchers proposes different models to answer these question.

It is impossible to explain every model. Hence, I introduce five prototype models, which illustrate their main idea. Many economists construct their models based on the mixture of these prototype idea. Hence, the result of this section can be considered as the basic idea rather than realistic.

Sticky Nominal Wage Model: Our short run analysis is the modern interpretation

of Keynes's idea in 1936. When he proposed his idea, he assumed that nominal wage is fixed. Let me assume that the nominal wage, W, is fixed in the short run. The bargaining process with workers may prevent a reduction of nominal wage when they are nor sure about the level of real wage. For several contractual reasons, it may be difficult to change the nominal wage quickly. Without asking its micro foundation, I assume the nominal wage is fixed.

Since labor demand curve is a decreasing in the real wage, if the nominal wage is rigid, labor demand must be an increasing in P: if $L = L\left(\frac{W}{P}\right)$, $L'\left(\frac{W}{P}\right) < 0$,

$$\frac{dL}{dP} = -L'\left(\frac{W}{P}\right)\frac{W}{P^2} > 0.$$

Since W is fixed, the labor market fails to equate demand for labor and supply of labor. That is, there are many unemployed workers in the labor market. Hence, production must be done by the number of worker demanded. It means that the labor demand function characterizes short run employment function. Intuitively, when output price goes up, it lowers the real wage rate. Therefore it lowers the marginal cost of employing a worker. Hence the firm will employ more.

Imperfect Information Model: Other economists emphasize other reasons than the sticky nominal wage as the reason of deviation from its long run equilibrium: imperfect information about price level. This model was originally proposed by Friedman (1968) and Lucas (1972). We explain the variants of it. Lucas considered a segmented market and information is perfect inside the market, but there is no informational linkage between the markets. He called each segmented market island.

Assume that firms and workers in the ith island observes output price P_i. Since the firms in the ith island faces price P_i, the marginal revenue of labor depends on P_i. Therefore, its labor demand is

$$L = L\left(\frac{W_i}{P_i}\right).$$

where W_i is nominal wage in the ith island.

On the other hand, the real wage depends on aggregate price, P. As the benchmark, let me first assume that everybody knows what is P. Then labor supply function is

$$\begin{aligned} N &= N\left(\frac{W_i}{P}\right), \\ &= N\left(\frac{W_i}{P_i}\frac{P_i}{P}\right). \end{aligned}$$

As you can see that the relative price $\frac{P_i}{P}$ increases workers' labor supply. When P_i is larger than average, the firm offers the relatively higher wage to attract more workers. Therefore, the real wage is high, and raises labor supply. On the plane of $\frac{W_i}{P_i} - L\ (N)$, an increase in the relative price $\frac{P_i}{P}$ shifts right labor supply curve. Now suppose that the overall price level P goes up. Then P_i goes up also. Hence it does not affect the relative price and supply decision does not change.

The next suppose nobody can observe aggregate price. Hence P must be inferred from observable price, P_i: $E\left(P|P_i\right)$. Hence, labor supply function is

$$N = N\left(\frac{W_i}{P_i}\frac{P_i}{E\left(P|P_i\right)}\right)$$

Suppose that the overall price level P goes up. Then P_i goes up also. Hence, the relative price does not change. However, workers do not know a change in aggregate price. Workers observe an increase in P_i and infer the possible source of the change. An observable price P_i goes up when not only the overall price level goes up, but also consumers change their taste. If customers buy their products more than before, it pushes up the relative price. Since workers cannot distinguish these two possibility, when they observes a rise in P_i, they interpret that it partially came from a change in aggregate price and partly came from a change in taste. Hence, $\frac{P_i}{E(P|P_i)}$ increases, which shifts right labor supply curve on the plane of $\frac{W_i}{P_i} - L\ (N)$. Because labor supply increases in price P, the number of employed workers increases in price P on the equilibrium.

Note that real wage goes down. When $\frac{P_i}{E(P|P_i)}$ increases, supply curve shifts right on the plain of $\frac{W_i}{P_i} - L\ (N)$. Since every movement of P_i came from the movement of P, this reflects the movement of real wage.

Discussions: Both models produce an increase in short run aggregate supply curve. Note that real wage is constant in the long run. The nominal wage can change in the long run. Workers can receive correct information in the long run. Then, there is no reason that labor market cannot equate demand to supply.

However, policy implication may change. First, the first model brings involuntary unemployment; the second model not. In the first model, there are workers who are willing to work with lower wage. However, the high nominal wage prevents the workers from working. In the second model, all workers and firms agree on the market price. Hence the labor market is cleared given their perception.

Second, consider active stabilization policy. Suppose that government can increase money supply and government expenditure in order to raise GDP with inflation in the short run. First model predicts that it has a long run effect since the first model implicitly assume that initial GDP is much less than equilibrium GDP and sticky wage prevents economy from recovering. Hence this model may fits the great depression.

Consider the second model. If there is no clear rule for the stabilization policy, sometimes active stabilization policy itself brings the uncertain movement of price. It may increase workers' further misperception. It increases GDP in the short run. However, that is because workers work more than they would do if they know correct price. Hence, it can reduce welfare. Hence, it the limited information is the main source of imperfection and government intervention causes the main source of uncertainty, a clear policy rule may be preferred.

A problem is that the real wage is counter cyclical in sticky price model and current version of information imperfection model. In the first model, when price goes up, since the nominal wage is constant, the real wage has to go down. In the second model, when price goes up, although the nominal wage go up, the real wage goes down. The reason is that since labor supply increases, it prevent nominal wage from going up till the real wage returns to the previous level. It is easy to see this intuition on the $\frac{W}{P} - L\ (N)$ plane. Since an increase in unexpected P shifts down the labor supply curve, it reduces the real wage. This prediction is against evidence. Empirical research suggests that the real wage is procyclical

Note that the different version of imperfect information model can avoid this problem. If firms also do not k

Real Shock: Kydland and Prescott (1982) proposed different approach to investigate the movement of business cycle. It was called real business cycle approach. Their purpose was to replicate the movement of macro data by using a standard neoclassical model. For simple analysis, initially they assume that there is no deviation of the number of employed workers from the long run equilibrium level. Hence, stabilization policy has no impact on GDP even in the short run.

One advantage of real business cycle approach is that they can explain procyclical movement of real wage. Consider the impact of productivity shock. That is, T changes in the short run. Since an increase in productivity raises the marginal productivity of labor, the labor demand function is a positive function of T,

$$L = L\left(w, T\right), L_T > 0$$

As usual, the labor supply function is an increasing function of w, $N = N\left(w\right)$. It is easy to see that a positive shock can shift demand function to the left and it increases the number of employed workers and real wage. Hence, this is procyclical.

In order to fit data better, real business cycle approach is moving to incorporate several nominal rigidity. Hence, many economists do not believe the purest form of real business cycle model. However, their approach to investigate data provide several useful insight and attracts many researchers.

Sticky Output Price Model: Another way to introduce procyclical movement of real wage is to introduce a sticky output price. I will discuss the essence of Blanchard and Kiyotaki (1987). When a market is competitive, we expect that the price will

adjust demand and supply. Hence, the price cannot be sticky. In order to consider a sticky price model, we need to assume an imperfect competition in goods market. Now assume that a firm has the monopoly power on its goods. Then it is known that the firm will set its price above its marginal cost. Hence, if the output price is fixed for a short period and aggregate demand increases, the additional benefit of production is always greater than the additional cost. Hence, the firm will increase its output until it meets the aggregate demand. In order to increase its output, the firm must increase its employment. This is the case, employment is perfectly elastic to its price.

$$E'(P) = \infty.$$

It is easy to see that the real wage is procyclical in this case. When aggregate demand increases, the price stays the same, but the number of employed workers increases. In order to attract more workers the firms must pay the higher nominal wage. Hence, the real wage increases.

The main question is why the monopolistic firm does not have any incentive to change its price. Consider a monopolistic competitive market. That is, each firms produce a differentiated good and has the monopoly power for their good. However, each firm is also competing each other since if they charge the higher price, customers may switch to other companies. More concretely, the ith firm faces the following demand function:

$$Y_i = Y_i^d\left(\frac{P_i}{P}, \frac{M^s}{P}, G\right),\ Y_{iP}^d(\cdot) < 0.$$

where Y_i and P_i is aggregate demand and price level for the ith firm. Assume that price index $P = \frac{\sum_i^n P_i}{n}$. Hence, when a firm increases its price relative to the average price level, the demand for the company decreases. Hence, without the loss of generality, the firm's profit can be written as

$$\pi_i\left(\frac{P_i}{P}, \frac{M^s}{P}, G\right).$$

The firm chooses P_i to maximize its profits given P, M^s and G. Since there are many firms in the market the firm are assumed to ignore the fact their choice of price affect overall price, P. Hence, its first order condition is

$$\pi_{iP}\left(\frac{P_i(M^s, G)}{P}, \frac{M^s}{P}, G\right)\frac{1}{P} = 0.$$

where $P_i(M^s, G)$ is the optimal price given a current money supply, M^s. This first order condition implies that the additional benefit from changing its price is 0 when the firm chooses its optimal prices. Suppose that when government slightly increase money supply or government expenditure. It increases aggregate demand. The firm's optimal decision is to reduce its price to capture more customers. However, we know that the additional benefit from changing price is small around its optimal

price. Hence, if a change in government policy is small, it does not affect the firm's profit very much. Hence, if there is small cost for changing price, the price will stay the same. Researchers call this cost menu cost.

Note that since nobody changes the price, real money supply increases. Hence, it raises aggregate demand and GDP increases. It is well-known that a monopoly produces less than optimal in order to keep its price high. Hence, an increase in government expenditure and money supply is welfare improving.

Coordination Failure: The last approach is to view recession as the failure of coordination. In order to see this insight, consider previous monopolistic competition model. In that model, if every firm lowers its price, it raises real money supply and increases aggregate demand. Hence, they can produce more. However, as I discussed before, if only one firm intends to lowers its price, it lowers its profits due to small menu cost. Hence, recession sustains. In this case, monetary policy and fiscal policy can be used to escape from recession. This view is summarized by Cooper and John (1988).

Discussions: I have shown five prototype idea. But all model are too week to explain slow adjustment of price. Nowadays it is relatively easy to access public information. Hence, it is difficult to believe that informational perfection cause so slow movement of price. Similarly, empirical research shows that menu cost must be too large in order to explain slow movement. Hence, there must have another source of distortion to prevent price from quickly adjusting demand and supply. Some researchers view real wage rigidity as one of the candidates. The rigidity of real wage will be reviewed by the next section.

5.6 Stabilization Policy and Unemployment

Let me discuss the impact of stabilization policy on unemployment rate. As I discussed before, stabilization policy has no impact on GDP in the long run. Since both capital market and labor market are cleared in the long run, supply side determines GDP. Although economists expect that labor market is cleared in the long run, it does not mean that we expect that there is no unemployment in the long run. A worker may seek the better job opportunity; when a firm is bankrupt, many employees must seek new jobs. Economists call the unemployment rate in the long run *the natural rate of unemployment*. Let me first discuss what determines the natural rate of unemployment, and then come back to unemployment in the short run. Unemployment in the short run can be seen the deviation of unemployment rate from the natural rate of unemployment. I discuss that the unemployment in the short run can be mitigated by stabilization policy.

Unemployment in the Long Run: In the long run, we can classify two types of unemployment: frictional unemployment and wait unemployment. Frictional unemployment occurs because finding a job takes time. When a worker lost his job, it is not easy to find his suitable job. Nobody knows where is a job offer, what is skill requirement and how much does he expect to earn. It causes a temporal unemployment. Wait unemployment occurs since real wage is sticky and does not adjust labor market. In this case, a job rationing occurs since real wage cannot equate demand and supply.

Frictional Unemployment: Let me construct a simple model of frictional unemployment. Suppose that s fraction of employed workers are separated. When a worker wishes to have the better job, he may quit. On the other hand, when a firm fires a worker, he must go to unemployment pool. On the other hand, some leave from unemployment pool. When unemployment workers meet a job offer they want, they will leave unemployment pool. I assume that m proportion of unemployment workers meets such a job. Given this simple assumption, the dynamics of unemployment is

$$\dot{U} = sE - mU,$$

where U is the number of unemployed workers and E is the number of employed workers. The parameter m can be interpreted as a poison arrival rate of a job opportunity. When m is infinite, unemployed workers can meet an offer in any small period, and unemployment rate is 0. This model assumes m is finite and it means that workers must spend some time to find a job.

As I discussed in Chapter 2, unemployed workers and employed workers consists of labor force:

$$N = E + U$$

where N denotes labor force. Hence

$$\begin{aligned} \dot{U} &= s(N - U) - mU \\ &= sN - (s + m)U \end{aligned}$$

I assume $\dot{U} = 0$ on the steady state. Then the steady state unemployment rate is

$$\begin{aligned} u &\equiv \frac{U}{N} \\ &= \frac{s}{s + m} \end{aligned}$$

Hence, the natural rate of unemployment can be determined by the separation rate and meeting rate. It shows that when the separation rate increases, unemployment rate increases and when the meeting rate increases, unemployment decreases.

Separation rate may increase when economy is changing. People changes their taste and it reduces demand for an industry. Emergence of new technology may

destroy job opportunities for old skill. Thus, a change in economic environment may cause mismatch in the labor market, which makes it difficult to solve frictional unemployment.

However, a change in economy increases meeting rate. When a new business demands more people, it will increase m and reduce frictional unemployment. Hence, the net effect is ambiguous. However, when mismatch is sever, a change in economy will have distributional effect: some are fired; others get a job.

Several public policy can affect m. Obvious policy is that government employment agencies can disseminate information about job vacancies. Retraining programs can ease mismatch in the labor market. The other important issues are unemployment insurance. When the meeting rate depends on an unemployed worker's effort, a generous unemployment insurance lowers the unemployed worker's incentive. It decreases m. Government must consider this effect of unemployment insurance when it is discussed.

Wait Unemployment: There are several reasons the real wage is sticky. If real wage does not fall down to the equilibrium level, real wage cannot equate demand to supply. Therefore, we will observe excess supply of workers. Hence there is some rationing mechanism there. But what prevent real wage from falling down.

There are three candidates to explain why the real wage does not fall down. Minimum wage laws prevent may real wage from falling to equilibrium levels. A strong labor union keeps real wage high so that union members in a firm can enjoy its high wage. The last theory is efficiency wage theory. It says that high wage keeps high productivity. Let me explain the third theory.

There are several reasons why high wage brings high productivity. First, high wage can increase workers' food consumption, which makes workers healthy and therefore productive. This explanation may be true in developing countries, but not developed countries. Second, a higher wage can raise workers' incentive to work when managers cannot monitor it. When wage is higher than equilibrium wage, workers in the firm receive rent. Since workers do not loose this rent, they will work harder in order to avoid the risk of being fired. Third, high wage has a screening effect. If able person has high reservation wage because he can receive better offer from others, but if a firm does not know who is able person, offering a high wage raises the average quality of applicants. Forth, high wage prevents skilled workers from quitting a job. When wage is higher than equilibrium wage, skilled workers receives rent. Therefore, they are less likely to quit the job. For every example, the high wage brings high productivity to the firm.

Let me provide more concrete model of efficiency wage theory. Suppose that productivity is a positive function of real wage:

$$T = T(w)$$

As I discussed before, higher wage makes workers more productive. Unemployment

rate affects productivity because when workers are fired or quit, they need to suffer from the temporal reduction of wage due to search cost in the labor market. Hence, a firm's profit is

$$F\left(K, T\left(w\right) L\right) - rK - wL$$

The firm chooses, K, L and w. The first order conditions with rental market clearing condition, $K = A$, are

$$\begin{aligned} r &= F_K\left(A, T\left(w\right) L\right) && (35) \\ w &= F_L\left(A, T\left(w\right) L\right) T\left(w\right) && (36) \\ L &= F_L\left(A, T\left(w\right) L\right) L T_w\left(w\right) && (37) \end{aligned}$$

Substituting equation (**??**) into equation (37),

$$T_w\left(w\right) = \frac{T\left(w\right)}{w}$$

It means that the marginal productivity of wage must be equal to productivity per wage. We can easily see the intuition on the plane of $T - w$. When T has S shape, productivity per wage is maximized when the marginal productivity equal productivity per wage.

Since $u = \frac{N-L}{N}$, $L = \left(1 - u\right) N$. Given w, the following condition determine u

$$w = F_L\left(A, T\left(w\right)\left(1 - u\right) N\right) T\left(w\right)$$

Hence, in general economy can sustain both high wage and unemployment in the long run.

Unemployment in the Short Run: In the short run the number of employed workers may deviate from the long run equilibrium level. Suppose that employment function is written as

$$E = E\left(\frac{P_t}{E\left(P_t|\Omega_t\right)}\right)$$

This is consistent with imperfect information model. Since unemployment rate is defined by $u = \frac{N-E}{N}$, unemployment decreases in $\frac{P_t}{E(P_t|\Omega_t)}$

$$u = u\left(\frac{P_t}{E\left(P_t|\Omega_t\right)}\right)$$

If $P_t = E\left(P_t|\Omega_t\right)$, then $u = u^n$, where u^n is the natural rate of unemployment. Assume that

$$u_t = u^n - \alpha \log \frac{P_t}{E\left(P_t|\Omega_t\right)}$$

Then

$$\begin{aligned} u_t &= u^n - \alpha \left[\log P_t - \log P_{t-1} - \left(\log E \left(P_t | \Omega_t \right) - \log P_{t-1} \right) \right] \\ &= u^n - \alpha \left[\pi_t - \pi_t^e \right] \end{aligned} \tag{38}$$

where $\pi_t^e = \log E \left(P_t | \Omega_t \right) - \log P_{t-1}$. This equation is called Phillips curve. Phillips curve shows that the short run negative relationship between unemployment rate and inflation rate. Note that when there is no inflation, the deviation from the natural rate of unemployment is caused by purely informational disturbance. If economy is quite and there is no uncertainty, unemployment rate will converge to the natural rate of unemployment.

Consider policy implication of Philips curve. If government wants to reduce unemployment, it must accept the higher inflation rate. That is, a trade off between unemployment and inflation. During 60s, it was believed that this philips curve is stable. Many governments conducted an active fiscal policy at the expense of inflation rate. But as you can see in the graph, Philips curve shifted up and it is quite unstable in the long run.

5.7 Commitment

When government conduct an active stabilization policy, a new question arises: can I trust government. When government must kill high inflation, government may need to accept high unemployment. However, politicians have always pressure from their supporter. Hence, it may be difficult to conduct timely policy. There are several games between private sector and politicians. It may cause political business cycle: politician may want to have boom just before their election.

In order to avoid this difficulty, many countries give central bank discretion of monetary policy. In this way, monetary policy can be conducted without warring about political intervention. Of course, fiscal policy may offset the impact of monetary policy. If so, independence of central bank has limited impact. But for the moment, I assume that central bank is completely independent of government. They can choose timely monetary policy. Should they conduct an active stabilization policy? Not quite. Let me illustrate the nature of the problem.

Suppose that Bank of Japan wants to minimize the following loss function:

$$L \left(u, \pi \right) = u_t + \frac{\gamma}{2} \pi_t^2.$$

Suppose that Bank of Japan directly control inflation, but not unemployment rate. Hence, they need to choose π_t in order to minimize the loss function given equation (38).

Let me consider the best strategy for Bank of Japan. Note that the lower the expected inflation, the lower the unemployment rate. Hence, Bank of Japan want

public to believe low inflation today. To do so, assume that they announce $\pi_t = 0$ and assume that public believes this announcement. Then

$$u_t = u^n - \alpha\pi_t.$$

Substituting this equation into the loss function of Bank of Japan,

$$L(u, \pi) = u^n - \alpha\pi_t + \frac{\gamma}{2}\pi_t^2$$

Therefore, the first order condition implies

$$\alpha = \gamma\pi_t$$

and the optimal choice is

$$\pi_t = \frac{\alpha}{\gamma} > 0.$$

Hence, after announcing $\pi_t = 0$, Bank of Japan always have an incentive to break the announcement and to choose a positive inflation in order to reduce unemployment. This is called *time inconsistency* problem.

Once, public knows that Bank of Japan cannot commit their announcement and its best strategy is $\pi_t = \frac{\alpha}{\gamma}$, public cannot trust Bank of Japan and correctly predict that inflation rate is $\frac{\alpha}{\gamma}$. $\pi_t^e = \frac{\alpha}{\gamma}$. Hence,

$$u_t = u^n - \alpha\left[\pi_t - \frac{\alpha}{\gamma}\right],$$

and

$$L(u, \pi) = u^n - \alpha\left[\pi_t - \frac{\alpha}{\gamma}\right] + \frac{\gamma}{2}\pi_t^2.$$

In this case the best strategy for Bank of Japan is again

$$\pi_t = \frac{\alpha}{\gamma}.$$

Therefore, this is consistent with Public's expectation and we can sustain this equilibrium. In this case, unemployment rate is the same as the natural rate of unemployment.

$$u_t = u^n.$$

Hence an active stabilization policy eventually brings a positive inflation and the natural rate of unemployment.

Now consider a passive stabilization policy. That is, Bank of Japan announce the rule of policy and commits the rule. Assume that this commitment is possible. Optimally, Bank of Japan announce $\pi_t = 0$. Since this policy is committed, public believe inflation rate is 0: $\pi_t^e = 0$. Then $u_t = u^n$. Note that a passive stabilization policy brings the better result for Bank of Japan. Because of time inconsistency problem, public cannot fully trust government's announcement unless they actually commit the policy. In this case, more government's discretionary power makes worse situation.

5.8 Limitations

I proposed a static aggregate supply and aggregate demand model to analyze the impact of fiscal policy and monetary policy. However, because it is a static model, there are issues which we cannot argue by the previous model. We need a dynamic model to address this issues. Because of this reason, we need to be careful about the usage of the previous model.

For example, an increase in government expenditure raises GDP in the short run. However, we know that it lowers investment because of crowding out effect. However, a lower investment can reduce capital accumulation. It might hamper economy in the long run. Of course the mixture of a fiscal and monetary policy may prevent this problem: if government expands government expenditure and increases money supply, they may be able to raise GDP without raising the real interest rate. However, a rise in inflation is another cost of this policy.

Another problem of static model is we cannot argue several careful policy changes. What happens if government increases expenditure today and commit to lower it later. What is the temporal effect and what is the permanent effect. How long lasts the short run impact of one policy change? What is the best timing of changing government policy? We cannot answer these important issues in this static model.

In the next chapter, I will discuss one important dynamic issue out of several others: finance by a bond. If government issues bond, consumers do not need to suffer from the reduction of current disposable income. Hence there may not have any crowding out effect. In order to emphasize the importance of micro foundation, and the forward looking individual behavior, this issue is suitable to discuss in the next chapter.

6 Lucas's Critique and Micro Foundation

In the last section of the previous chapter, I pointed out that we cannot analysis dynamic issues in the aggregate demand and aggregate supply model. Another important problem of the aggregate demand and the aggregate supply model is the lack of micro foundation. I assumed the properties of the saving function, the investment function and the money demand function. But these functions must be the results of individual behaviors, therefore a change in environment may affect the property of each function. In particular, if a future policy change itself can affect the functional form, it is difficult to predict the impact of a change in policy based on the estimates of the parameter of each function. This is the essence of Lucas's critique (1976). Lucas's critique had a huge impact on macroeconomist's research strategy. I will discuss the implication of Lucas's critique in the next section.

Following Lucas's critique, many macroeconomists pay more attention to the micro foundation of the consumer's behavior and firm's behavior, and derives consumption function, investment function and money demand function. I would like to

review these discussions. The section 2 reviews the literature for consumption, and the section 3 reviews the literature for investment.

6.1 Lucas's Critique (1976)

I would like to show the idea of Lucas's critique by using a simple consumer's decision problem. This example clarify why economists care about micro foundation.

Let me consider the following consumption function This is a typical consumption function that many economists estimated for their policy analysis in 60s:

$$C = \phi_0 + \phi_1 Y^d + \varepsilon \tag{39}$$

where Y^d is disposable income.

Once you obtain the parameter ϕ_1 and ϕ_2, it is simulated to see its policy implications. Question arises: how much can trust these estimated parameters? Lucas (1976) said that it might be good for short run prediction. But it cannot be neither a basis for a policy evaluation nor a long run prediction. Why? Let me summarize that Lucas's points here by using a simple example.

Consider the following consumer's decision problem.

$$\begin{aligned} & \max_{C_t, C_{t+1}} \left\{ u\left(C_t\right) + \beta E\left[u\left(C_{t+1}\right) | \Omega_t \right] \right\} \\ s.t.\ Y_t^d &= C_t + S_t \\ C_{t+1} &= \left(1 + i_t\right) S_t + Y_{t+1}^d \end{aligned}$$

Substituting the budget constraints into objective function, the original problem is rewritten as

$$\max_{NS_t} \left\{ u\left(Y_t^d - S_t\right) + \beta E\left[u\left[\left(1 + i_t\right) S_t + Y_{t+1}^d \right] | \Omega_t \right] \right\}.$$

Therefore the first order condition is

$$u'\left(C_t\right) = \beta\left(1 + i_t\right) E\left[u'\left(C_{t+1}\right) | \Omega_t \right].$$

For my simple analysis, assume that $u\left(C\right) = aC - \frac{b}{2}C^2$. Then $u'\left(C\right) = a - bC$. Hence the first order condition becomes

$$a - bC_t = \beta\left(1 + i_t\right)\left(a - bE\left[C_{t+1} | \Omega_t\right]\right) \tag{40}$$

From two budget constraints,

$$C_{t+1} = \left(1 + i_t\right)\left(Y_t^d - C_t\right) + Y_{t+1}^d \tag{41}$$

Substituting this equation into the first order condition,

$$\begin{aligned} a - bC_t &= \beta\left(1 + i_t\right)\left[a - b\left(\left(1 + i_t\right)\left(Y_t^d - C_t\right) + E\left[Y_{t+1}^d | \Omega_t\right]\right)\right], \\ bC_t\left[1 + \beta\left(1 + i_t\right)^2\right] &= a\left[1 - \beta\left(1 + i_t\right)\right] + b\beta\left(1 + i_t\right)^2 Y_t^d + b\beta\left(1 + i_t\right) E\left[Y_{t+1}^d | \Omega_t\right]. \end{aligned}$$

Hence I can derive a consumption function:

$$\begin{aligned} C_t &= A + BY_t^p \qquad (42) \\ A &= \frac{a\left[1-\beta\left(1+i_t\right)\right]}{b\left[1+\beta\left(1+i_t\right)^2\right]}, \\ B &= \frac{\beta\left(1+i_t\right)^2}{1+\beta\left(1+i_t\right)^2}, \\ Y_t^p &= Y_t^d + \frac{E\left[Y_{t+1}^d|\Omega_t\right]}{1+i_t}. \end{aligned}$$

Note that consumption today depends on the consumer's expectation about the future after tax income. If consumer's expectation is subjective, this equation does not have any empirical implication. So typically economists assume that consumers subjective expectation is consistent with its objective stochastic process. In other words, knowing objective distribution, consumer rationally expect its realization. It is called rational expectation hypothesis.

Let me assume that $Y_{t+1}^d = xY_t^d$, where x is unpredictable noise with mean x^e. Assume that an agent knows this true distribution. Then

$$\begin{aligned} Y_t^p &= Y_t^d + \frac{E\left[Y_{t+1}^d|\Omega_t\right]}{1+i_t} \\ &= Y_t^d + \frac{x^e Y_t^d}{1+i_t} \\ &= \left[1+\frac{x^e}{1+i_t}\right] Y_t^d \end{aligned}$$

the empirically testable equation is

$$\begin{aligned} C_t &= \phi_0 + \phi_1 Y_t^d + \varepsilon_t, \qquad (43) \\ where\ \phi_0 &= A, \\ \phi_1 &= B\left[1+\frac{x^e}{1+i_t}\right] \end{aligned}$$

Note that x^e can change ϕ_1. Based on the estimated ϕ_0 and ϕ_1, a policy maker can simulate the impact of tax policy on aggregate consumption. But in fact, the future tax change affects consumers' expectation x^e and changes ϕ_1 itself. Hence we cannot trust any estimations ϕ_0 and ϕ_1 for a basis of a policy evaluation. If you consider general equilibrium, the results are more devastated. Since most likely policy change will change i_t, it changes parameter ϕ_0 and ϕ_1. In other word, the parameter that is estimated from reduced form estimation is not robust.

Reaction to Lucas Critique: I will classify several reactions to Lucas's critique.

I emphasize that the difference reaction depends on the different view about the relationship between theory and observation.

Time series analysis: Time series analysis assumes that macro variables follows particular stochastic process without any theoretical justification. They are skeptical about economic theories. Since economy is so complicated, a simple theory cannot explain evidence. So instead of following the prediction of theory, it is more productive to obtain more information from data itself.

But obviously Lucas critique is still valid for time series approach. So they seems not to consider Lucas critique as a serious problem.

Structural Estimation: If we can derive a closed form solution like the above model, we can estimate equation (43) with data about C_t, Y_t, i_t and x^e. Then we may be able to identify parameters a, b and β in above example. The basic assumption is that technology and preference does not change for a period. At least we assume that policies cannot change technology and preference. If we can derive testable equations, from individual behaviors, it may be possible to identify technology parameters and preference parameters. These parameters are assumed to be constant over time and across regions. So we can trust these estimated parameters. Each theory can be statistically tested by using econometrics.

The problem of this approach is that restrictive assumptions. In the above model, I need to assume a quadratic utility function in order to derive above equations. Similarly, because it is difficult to solve a closed form solution, many structural estimations was done only for individual behaviors, but not for general equilibrium solution. Hence, we cannot see the quantitative impact of interaction with markets. In the above example, we never know that the policy impact on consumption through a change in interest rate.

Because it is difficult to derive a closed form solution in general, two different approach are taken by reserchers.

Micro foundation and the selection of variables: Some economists constructs macro models based on individual maximization problem. The models are used to derive empirically testable implication, or to select independent variables. For example, the above model predicts consumption C_t depends on Y_t, τ_t, i_t and x^e. Then they do a linear approximation and conduct regression analysis in order to obtain economically meaningful information.

Lucas critique can be applied to this approach, too. Although it gives rough idea about the relationship between data and theory, the approximation error might be important. The above example shows a nonlinear relationship between C_t and other independent variables. Nobody tells that how the approximation errors can affect the impact of policy.

Calibration: Calibration approach takes model more seriously. In particular, they consider interactions through the general equilibrium. First, they roughly estimate reasonable parameter values from data. In order to identify its values, they use theoretically predicted relationship between parameters and data. But there is no

statistical inference. Then they try to simulate model to see how much model fits observable data. They do not have any statistical criterion to judge the fit of model. It can be seen as quantitative exercise or experiment of the theory. They believe that theory differs from reality. A statistical rejection of a certain theory does not imply that the theory is bad. Each theory is aimed to capture a particular problem. It cannot fully describe reality. They want to roughly see how much theory can explain data. If a theory cannot explain data with any "reasonable" parameters, they reject the theory. So researchers must put more weight on their subjective judgment in order to select a better theory. Hence, the better theory can be selected only by consensus.

This approach most seriously takes Lucas' critique. However, An obvious week point is the lack of statistical criteria. It is difficult to judge how informative their practice is.

Remarks: Although I have tried to classify economists' several research plans after Lucas critique. I have three warning. Firstly, these classifications are obviously dogmatic. Reality is mixed. Do not take this classification literally. It just helps clarifying its main difference. Secondly, many researchers choose different techniques depending on its purpose. That is, researchers are more flexible.

Thirdly, Redman (1991) criticizes based as follows. On one hand, when economists talk about empirical test, it is just an excuse to defend their theory: since it passes empirical tests, it was supported by data. Hence, economists do not know the importance of criticisms, which is the spirits of Popper's falsification. On one hand, when economists talk about consensus, they intentionally try to construct the consensus. But true consensus must occur through natural selection through competition, which was emphasized by Kuhn. She believes that economists still do not have any consensus to be science.

Which position will you take? How will you responds Redman's criticism against economics? It is up to you. As you can see, the relationship between theory and data is not simple. Therefore, different researchers take different position about this issue, and the different position affects their view about economics. However, although the approach is different, after Lucas's critique most of macroeconomists agree that it is important to examine the micro foundation of macroeconomics. I want to review its fruits in the next two sections.

6.2 Consumption

In this section, I would like to discuss how consumers determine their consumption. I would like to start with the model in the previous section. It brings the permanent income hypothesis. The permanent income hypothesis has an strong implication on the behavior of consumption: the temporal movement of income has little impact on consumption. Empirical evidence does not support this evidence very much. I points

out two possible reasons: a high serial correlation of income shock, and a liquidity constraint. Then I apply the permanent income hypothesis to analyze the impact of debt financing. Instead of financing government expenditure by taxes, government can finance it by issuing a bond. I discuss how and when this policy might affect real economy.

Permanent Income Hypothesis: Let me examine consumption function. Equation (42) implies that consumption is determined by the expected life time permanent income, Y_t^p, but not current income, Y_t^d, which I have assumed before. This is a generalized version of permanent income hypothesis. That is, rational agents are concerned not only about today, but also about the future.

A permanent income hypothesis has an strong implication that differs from a traditional consumption function: a change in current income has a small impact on a change in consumption. When consumers observe a boom and higher income than usual, a traditional consumption function predicts consumption must go up. On the other hand, the permanent income hypothesis predicts that consumption increases only if it changes the expected permanent income. If consumers believe that this boom is temporal and the recession will come soon, and that their permanent income will not change, consumption must stays the same. Therefore, given this view, if consumers believe that the short run stabilization policy does not have a long run impact, consumers may not change their consumption plan and government may fail to push up aggregate demand.

One of empirically testable prediction of the permanent income hypothesis is consumption smoothing. Because consumption changes less than income does, consumption does not fluctuate very much. Since the marginal benefit from consumption is decreasing, consumers prefer stable consumption to unstable consumption. Therefore, if it is possible, consumers have always incentive to smooth their consumption. When consumption depends on the permanent income, it is possible to smooth consumption by exchanging consumption today and tomorrow.

To see this implication formally, it is derived from equation (40) that

$$\begin{aligned} E\left[C_{t+1}|\Omega_t\right] &= \frac{a}{b} - \frac{a - bC_t}{b\beta\left(1+i_t\right)} \\ &= \frac{a}{b}\left[1 - \frac{1}{\beta\left(1+i_t\right)}\right] + \frac{C_t}{\beta\left(1+i_t\right)} \end{aligned}$$

Assume that i_t is constant, which is satisfied by steady state. Then the equation implies that the expected change in consumption is fairly constant on the steady state. Another interpretation is that current consumption is the best predictor of the future consumption. Once we observe current consumption, we do not need any other information to predict C_t. In particular, if $\beta = \frac{1}{1+i_t}$, then

$$E\left[C_{t+1}|\Omega_t\right] = C_t. \tag{44}$$

Consumption follows the martingale process. That is, the predicted value of the future consumption is just current consumption, and a change in consumption is unpredictable.

$$E\left[\Delta C_t|\Omega_t\right]=0$$

In this extreme case, the expected consumption is the same over time.

Hall (1978) tested this observation. He cannot reject the hypothesis that lagged values of either income and consumption cannot predict a change in consumption. This result supports the implication of the permanent income hypothesis. After Hall (1978), much empirical research was conducted for this issue. For example, Campbell and Mankiew (1989) did the following regression:

$$\begin{aligned}\Delta C_t &= \lambda\Delta Y_t^d+v_t,\\ where\ v_t &= (1-\lambda)\,\varepsilon_t,\end{aligned}$$

where ε_t is the change in consumers' prediction of their permanent income. Hence, if the permanent income hypothesis is right, λ is close to 0, and if the traditional theory is correct, λ is close to 1. Since v_t and ΔY_t^d is correlated, the classical regression analysis is biased. They conducted instrumental variables approach. They found $\lambda=0.42\tilde{}0.52$. This result indicates that consumption partially responds to disposable income.

Permanent Income Hypothesis and Traditional Consumption Function: If a change in current income has an impact on a change in consumption, what explains this behavior. I will propose two possible explanations: a high serial correlation of income; a liquidity constraint.

Serially Correlated Income: When today's income is highly correlated with the future income, consumers can easily predict the steam of the future income based on the current income. In the labor market, when you are promoted, consumers typically expect not only a high income today, but also a high income in the future. Hence, a change in current income can be seen as the change in the permanent income.

Formally, assume that

$$Y_{t+1}^d=(1-c)\,\bar{Y}+cY_t^d+\varepsilon_t,\ \varepsilon_t\tilde{}N\left(0,\sigma^2\right).$$

where $\bar{Y}$ is the average income and $c\in[0,1]$. If c is close to 1, the large proportion of disposable income at data $t+1$ is explained by current disposable income Y_t^d. If c is close to 0, the current income has little impact on the future income and the main part of the future income would be explained by the average income, $\bar{Y}$. Hence, the parameter c can be considered as the measure of the persistency of income.

Substituting this assumption into equation (42),

$$\begin{aligned} C_t &= A' + B' Y_t^d \\ where\ A' &= A + \frac{(1-c)\,B}{(1+i_t)}\bar{Y}, \\ B' &= \left[1 + \frac{c}{(1+i_t)}\right] B. \end{aligned}$$

As you can see, if c is large, $B^{'}$ is large and $A^{'}$ is small. Hence, the consumption is more sensitive to the current income. However, if c is small, $B^{'}$ is small and A' is large. Hence, consumption is insensitive to the current income and consumption is fairly constant.

Liquidity Constraint: The permanent income hypothesis implicitly assumes that every consumer can borrow money as long as they can return it in the future. But if some consumers cannot borrow enough money, they cannot buy consumption goods more than their income. Therefore, current disposable income limits consumption:

$$C_t \leq Y_t^d.$$

If many consumers are constrained by current income, and if current income increases, then obviously, consumers will increase their income. In this way, a change in current income may affect a change in consumption.

Recardian Equivalence and Government Debt: One of the interesting application of the permanent income hypothesis is government debt. When government finances its expenditure by taxes, I have shown that it increases the tax burden and reduces private expenditure. An alternative idea is government issuing bond. In this way, it does not increase tax burden today. However, if consumers care about their permanent income, they are worried not only about today's income, but tomorrow's income. Since today's debt can be seen as the future tax burden, it does not change their permanent income. Therefore, consumers do not change their consumption decision. This is called Recardian Equivalence.

Let me formally analyze Recardian Equivalence. Assume that in order to finance government expenditure, G_t, government either imposes a lump sum tax: τ_t: $Y_t^d = Y_t - \tau_t$ or issues a bond, B_t. Assume that when government issues a bond at date t, it must pay back to consumers at date $t+1$. Then assuming that population is 1 for a simple analysis, the government's budget constraint is

$$B_t = G_t - \tau_t, \tag{45}$$

$$G_{t+1} = \tau_{t+1} - (1+i_t)\,B_t. \tag{46}$$

Substitute equation (45) into equation (46):

$$\begin{aligned} G_{t+1} &= \tau_{t+1} - (1+i_t)(G_t - \tau_t) \\ \tau_t + \frac{\tau_{t+1}}{1+i_t} &= G_t + \frac{G_{t+1}}{1+i_t}. \end{aligned} \tag{47}$$

This equation shows that the present value of tax revenue must be equal to the present value of government expenditure. When consumers invested the bond, consumers' budget constraint changes as follows:

$$Y_t^d = C_t + S_t + B_t, \tag{48}$$

$$C_{t+1} = (1+i_t)(S_t + B_t) + Y_{t+1}^d. \tag{49}$$

Substitute equation (48) into equation (49),

$$\begin{aligned} C_{t+1} &= (1+i_t)\left(Y_t^d - C_t\right) + Y_{t+1}^d, \\ C_t + \frac{C_{t+1}}{1+i_t} &= Y_t^d + \frac{Y_{t+1}^d}{1+i_t}. \end{aligned} \tag{50}$$

This equation shows that the present value of consumption is equal to the present value of disposable income, which is permanent income. Since $Y_t^d = Y_t - \tau_t$, it is shown from equations (47) that

$$\begin{aligned} Y_t^d + \frac{Y_{t+1}^d}{1+i_t} &= Y_t + \frac{Y_{t+1}}{1+i_t} - \left(\tau_t + \frac{\tau_{t+1}}{1+i_t}\right), \\ &= Y_t + \frac{Y_{t+1}}{1+i_t} - \left(G_t + \frac{G_{t+1}}{1+i_t}\right). \end{aligned}$$

Note that the budget constraint does not depend on neither tax nor bond. That is, issuing bond does not change consumers' permanent income, therefore it does not change consumption.

Deviation from Recardian Equivalence: Recardian Equivalence gives us theoretical possibility that finance by bond has no real impact on economy. Because of the importance of the result, many researchers investigated the condition with which Recardian Equivalence does not hold. Here are several such conditions.

1. Transfer among generations: one possibility is that new generations come into economy. If parents do not care about their children, parents can enjoy low tax today and enforce their children to pay for them. Hence, the issue is how much parents care about their children. If they care their children like themselves, Recardian Equivalence can still hold. However, if they do not care their children, an increase in government debt might induce their demand.

2. Liquidity Constraint: As I argued before, one way to deviate from the permanent income hypothesis is a liquidity constraint. If there is liquidity constraints, the reduction of tax can increase disposable income and raises consumption. Although this is quite intuitive argument, Hayashi (1987) shows that if we consider general equilibrium effect, since liquidity constraint is not exogenous, we can construct an example that Recardian Equivalence holds even when consumers face liquidity constraints. Hence, the argument is not so simple. However, liquidity constraints still give us one important path of deviating from Recardian Equivalence.

3. Distortional tax: As you can see, Recardian Equivalence results come from budget constraint. When a tax is lump sum, it is just fixed cost. It does not affect the marginal benefit and cost of consumption. Therefore, it does not change consumption. However, if a tax change the marginal benefit or cost of consumption, it affects consumer's decision and affect consumption. In particular, if government raises capital income tax, consumers are discouraged to save and increases consumption. In this case, obviously tax schedule matters.

7 Last Words

Congratulation. You finish this lecture. It has covered broad topics of macroeconomics. I can guarantee that you are ready to study graduate macroeconomics. It may be rather technical. However, it is necessary for researchers to know these technical issues. However, the intuition of many important discussions are covered in this lecture note.

Another limitation of this lecture note is that I do not cover open macroeconomics. As you can see, there is no foreign country in this lecture note. Hence, you had better study it in other classes.

Finally, this lecture note summarize the main discussions of macroeconomics. I suggest you read data and see how reality differs from theories. You should ask why it differs? How can you modify theories? Observing difference between theories and data reads you what to do the next.